Optimal Kernel Parameters of Smooth-Windowed Wigner-Ville Distribution for Power Quality Analysis

A.R. Abdullah[*1], N.H.T.H.Ahmad[2], A.Z.Shameri [3], N.A Abidullah [4], M.H. Jopri[5], E.F.Shair[6]

[1, 2, 4, 6]Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Malaysia

[3]Faculty of Electrical Engineering Universiti Teknologi Malaysia, 81310 Skudai, Malaysia

[5]Faculty of Engineering Technology, Universiti Teknikal Malaysia Melaka, 75450 Ayer Keroh, Malaysia

[*1]abdulr@utem.edu.my; [2] fizah_jaa@yahoo.com; [3]zuri@fke.utm.my; [4]athira_abidullah@yahoocom; [5]hatta@utem.edu.my; [6]ezreen@utem.edu.my

***Abstract*- Bilinear time-frequency distributions (TFDs) are developed to represent time-varying signal jointly in time frequency representation (TFR). The TFDs offer a good time and frequency resolution, are appropriate to analyze power quality signals that consist of magnitude variation and multiple frequencies. However, the TFD suffers from cross-terms interferences because of their bilinear structures. In this paper, smooth-windowed Wigner-Ville distribution (SWWVD) is used to analyze power quality signals. The power quality signals are swell, sag, interruption, harmonic, interharmonic and transient. To get an accurate TFR, the parameters of the separable kernel are estimated from the signal. A set of performance measures are defined and used to compare the TFR for various kernel parameters. The comparison shows the signals with different parameters require different kernel settings in order to get the optimal TFR.**

Keywords*- *Time-frequency Analysis; Power Quality; Bilinear Transformation

I. INTRODUCTION

The power quality issue has become major concern for electric utilities and user .It‘s due to the growth of the electronic equipment used in industrial, commercial and domestic sector for example increasing used of solid state switching devices, nonlinear and power electronic switched, lighting control, computer and data processing [1-4].The main problem in industries is facing the distortion in electric supply. The low power quality can affect the lifetimes of loads, damaged, malfunction, instability, interruption and also affect the performance and economy. Thus, an automated monitoring system which is capable to estimate the presence of disturbance, accurately classify and also, characterize the disturbances is needed in order to improve the quality of signal and rectify the failure [3-5].

Time-frequency distributions (TFDs) are widely used to analyze power quality signal which represents the signal in jointly time-frequency representation (TFR) [3]. Previously, a popular method of linear TFD is short time Fourier transform (STFT). It uses a fixed analysis window [4] but has a major disadvantage that there is a compromise between time and frequency resolution. Alternatively, Wigner-Ville distribution, a bilinear TFD is a possible solution. It offers good time and frequency resolution but suffers from the presence of cross-terms interferences [4].This inhibits interpretation of its TFR, especially when the signal has multiple frequency components.

This paper focuses on application of the smooth-windowed Wigner-Ville distribution (SWWVD) to analyze power quality signals. The power quality signals are swell, sag, interruption, harmonic, interharmonic and transient. This TFD consists of a separable kernel whose parameters are estimated from the time-lag signal characteristics. From the appropriate choice of the kernel parameters, the auto-terms are preserved and the cross-terms removed. A set of performance measures are defined based on the main-lobe width (MLW), peak-to-side lobe ratio (PSLR), signal-to-cross-terms ratio (SCR) and absolute percentage error (APE) to quantify the accuracy of the resulting TFR.

II. SIGNAL MODEL

Power quality signals are divided into three classes: voltage variation for swell, sag and interruption signal, waveform distortion for harmonic and interharmonic signal, and transient signal. Signal models of the voltage variation, waveform distortion and transient signal are formed based on IEEE Std. 1159-1995 [5] and can be defined as

$$z_{vv}(t) = e^{j2\pi f_1 t} \sum_{k=1}^{3} A_k \Pi_k (t - t_{k-1}) \tag{1}$$

$$z_{wd}(t) = e^{j2\pi f_1 t} + Ae^{j2\pi f_2 t} \tag{2}$$

$$z_{trans}(t) = e^{j2\pi f_1 t}\sum_{k=1}^{3}\Pi_k(t-t_{k-1}) + Ae^{-1.25(t-t_1)/t_d}e^{j2\pi f_2(t-t_1)}\Pi_2(t-t_1) \tag{3}$$

$$\begin{aligned}\Pi_k(t) &= 1 \qquad \text{for } 0 \le t \le t_k - t_{k-1} \\ &= 0 \qquad \text{elsewhere}\end{aligned} \tag{4}$$

where k is the signal component sequence, A_k is the signal component amplitude, f_1 and f_2 are the signal frequency, t is the time and $\Pi(t)$ is a box function of the signal.

In this analysis, f_1, t_0, t_1 and t_3 are set at 50 Hz, 0 ms, 100 ms and 200 ms and other parameters are defined as below:

1. Swell: $A_1 = A_3 = 1$, $A_2 = 1.2$, t_2 = 140 ms
2. Sag: $A_1 = A_3 = 1$, $A_2 = 0.8$, t_2 = 140 ms
3. Interruption: $A_1 = A_3 = 1$, $A_2 = 0$, t_2 =140 ms
4. Harmonic: $A = 0.25$, f_2 = 250 Hz
5. Interharmonic: $A = 0.25$, f_2 = 275 Hz
6. Transient: $A = 0.5$, f_2 = 1000 Hz, t_d =15 ms, t_2 =115 ms

III. SMOOTH-WINDOWED WIGNER-VILLE DISTRIBUTION

The smooth-windowed Wigner-Ville distribution (SWWVD) [6] can be expressed as

$$\rho_{z,swwvd}(t, f) = \int_{-\infty}^{\infty} H(t) \underset{(t)}{*} K_z(t,\tau)w(\tau)e^{-j2\pi f\tau}d\tau \tag{5}$$

where $H(t)$ is the time-smooth (TS) function , $w(\tau)$ is the lag-window function. $K_z(t,\tau)$ is the bilinear product of the signal of interest, $z(t)$, and is further defined as

$$K_z(t,\tau) = z(t+\tau/2)z^*(t-\tau/2) \tag{6}$$

In this paper, Hamming window is used as the lag-window and raised-cosine pulse as the TS function [6]. It defined as

$$w(\tau) = 0.54 + 0.64\cos\frac{\pi\tau}{T_g}, \qquad |\tau| \le T_g \tag{7}$$

$$\begin{aligned}H(t) &= 1+\cos(\pi t/T_{sm}) \qquad 0 \le t \le T_{sm} \\ &= 0 \qquad \text{elsewhere}\end{aligned} \tag{8}$$

The Doppler representation of this TS function obtained from the Fourier transform with respect to time is

$$h(v) = \frac{\sin(\pi v T_{sm})}{\pi v T_{sm}} + \frac{1}{2}\frac{\sin(\pi(v-1/2T_{sm}))}{\pi(v-1/2T_{sm})} + \frac{1}{2}\frac{\sin(\pi(v+1/2T_{sm}))}{\pi(v+1/2T_{sm})} \tag{9}$$

It is a low-pass filter in the Doppler domain where the cutoff Doppler frequency is

$$\upsilon_c = 3/2T_{sm} \tag{10}$$

IV. TIME-LAG SIGNAL CHARACTERISTIC

The bilinear product in (6) is represented in time-lag representation which is used to determine the separable kernel parameters of the SWWVD. Generally, the time-lag representation can be defined in terms of auto-terms and cross-terms as

$$K_z(t,\tau) = K_{z,auto}(t,\tau) + K_{z,cross}(t,\tau) \tag{11}$$

Voltage variation signal has a variation in the root mean square (RMS) value from nominal voltage [5]. The auto-terms and cross-terms for the signal in (1) are expressed as

$$K_{auto,vv}(t,\tau)=\sum_{k=1}^{3}A_k^2 e^{j2\pi f_1\tau}K_{\Pi_{k,k}}(t,\tau) \tag{12}$$

$$K_{cross,vv}(t,\tau)=\sum_{\substack{k=1\\k\neq l}}^{3}\sum_{l=1}^{3}A_k A_l e^{j2\pi f_1\tau}K_{\Pi_{k,l}}(t,\tau) \tag{13}$$

where k and l represent the signal component, A_k and A_l are the signal components amplitude and the bilinear product of the box function, $\Pi(t)$, is defined as

$$K_{\Pi_{k,l}}(t,\tau)=\Pi_k(t+\tau/2-t_k)\Pi_l(t-\tau/2-t_l) \tag{14}$$

Equations (12) and (13) show that the auto-terms lie along the time axis and are centered at $\tau = 0$, while the cross-terms are elsewhere. Fig. 1 shows the time-lag signal representation and the auto-terms are highlighted in green while the cross-terms are densely dotted. Thus, to preserve the auto-terms and suppress the cross-terms, the lag-window in (7) should cover all the auto-terms while removing the cross-terms as much as possible. The lag-window width can be set as

$$\left|T_g\right|\leq t_2-t_1 \tag{15}$$

Since the cross-terms do not have Doppler-frequency terms as shown in (13), the TS function used in (8) is an impulse function. Therefore, only a lag-window is required and the resulting is known as the windowed Wigner-Ville distribution (WWVD).

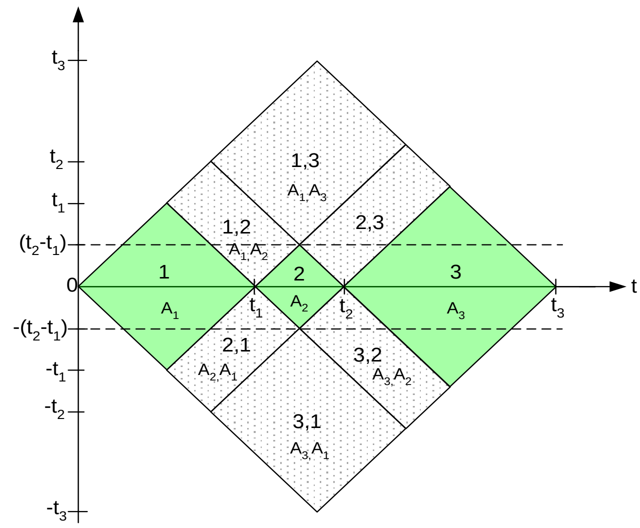


Fig. 1 Bilinear product of the voltage variation

Waveform distortion signal is a steady state signal which consists of multiple frequency components [7]. The auto-terms and cross-terms of the signal in (2) are defined as

$$K_{auto,zwd}(t,\tau)=e^{j2\pi f_1\tau}+A^2e^{j2\pi f_2\tau} \tag{16}$$

$$K_{cross,zwd}(t,\tau)=2Ae^{j2\pi\left(\frac{f_2+f_1}{2}\right)\tau}\cos(2\pi(f_2-f_1)t) \tag{17}$$

As shown in (17), the cross-terms have a Doppler-frequency component at $\upsilon = (f_2 - f_1)$. To remove the cross-terms, the TS function is used with the Doppler cutoff frequency set as $\upsilon_c \leq |f_2 - f_1|$. Thus, the TS function parameter, T_{sm}, can be set as

$$T_{sm}\geq\frac{3}{2\left|f_2-f_1\right|} \tag{18}$$

In addition, the lag-window is also used to obtain desirable lag-frequency resolution in the TFR. By choosing the lag-frequency resolution less or equal to $f_1/2$, the resulting TFR is able to differentiate harmonic and interharmonic frequency components. Higher lag-frequency resolution can be obtained with higher T_g but it results in increased computation complexity and memory size to calculate the TFR.

Transient signal is a sudden signal which changes in steady state condition at non-fundamental frequency [5]. From the signal model in (3), the auto-terms and cross-terms are expressed as

$$K_{auto,trans}(t,\tau)=\sum_{k=1}^{3}e^{j2\pi f_1\tau}K_{\Pi_{k,k}}(t,\tau)+A^2e^{-2.5(t-t_1)}e^{j2\pi f_2\tau}K_{\Pi_{2,2}}(t,\tau) \tag{19}$$

$$\begin{aligned}K_{trans,cross}(t,\tau)=&\sum_{\substack{k=1\\k\neq l}}^{3}\sum_{l=1}^{3}e^{j2\pi f_1\tau}K_{\Pi_{k,l}}(t,\tau)+Ae^{-1.25(t+\tau/2-t_1)}\\&\left(\sum_{l=1}^{3}e^{j2\pi(f_2-f_1)t-f_2t_1}e^{j2\pi(f_2+f_1)\tau/2}K_{\Pi_{2,l}}(t,\tau)\right.\\&\left.+\sum_{k=1}^{3}e^{-j2\pi(f_2-f_1)t-f_2t_1}e^{j2\pi(f_2+f_1)\tau/2}K_{\Pi_{k,2}}(t,\tau)\right)\end{aligned} \tag{20}$$

The locations of the auto-terms in (19) and the cross-terms in (20) are shown in Fig. 2. Similar to the voltage variation signal, the lag-window width is set at $|T_g| \le (t_2 - t_1)$ to remove the cross-terms located away from the time axis and to preserve the auto-terms lying along the time axis. For this signal, the TS function is needed to remove the remaining cross-terms that have Doppler-frequency component. The cutoff Doppler-frequency of the TS function is set at $\upsilon_c \le |f_2 - f_1|$ by setting T_{sm} as in (18).

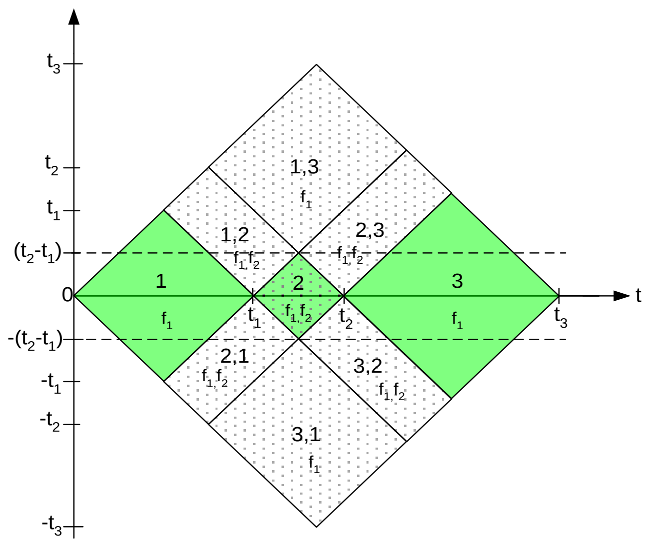


Fig. 2 Bilinear product of the transient signal

V. PERFORMANCE MEASUREMENT

In this paper, the power quality signals are analyzed using SWWVD with various kernel parameters. The performance of TFR is compared in terms of main-lobe width (MLW), peak-to-side lobe ratio (PSLR), signal-to-cross-terms ratio (SCR) and absolute percentage error (APE). These measurements are adopted to evaluate the performance of the TFD in terms of its concentration, accuracy, resolution, and interference minimization [6].

MLW and PSLR are from the power spectrum which is derived from the frequency marginal of the TFR [6] as shown in Fig. 3. MLW is the width at 3dB below the peak of the power spectrum, while PSLR is the power ratio between the peak and the highest side-lobe calculated in dB. Low MLW indicates good frequency resolution that gives the ability to resolve closely-spaced sinusoids. PSLR should be as high as possible to resolve signal of various magnitudes.

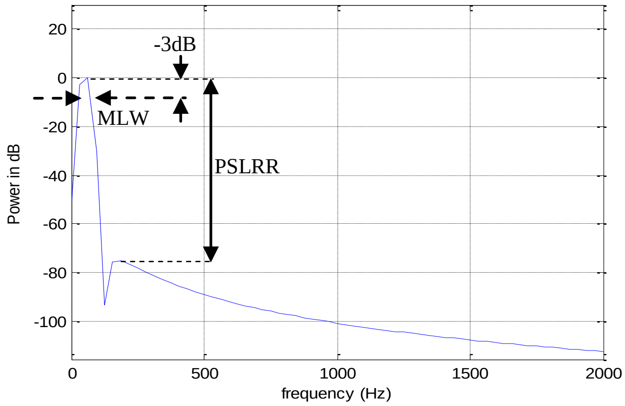


Fig. 3 Performance measures used in the analysis

SCR is a ratio of signal to cross-terms power in dB. High SCR indicates high cross-terms suppression in the TFR. It can be calculated by

$$\mathrm{SCR} = 10\log(\text{signal power/cross} - \text{terms power}) \tag{21}$$

Besides the MLW, PSLR and SCR, APE is also applied to quantify the accuracy of the signal measurement. Details on this performance measure are discussed in [8] and can be expressed as

$$\mathrm{APE} = \frac{x_i - x_m}{x_i} x100\% \tag{22}$$

where x_i is actual value and x_m is measured value. Low APE shows high accuracy of the TFR. In general, an optimal kernel of TFD should have low MLW and APE while high PSLR and SCR.

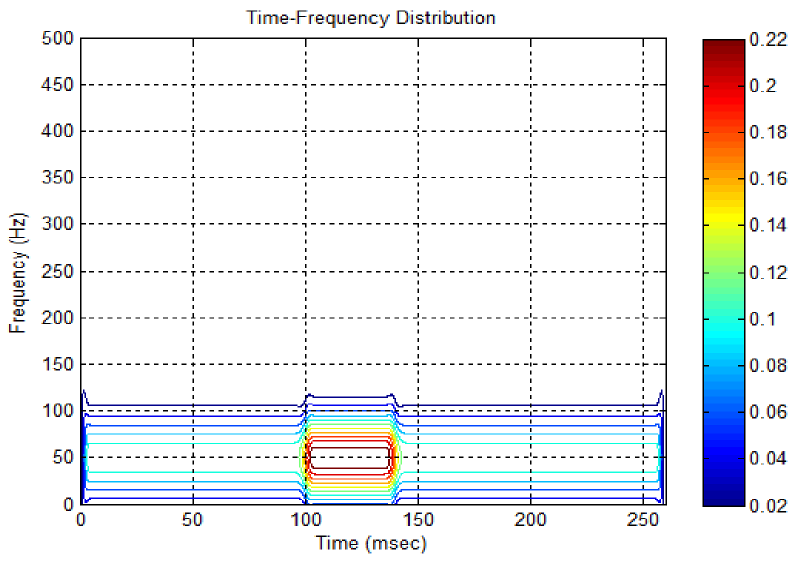
VI. RESULTS

The best kernel parameters for the voltage variation signal are at T_g = 10 ms and T_{sm} = 0 ms. A bigger T_g will results in more cross-terms while higher T_{sm} is not necessary because the signal has no Doppler-frequency component. For the waveform distortion signal, the optimal T_g is 40ms,sufficient to differentiate between harmonic and interharmonic signals. The T_{sm} for harmonic and interharmonic depends on frequency difference in the signal components. From the signal model described in Section II, the optimal T_{sm} for harmonic and interharmonic signals are 7.5 and 6.67 ms. The best parameters for transient signal are T_g = 10 ms and T_{sm} = 1.578 ms because the transient duration is 15 ms and Doppler-frequency estimated from the time-lag representation is 525 Hz. The performance of SWWVD for various kernel parameters is shown in Table I.

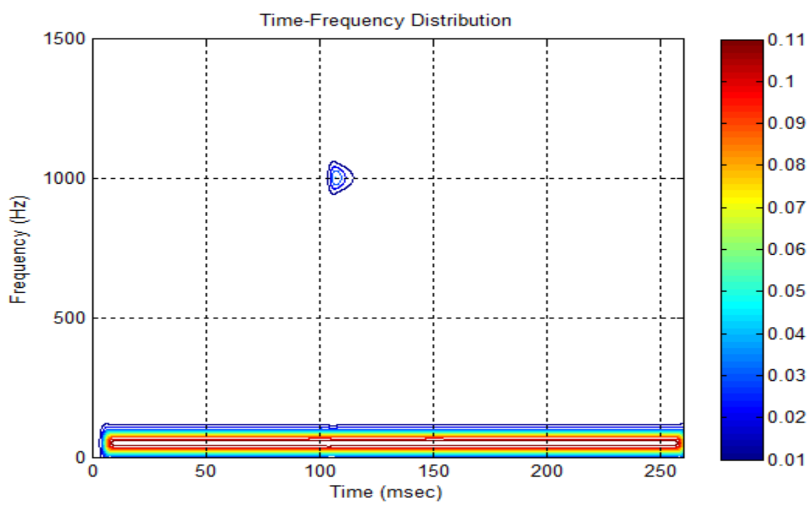
Fig. 4 shows the example of TFRs for the swell, transient and harmonic signals.

TABLE 1 PERFORMANCE COMPARISON OF SWWVD WITH VARIOUS KERNEL PARAMETER

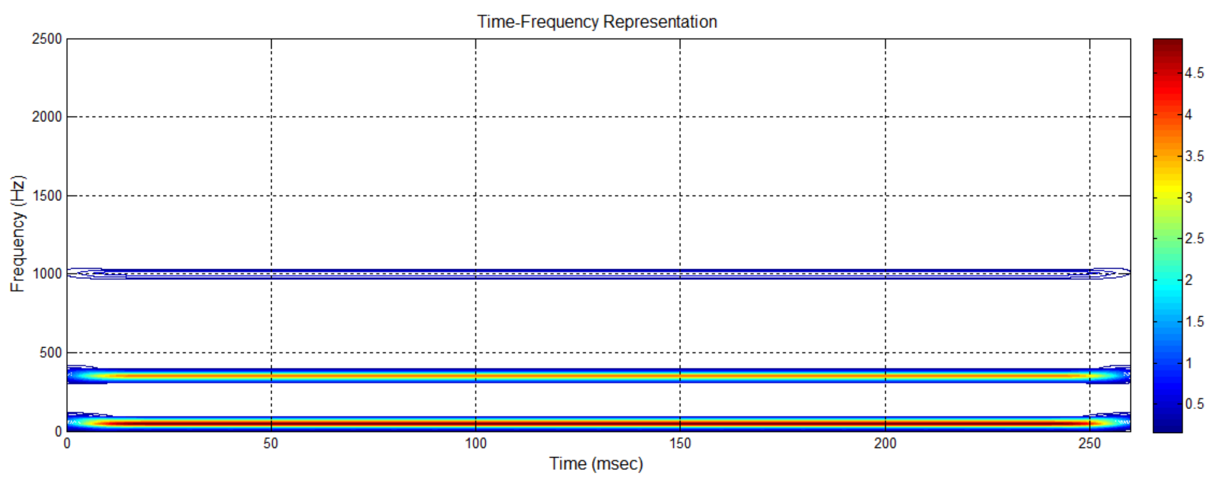
Kernel Parameters	Performance measures	Voltage variation			Waveform distortion		Transient
		Swell	Sag	Interruption	Harmonic	Interharmonic	
T_g=10ms T_{sm}=0ms	MLW (Hz) PSLR (dB) SCR (dB) APE (%)	25 614.82 15.641 0.2083	25 614.82 17.799 0.625	25 614.82 55.446 0.625	25 623.12 4.4785 0.3755	25 50.795 4.57578 100	25 19.102 13.764 55
T_g =10ms T_{sm} =1.578ms	MLW (Hz) PSLR (dB) SCR (dB) APE (%)	25 218.09 13.491 3.125	25 216.68 15.038 12.708	25 198.78 35.107 100	25 18.835 5.6064 55.544	25 49.889 5.7679 100	25 86.259 14.171 1.6667
T_g =40ms T_{sm} =6.67ms	MLW (Hz) PSLR (dB) SCR (dB) APE (%)	6.25 120.03 6.7221 20.416	6.25 100.92 8.4121 35.417	6.25 81.315 28.701 100	6.25 51.168 28.570 20.142	6.25 655.78 24.256 0.125	6.25 129.74 7.7198 90.556
T_g =40ms T_{sm} =7.5ms	MLW (Hz) PSLR (dB) SCR (dB) APE (%)	6.25 122.91 6.6568 21.458	6.25 102.56 8.3551 36.667	6.25 111.22 28.600 100	6.25 664.29 41.739 0.18930	6.25 651.32 32.356 0.125	6.25 131.79 7.4996 97.778



(a) Swell signal (T_g=10ms, T_{sm}=0ms)

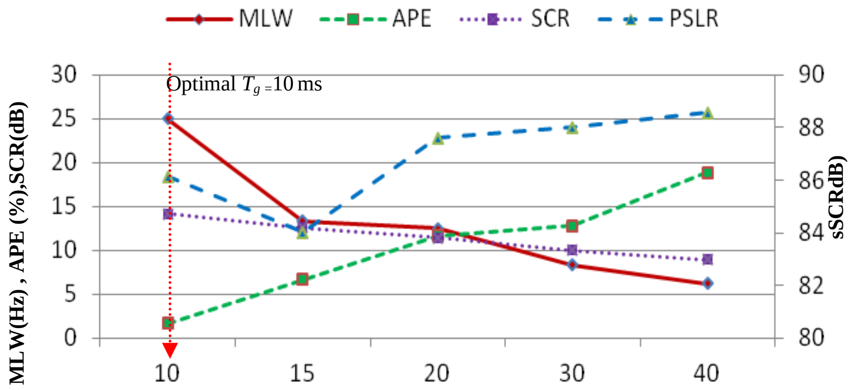


(b) Transient signal (T_g=10ms, T_{sm}=0ms)

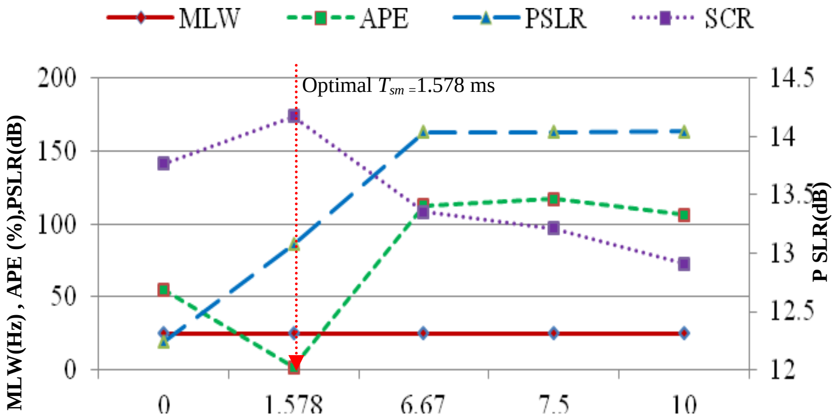


(c) Harmonic signal (T_g=10ms, T_{sm}=0ms)

Fig. 4 The TFRs of power quality signals



(a) MLW, APE, PSLR and SCR at optimal T_{sm} with various T_g



(b) MLW, APE, PSLR and SCR at optimal T_g with various T_{sm}

Fig. 5 Performance of TFR using SWWVD with various kernel parameter for transient signal.

In the contour plots, the red color represents the highest power while the blue color represents the lowest power.

Fig. 4a shows that the swell signal occurs between 100 and 140 ms while its frequency is 50 Hz. For the transient signal, its duration is between 100 and 115 ms and its frequency is 1000 Hz as shown in Fig. 4b.Meanwhile for the harmonic signal in Fig. 4c it consists three frequency component which is 50Hz, 350 Hz and 1000 Hz.Thus, the true characteristics of the signal is represent in TFR.

Fig. 5 shows the performance measure for transient signal at. optimal T_{sm} with various T_g and optimal T_g with various T_{sm}.. The optimal kernel parameter for transient signal is obtained from Table 1 which is T_g =10 ms and T_{sm} = 1.578ms. Fig. 5a shows that at optimal T_{sm} and when T_g is set higher, the MLW is smaller indicating a higher frequency resolution of the TFR However, it suffers from the reduction of the cross-terms suppression which results in smaller. This because higher T_g covers more adjacent cross-terms in lag-axis in the bilinear product. As a result, the APE is higher which presents lower accuracy of the signal characteristic measurement.

Fig. 5b shows the optimal T_g and when T_{sm} is set higher than its optimal value, the SCR increases while the MLW remains constant. It indicates that higher T_{sm} improves the cross-terms suppression and does not give any effect to the frequency resolution. However, the APE is also higher, which shows that the time resolution of the TFR is lower. This is because the application of TS function with higher T_{sm} increases the smearing of the auto-terms in time domain. Thus, there is a compromise between cross-terms suppression and time resolution to obtain optimal TFR.

VII. CONCLUSIONS

The TFR for power quality signals can be obtained using the SWWVD. For optimal TFR, the kernel parameters can be determined mathematically from the signal characteristics in time-lag representation with the objective to remove cross-terms and preserve auto-terms in the bilinear product. A set of performance measures were defined to compare the TFRs.

For voltage variation signal, only the T_g is adjusted to obtain accurate TFR while the waveform distortion and transient signals require the adjustment of both T_g and T_{sm}. From the TFR, the estimated signal parameters can be used to classify the power quality events signals.

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A.R Abdullah was born in Kedah, Malaysia in 1979. He received his B. Eng., Master, PhD Degree from University of Technology Malaysia in 2001, 2004 and 2011 in Electrical Engineering and Digital Signal Processing. He is currently a Senior Lecturer and Coordinator of Center of Excellent Robotic and Industrial Automation (CeRiA) in Universiti Teknikal Malaysia Melaka (UTeM).

N.H.T.H Ahmad was born in Terengganu Malaysia in 1986. She received diploma and B.Eng from Universiti Teknikal Malaysia Melaka in 2007 and 2010 respectively in Electrical Engineering and Control, Instrumentation and Automation. She is now continues her Master degree at same university in Digital Signal Processing.

M.H Jopri was born in Johor, Malaysia in 1979. He received his B.Sc from University of Technology Malaysia in 2004 and Msc. in Electrical Power Engineering from Rheinisch-Westfälische Technische Hochschule Aachen (RWTH), Germany in 2011. Since 2005, he has been an academia staff in the Universiti Teknikal Malaysia Melaka (UTeM). His fields of interests are power quality, motor drives and power electronics.