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Observer-Based Output Feedback Control with Linear Quadratic Performance

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Abstract

This paper discusses about the use of LQ performance optimality in the design of observer-based output feedback control for uncertain system with exogenous disturbance. The variable structure control law is adopted to compensate the uncertainty and disturbance. The separation principle is considered in the separate design of the feedback control law and observer. The design focuses on the SISO system and does not require the 'Kimura-Davison' conditions to be satisfied. An exploitation of LQR based method offers design freedom and less computational complexities and minimize the driving energy of the controller.

Keywords: LQR; Sliding Mode, Observer.Introduction

1. Introduction

In practice, measuring systems states are not always feasible and hence, not available for feedback[1-2], we consider the observer-based output feedback control for uncertain systems with exogenous disturbance. Using output information for control design require that the triple system matrices to be minimal realization. Astatic output feedback control offer simple design. When the system output matrix is non-singular square matrix, static output feedback becomes direct state feedback. However, such matrix is normally rectangular in practice and the control scheme seems impractical and incur dimensionality problem which would require that the Kimura-Davison conditions to be satisfied [3-4].Alternatively, a dynamic compensator or observer-based output feedback offer feasible technique when the Kimura-Davison condition does not meet. The compensator is augmented to the control system to introduce extra dynamics and hence providing additional degrees of freedom [5].The observer could estimates states using output information if system states are not available or measurable. However, when the observer-based sliding mode controller is used, the assumption of matching conditions invoked. This conditions has been studied in [6].Another approach of output feedback control, a conservative Proportional-Derivative (PD) linear observer suggested in [7-8] require the exact model of Lagrangian system and the dynamics model has to be identified precisely.

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In practical control system, the design procedure becomes crucial when the control input is bounded to a certain limits. In some industrial cases are DC drives systems which the constraints are due to the physical limitation of the motor drive such as converter protection, magnetic saturation and motor overheating that make the current command is limited to an admissible set of input[9].For another case such as electric vehicles where the controlled variable is speed, the motor torque may be bounded hence degrade system performance. Many approaches have been developed to deal with input-constraint or bounded-control problem as in [10] and in references therein. However, many of them assume that the states are available. The most classical approach is an anti-windup scheme [11-14], which the augmentation to the nominal controller when dealing with windup phenomena due to input-constraint or actuator saturation. Having motivated by [15],[16] and [17], we developed sliding mode observer for uncertain system. We replenish the explicit design procedure through LQ performance and clear stability proving. To the end, we proved the effectiveness of our method as a comparison with [16] and [17] through numerical example similar to [16] plus additional verification on the set point tracking output feedback.In our method, LQR minimized the driving force so that it lies within admissible set of input control bound and at the same time preserving systems performance. The sliding mode full-order states estimator is developed via LQR to ensure the estimation error dynamics is minimized. Hence, the developed output feedback control is dynamics one and supersede the drawback of static output feedback control such as the requirement of Kimura-Davison conditions to be satisfied. We adopt separation principle to independently design feedback control law and observer.

2. Problem Formulation

Consider the nonlinear system

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t, x, u) + H\xi(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where the system matrices $A \in \mathcal{R}^{nxn}$, $B \in \mathcal{R}^{nxm}$ and $C \in \mathcal{R}^{mxn}$. The state variable $x \in \mathcal{R}^{n}$, the control $u \in \mathcal{R}^{m}$ and the output $y \in \mathcal{R}^{p}$. Matrix $H \in \mathcal{R}^{nxm}$ is the perturbation input map and $\xi \in \mathcal{R}^{m}$ is the input disturbance generated by unknown exogenous signal. The uncertain nonlinear term is given by function $f(t, x, u): \mathcal{R}x\mathcal{R}^{n} \mapsto \mathcal{R}^{n}$satisfies Lipschitz condition. The following assumptions are satisfied:

A1) states x is not available, but output y is available to measure.
A2) the number of input channel equals the number of output channel (i.e, $m = p$)
A3) the triple(A, B, C) is minimal realization.
A4) the disturbance $\xi(\cdot)$ is bounded. i.e there exists a positive real number ρ such that $\|\xi\| \le \rho$.
A5) for a defined Hurwitz system matrix $A_o \in \mathcal{R}^{nxn}$, there exist two symmetric positive definite matrices $P \in \mathcal{R}^{nxn}$ and $Q \in \mathcal{R}^{nxn}$ such that P satisfies structural constraint $PH = C^T F^T$, with some selected matrix $F \in \mathcal{R}^{mxp}$.
A6) it is able to regulate estimated states of system (1) such that during successive regulation, $\xi(\cdot)$and $f(\cdot)$ are eliminated and insignificant to closed loop interconnection.

In design process, the separation principle holds. That is the design of a feedback controller and the observer is done independently. The following will be used to establish our main result:

Lemma 1[18] given the convex nonlinear inequalities

$$R(x) > 0, \qquad Q(x) - S(x)R(x)^{-1}S(x)^T > 0,$$

Where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depend affinely on x. Then this set of convex nonlinear inequalities can be converted into the equivalent LMI

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} > 0.$$

Lemma2[19] for a function $F(\cdot): X \in \mathcal{R}^{n} \mapsto \mathcal{R}^{m}$ satisfies Lipschitz conditions, then, there exists a positive Lipschitz constant γ, such that

$$|F(x) - F(\hat{x})| \le \gamma |x - \hat{x}|.$$

Theorem 1 (Theorem 1 of [20]) The systems described by the differential equation

$$\frac{dx(t)}{dt} = Ax(t) + f(x(t),t)$$

Where $A \in \mathcal{R}^{nxn}$ is stable nominal matrix, and $f(x(t),t)$is an unstructured perturbing function with the property of $f(0) = 0 \ \forall t$, is stable if

$$\frac{\|f(z,t)\|}{\|z\|} \leq \Lambda \equiv \frac{\sigma_{min}(Q)}{\sigma_{max}(P)} \ , \qquad \forall \, ((z \neq 0), t) \ \in \mathcal{R}^{n+1}$$

Where$Q \in \mathcal{R}^{nxn}$ and $P \in \mathcal{R}^{nxn}$ are positive definite matrices that fulfills the Lyapunov equation $A^T P + PA = -2Q$. $\Lambda$is quantitative robustness-bound measure, $\sigma_{min}(\cdot)$ and $\sigma_{max}(\cdot)$ are the maximum and minimum singular value respectively.

3. Main Result

3.1 Full-state observer

Our first objective is to develop a robust nonlinear observer from the measured input and output signals to estimate systems states. Let an observer for system (1) and (2) be in the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G(y - \hat{y}) \ + f(t,\hat{x},u) + Hv \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{4}$$

Where $G \in \mathcal{R}^{nxp}$ is the gain for estimation correction term, $v \in \mathcal{R}^m$ is an external discontinuous control law, and $H \in \mathcal{R}^{nxm}$ is the control injection map such that CH is nonsingular. Having only estimated states $\hat{x}$ in our possession, our first intention is to design the observersuch that the linear quadratic performance index in (5) is minimized.

$$J = \int_0^\infty (\hat{x}^T(t)Q\hat{x}(t) + u^T(t)Ru(t))dt \ \ , \ \forall \, x \neq 0 \tag{5}$$

With Q and R are square positive definite matrices.Term $\hat{x}^T Q\hat{x}$ is to solve state regulation problem. Term $u^T Ru$ is to minimize the energy$u(t)$where the control input is bounded such that $u_{min} \leq u(t) \leq u_{max}$. Via term $u^T Ru$, a designed feedback gain $K = R^{-1}B^T P$ produces minimum energy $u = -K\hat{x}$to drives the estimated states toward origin. Hence reducing estimation error and fulfill the output regulation of the actual system (1) and (2). Considering (A6) holds, therefore

$$J = \int_0^\infty (\hat{x}^T Q\hat{x} + (-K\hat{x})^T R(-K\hat{x}))dt. \tag{6}$$

simple derivation will arrive at the reduced Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{7}$$

Proposition 1: Linear quadratic performance index in (5) quadratically stabilize observer (3) and (4) for nonlinear uncertain system in (1) and (2); i.e the estimated states converged to zero for any arbitrarily initial states.

Proof:

Consider a candidate Lyapunov function:

$$V = \hat{x}(t)^T P\hat{x}(t) \tag{8}$$

Taking the derivative along system trajectory yields

$$\dot{V} = \hat{x}^T(PA + A^T P - PBK - K^T B^T P)\hat{x} \tag{9}$$

Equating with (6) results in

$$\dot{V} = \hat{x}^T(A^T P + PA - PBR^{-1}B^T P + Q)\hat{x} < 0 \tag{10}$$

Complete the proof

We give choice for searching an optimize value of $R$and Q. We use LQR to obtain minimal energy. One may use LMI for design freedom. With *P* is the solution of reduced Riccati equation (7), as long as (10) is valid and by Lemma 1, the Riccati equation can be represented in LMI.

$$\begin{pmatrix} A^T P + PA + Q & P \\ P & BR^{-1}B^T \end{pmatrix} < 0 \tag{11}$$

Proposition 2For $K \in \mathcal{R}^{mxn}$, the control feedback law $u = -K\hat{x}$ and observer gain $G \in \mathcal{R}^{nxp}$ retain the stability of the closed loop system (1), (2), (3) and (4).

Proof:

Consider the error dynamics $\dot{e}(t) = \dot{\hat{x}}(t) - \dot{x}(t)$, and considering Lemma 2, the closed loop system can be written as

$$\dot{e}(t) = (A - GC)e(t) + \gamma|\hat{x} - x| + Hv - H\xi(t)$$

$$\Rightarrow \quad \dot{e}(\mathrm{t}) = (\mathrm{A} - \mathrm{GC} + \gamma \mathrm{I_n})e(\mathrm{t}) + \mathrm{Hv} - \mathrm{H}\xi(\mathrm{t}) \tag{12}$$

$$\dot{e}_{\mathrm{y}}(\mathrm{t}) = \mathrm{C}e_{\mathrm{y}}(\mathrm{t}) \tag{13}$$

With $= -K\hat{x}$, the actual system can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t,x,u) + H\xi(t)$$

$$\dot{x}(t) = (A - BK)x(t) - BKe(t) + f(t,x,u) + H\xi(t) \tag{14}$$

can be written in matrix form

$$\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A - BK & -BK \\ 0 & A - GC + \gamma I_n \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix} \tag{15}$$

By judicious choice of *G* and *K*, the stability of overall system is guaranteed by the stability of both matrices $(A - BK)$ and $(A - GC + \gamma I_n)$.Complete the proof.

3.1.1.Sliding Mode Control

There exists a sliding surface $\mathcal{L}_s = \{e \in \mathcal{R}^p : Fe = 0\}$ such that the error trajectory slides along it and remain thereafter when $\mathcal{L}_s$is reached. Assuming robustness during sliding motion is obtained, then the equivalent control during sliding motion is simply

$$\mathrm{v_{eq}} = -(\mathrm{FH})^{-1}\mathrm{FA_c e} \tag{16}$$

and produces a reduced order sliding motion $\dot{e}(t)$ which is independent of the control action

$$\dot{e}(\mathrm{t}) = (\mathrm{I_n} - \mathrm{H}(\mathrm{FH})^{-1}\mathrm{F})\mathrm{A_c}e(\mathrm{t}) \tag{17}$$

Motion of $\dot{e}(t)$ trajectory toward $\mathcal{L}_s$ is facilitated by a variable structure control law of the form

$$u(t) = -\mu F e(t) + u_n(t) \tag{18}$$

where μ is a positive design scalar and $u_n(t)$ is the nonlinear or discontinuous control vector given by

$$u_n(t) = v = \begin{cases} -\rho \dfrac{Fe}{\|Fe\|} & , e \neq 0 \\ 0 & , \text{otherwise} \end{cases} \tag{19}$$

Proposition 3: The variable structure control law in (19) quadratically stabilize the error dynamics in (12) and (13).

Proof:

By [16], [17] and Theorem 1, the condition for stability of error dynamics in (12) and (13) is

$$\Lambda \leq \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)} \tag{20}$$

Consider a candidate Lyapunovfunction $V = e(t)^T P e(t)$. Assumptions (A4) and (A5) hold, and using Lemma 2, the derivative along (t):

$$\dot{V} = -e(t)^T Q e(t) + 2\gamma\|P\| \cdot \|e(t)\|^2 + \cdots$$

$$\cdots 2\big(FCe(t)\big)^T (u_n(t) - \|\xi\|)$$

$$\Rightarrow \; < -e(t)^T Q e(t) + 2\gamma\|P\| \cdot \|e(t)\|^2 - 2\|Fe_y\|(u_n(t) - \|\xi\|)$$

$$\Rightarrow \; < -e(t)^T Q e(t) + 2\gamma\|P\| \cdot \|e(t)\|^2$$

$$\Rightarrow < -[\lambda_{min}(Q) - 2\gamma\lambda_{max}(P)]\|e(t)\|^2 < 0 \tag{21}$$

Complete the proof.

3.2 Command Following Output Feedback

General form of the state variable feedback compensator is in the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\mathfrak{u}(t) + G\mathcal{y}(t) + \cdots$$

$$\cdots f(t, \hat{x}, u) + Hv + Mr(t) \tag{22}$$

$$u(t) = \mathfrak{u}(t) + Nr(t) \tag{23}$$

Where $\mathcal{y}(t) = y(t) - C\hat{x}(t)$ and $\mathfrak{u}(t) = -K\hat{x}(t)$.In this case, M and N are the compensator parameters to be determined. Single input case, i.e$m = 1$give$r(t)$ a scalar, $M \in \mathcal{R}^{nxn}$ and N a scalar. Note that when $M = N = 0$, the closed loop system interconnection reduces to regulator case in the previous design. We choose the compensator as a feedback loop component, and suppose of the choice $M = BN$ produce the estimation error dynamics as

$$\dot{e}(t) = (A - GC)e(t) - (BN - M)r(t) + \cdots$$

$$\cdots Hv + f(t, \hat{x}, u) - f(t, x, u) - H\xi(t)$$

$$\Rightarrow \dot{e}(t) = (A - GC + \gamma I_n)e(t) \tag{24}$$

Then by proper setting of $= BN$, the estimation error is independent of the reference input $r(t)$. And the feedback compensator is given as

$$\dot{\hat{x}}(t) = (A - GC)\hat{x}(t) + Bu(t) + Gy + Hv + f(t, \hat{x}, u) \tag{25}$$

$$u(t) = \mathrm{u}(t) + Nr(t) \tag{26}$$

The stability of the closed-loop interconnection is guaranteed as long as matrix $(A - GC - BK)$ stable, i.e the command following is achieved. While the variable structure control law combat the uncertainties and external disturbance.

3.3 Numerical Example

Consider the nonlinear system[16], defined as

$$\mathrm{x(t)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x_1} \\ \mathrm{x_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{u(t)} + \begin{bmatrix} 0.05\mathrm{e}^{\|\mathrm{x_1}\|}\sin\mathrm{x_2} \\ 0 \end{bmatrix} + \cdots$$

$$\cdots \begin{bmatrix} 1 \\ 1 \end{bmatrix} (5\sin(1.5\mathrm{x_1}) - 10\mathrm{y(t)u(t)}\Gamma\mathrm{(t)})$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where $\Gamma(\mathrm{t})$ be the exogenous input disturbance and assume that the Lipschitz constant $\gamma = 0.1$. The proposed method gives the estimation corrective term $G = [63.852 \quad 538.52]$ and estimated state feedback gain$K = [1 \quad 1]$.System configuration for observer and set point output feedback tracking control are shown in Fig. 1 and Fig. 2 respectively. The estimated states and regulated output are depicted in Fig. 3 and Fig. 4 respectively. However, the set of output is not positively invariant. The energy needed while steering the states toward origin is depicted in Fig.5. Note that by LQR, the energy is limited between± 5. The set point tracking is shown in Fig. 6. The system able to asymptotically track the reference input of 10 unit and confirm the asymptotic tracking of the control algorithm.

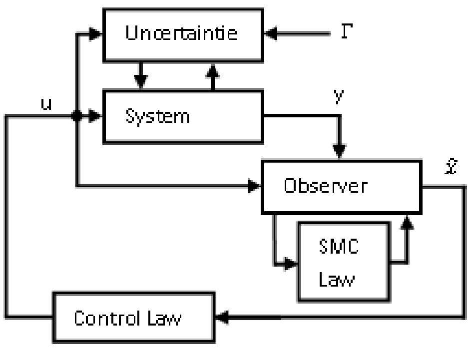

Fig. 1. Observer

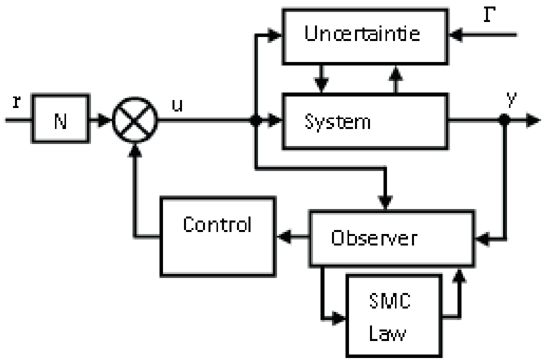

Fig. 2. Set point trajectory

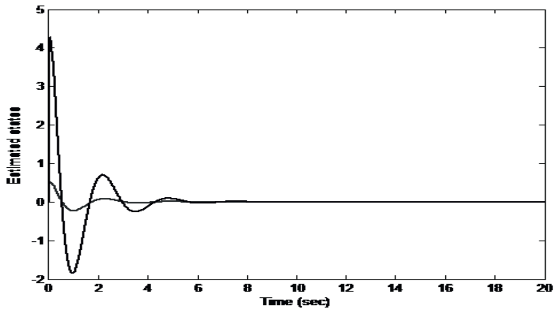

Fig. 3. Estimated states trajectory

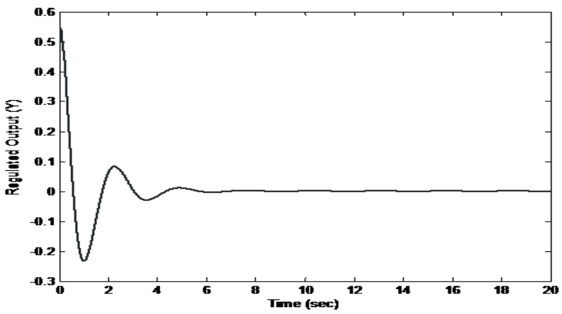

Fig. 4. Regulated output

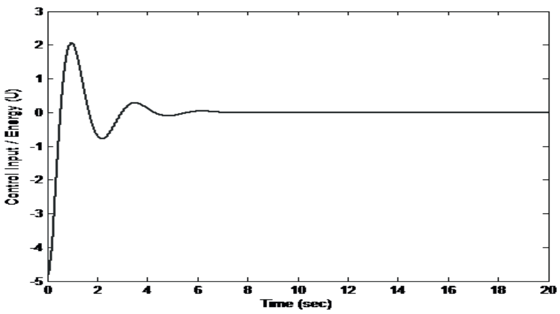

Fig. 5. Control signal

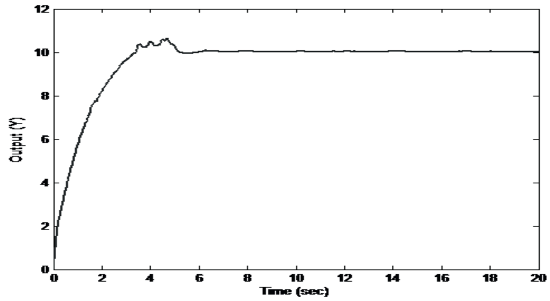

Fig. 6. Response to reference input

4. Concluding Remark

An observer-based output feedback control for uncertain systems with external disturbance has been designed. The use of LQR optimization minimized the energy while the uncertainty is combated by variable structure control law for robustness. Through the optimization via limited energy cost function, the control signal respect the bounded in allowable set of input signal

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