HYDROSTATIC UNBAB MAPPING FUNCTION MODIFICATION FOR TROPOSPHERIC DELAY

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ABSTRACT – There are many mapping function models in the form of continued fractions which need many operations in calculation. The mapping function need to be modified to allow faster calculation. UNBab mapping function needs 7 mathematical operations for getting the mapping function value. The UNBab mapping function is chosen to be modified due to its ability to calculate mapping function value down to 2 degree of elevation angle. Regression method can be used to modified the mapping function model and also can produce the same result. The calculation of sum of errors shows that the deviation of the modified models from the original models is not significant.

1. Introduction

Marini (1972), the founder of mapping function in term of continued fraction, states that the elevation angle \( \varepsilon \) dependence of any horizontally stratified atmosphere can be approximated by expanding in a continued fraction in term of \( 1/\sin \varepsilon \) [3]. The mapping function values is used for getting the total tropospheric delay due to its function as the coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay as given in equation (1) below [4]:

\[
TD = ZHDm_h(\varepsilon) + ZWDM_w(\varepsilon)
\] (1)

where:
ZHD - zenith hydrostatic delay (m)
ZWD - zenith wet delay (m)
\( m_h(\varepsilon) \) - hydrostatic mapping function
\( m_w(\varepsilon) \) - non-hydrostatic mapping function

2. Process of UNBab Mapping Function Modification

Guo (2003) from the University of New Brunswick has established the \( UNBab(E) \) mapping function model. This model has 7 operations in a form of continued fraction. The hydrostatic component of \( UNBab(E) \) is written as [1]:

\[
UNBab_h(E) = \frac{1 + \frac{a_h}{1 + b_h}}{\sin E + \frac{a_h}{\sin E + b_h}}
\] (2)

where, \( E \) : elevation angle

The parameters \( a_h \) and \( b_h \) for the hydrostatic function are:

\[
a_h = (1.53804 - 0.039491 H + 0.17020 \cos \phi) / 1000.
b_h = (50.0724 - 0.814759 H + 2.35232 \cos \phi) / 1000.
\]

where \( H = 0.1 \text{ km} \) and \( \phi \) is 45 degrees.

The mapping function for the original \( UNBab_h(E) \), named as P shown in Figure 1.

![Figure 1: Graph of mapping functions for P](image)

The mapping function give a shape of hyperbola and equation (2) can be modified as:

\[
P1(E) = AE^g
\] (3)
where,
\[ P1(E) : \text{simplified } UNBab_h(E) \]
\[ A, B : \text{constant} \]
\[ E : \text{elevation angle (degree).} \]

In logarithm scale, equation (3) can be written as:
\[ \log_{10} P1 = B \log_{10} E + \log_{10} A \]  
(4)

By linear regression method, equation (4) becomes:
\[ \log_{10} P1 = -0.8924 \log_{10} E + 1.6604 \]  
(5)
where, \( B = -0.8924 \) and \( \log_{10} A = 1.6604 \) which gives \( A = 45.751 \).

Therefore, equation (5) becomes:
\[ P1(E) = 45.751E^{-0.8924} \]  
(6)

By regression method, polynomial equation, log P2 can be generated from the original model (log P) in a form of quadratic equation as given below:
\[ \log_{10} P2 = 0.1575(\log_{10} E)^2 - 1.2761(\log_{10} E) + 1.8509 \]  
(7)

The graphs of P, P1 and P2 can be shown in Figure 2.

![Graphs of P, P1 and P2 for UNBab_h(E) mapping function](image)

Figure 2: Graphs of P, P1 and P2 for UNBab_h(E) mapping function

3. Calculation of Sum of Error For UNBab_h(E)

Sum of error (SOE) method can be used to show how the modified models deviate from the original model. Smaller deviation is better, which shows that the modified model is closer to the original model.

Table 1: Sum of error between UNBab_h(E), P and modified models P1 & P2

<table>
<thead>
<tr>
<th>E</th>
<th>P</th>
<th>P1</th>
<th>ABS (P-P1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>28.628</td>
<td>24.646</td>
<td>3.982</td>
</tr>
<tr>
<td>5</td>
<td>11.470</td>
<td>10.880</td>
<td>0.590</td>
</tr>
<tr>
<td>10</td>
<td>5.758</td>
<td>5.861</td>
<td>0.103</td>
</tr>
<tr>
<td>15</td>
<td>3.863</td>
<td>4.082</td>
<td>0.218</td>
</tr>
<tr>
<td>20</td>
<td>2.924</td>
<td>3.157</td>
<td>0.234</td>
</tr>
<tr>
<td>25</td>
<td>2.366</td>
<td>2.587</td>
<td>0.221</td>
</tr>
<tr>
<td>30</td>
<td>2.000</td>
<td>2.199</td>
<td>0.199</td>
</tr>
<tr>
<td>35</td>
<td>1.743</td>
<td>1.916</td>
<td>0.173</td>
</tr>
<tr>
<td>40</td>
<td>1.556</td>
<td>1.701</td>
<td>0.145</td>
</tr>
<tr>
<td>45</td>
<td>1.414</td>
<td>1.531</td>
<td>0.117</td>
</tr>
<tr>
<td>50</td>
<td>1.305</td>
<td>1.394</td>
<td>0.088</td>
</tr>
<tr>
<td>55</td>
<td>1.221</td>
<td>1.280</td>
<td>0.059</td>
</tr>
<tr>
<td>60</td>
<td>1.155</td>
<td>1.185</td>
<td>0.030</td>
</tr>
<tr>
<td>65</td>
<td>1.103</td>
<td>1.103</td>
<td>0.000</td>
</tr>
<tr>
<td>70</td>
<td>1.084</td>
<td>1.032</td>
<td>0.032</td>
</tr>
<tr>
<td>75</td>
<td>1.035</td>
<td>0.971</td>
<td>0.065</td>
</tr>
<tr>
<td>80</td>
<td>1.015</td>
<td>0.916</td>
<td>0.099</td>
</tr>
<tr>
<td>85</td>
<td>1.004</td>
<td>0.868</td>
<td>0.136</td>
</tr>
<tr>
<td>90</td>
<td>1.000</td>
<td>0.825</td>
<td>0.175</td>
</tr>
<tr>
<td>TOTAL</td>
<td>71.625</td>
<td>68.135</td>
<td>4.87%</td>
</tr>
</tbody>
</table>

Table 1 shows that the percentage of the sum of error is only 4.87% which shows that the modified model is closer to the original model. Therefore, the difference of mapping function between the original model and the modified model is not significance. (less than 5% mapping function)

4. Computation Time For UNBab_h(E)

The computation time for calculating (100,000 cycles) the original model and also the modified model for UNBab_h(E) can be shown using CodeGear C++ Builder 2007 software [2].
Table 2: Comparison for the computation time

<table>
<thead>
<tr>
<th>Model</th>
<th>Computation time for original model (ms)</th>
<th>Computation time for modified model (ms)</th>
<th>Reduction of computation time. (times)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNBabₙ</td>
<td>281</td>
<td>110</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 2 shows that the computation time between the original model and modified model shows that the modified UNBabₙ(E) model is 2.6 times faster than the original model.

Table 3: Reduction percentage of the number of model operations

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of operations (Original model)</th>
<th>Number of operations (Modified model)</th>
<th>Operation reduction</th>
<th>Percentage of reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNBabₙ</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>71.4</td>
</tr>
</tbody>
</table>

From Table 3, the modified UNBabₙ(E) model can reduce the number of operations up to 71.4 percent compared to the original model.

5. Conclusion

The modified UNBabₙ(E) mapping function model can be used to replace the original model due to its simpler model, smaller sum of error values and also the shorter computation time compared to the original model. This study has a commercial value by replacing the continued fraction algorithm into either hyperbolic or linear form for its mapping function algorithm.

6. Acknowledgement

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References


