Faculty of Manufacturing Engineering

STICK SLIP FRICTION MODELS CONTROL DESIGN APPROACH FOR FRICTION COMPENSATION IN MACHINE TOOLS DRIVE SYSTEM

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STICK SLIP FRICTION MODELS CONTROL DESIGN APPROACH FOR FRICTION COMPENSATION IN MACHINE TOOLS DRIVE SYSTEM

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DECLARATION

I declare that this thesis entitle " Stick Slip Friction Models Control Design Approach for Friction Compensation in Machine Tools Drive System " is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Doctor of Philosophy.

Signature	:	....
Name	:	PROF. MADYA DR. ZAMBERI BIN JAMALUDIN
Date	:	....

DEDICATION

Thank you Allah for this journey in seeking of knowledge and Iman

To my loves,

Amirah Sofiyyah and Thoriq Ziyad

And

For those Moms

who loves their family more than their research

with tears and courage

who cherish their family milestone more than their research

with patience and perseverance

ABSTRACT

In machining process, positioning accuracy of the drives system is always the key element in producing good products with great precision and minimal or zero defects. Positioning accuracy of an electrical drive system is measured by two types of errors; tracking and contour errors. Reducing tracking error will reduce positioning error and thus increase motion accuracy. Meanwhile, reducing contour error will improve quality of machined surface that leads to improvement in overall precision. Position accuracy and precision are subjected to input disturbance acting on the drive system. A special form of error produced as a result of friction is quadrant glitches. Quadrant glitches are spikes occurred at each quadrant angle in a circular motion due to the effect of highly non-linear friction force acting on the feed drive mechanism influenced by pre-sliding friction characteristics at low velocity. At pre-sliding, friction is pre-dominantly a function of displacement that behaves as hysteresis function with non-local memory. This thesis aims at enhancing knowledge and contributes towards compensating quadrant glitches in circular motion for a ball screw driven XY milling positioning table by means of control design approach using enhanced friction force models. The objectives are to model friction behaviour, design and validate the friction compensation performance at low tracking velocity. Two models of friction forces were introduced; the Sigmoid-Like-Curve-Slip (SLCS) model and the Pseudo-Like-Curve-Slip (PLCS) model. Compensation via friction model based method was implemented in this thesis with different position controllers; namely, Proportional Integral Derivative Controller (PID), Cascade Proportional/Proportional-Integral (P/PI) Controller and Sliding Mode Controller (SMC). The effectiveness of the two proposed friction models were validated against the Generalized Maxwell Slip (GMS) friction model – a model known for effective friction compensation in pre-sliding regime. The numerical analyses and experimental validation performed showed improved performance with reduced contour errors. The SLCS model managed to produce a 99% reduction in the magnitude of the quadrant glitches when combined with cascade P/PI position controller at tracking velocity of 2 mm/s. For similar position controller, the PLCS model was able to produce a maximum quadrant glitches reduction of 70%. In comparison, the GMS model was only able to produce a maximum reduction of 40%. Also, both SLCS and PLCS models demonstrate better friction compensation performance when applied with cascade P/PI position controller compared to SMC. Whereas, PID controller has limited ability to sufficiently compensate quadrant glitches even with feedforward of friction models. In conclusion, this thesis has successfully presented significant improvement in accuracy of drives system made with implementation of the two new improved friction models combined with a cascade P/PI position controller. The new models are able to accurately describe friction behaviour in pre-sliding regime by providing smooth transition between pre-sliding and sliding regimes. However, further researches are desired in enhancing the capability of the friction compensation performance in terms of adaptive ability and robustness. Also, further analyses are necessary in the design of SMC robust controller for friction compensation.

ABSTRAK

Ketepatan posisi suatu sistem pemacu semasa proses pemesinan menjadi salah satu elemen utama di dalam menghasilkan produk yang tinggi kepersisan dan minima atau sifar kecacatan. Ketepatan posisi suatu sistem pemacu elektrik diukur oleh dua jenis ralat; ralat menjejak dan ralat kontur. Pengurangan ralat menjejak akan mengurangkan ralat posisi dan memperbaiki ketepatan pergerakan. Pengurangan ralat kontur pula akan memperbaiki kualiti permukaan produk seterusnya meningkatkan ketepatan. Ketepatan dan kepersisan posisi dipengaruhi oleh input gangguan yang bertindak pada sistem pemacu. Ralat khas yang terhasil akibat dari daya geseran adalah 'glic' sukuan. 'Glic' sukuan adalah pancang yang terhasil pada setiap sudut sukuan semasa gerakan membulat disebabkan oleh kesan daya geseran yang sangat tidak linear pada mekanisma sistem pemacu dengan dipengaruhi oleh karakteristik geseran pra-gelangsar semasa kelajuan rendah. Semasa pra-gelangsar, geseran adalah sebahagian besarnya menjadi fungsi kepada nilai anjakan dan berkelakuan seperti histeresis dengan memori bukan setempat. Tesis ini menyasarkan peningkatan pengetahuan dan sumbangan ke arah pengurangan 'glic' sukuan semasa pergerakan membulat oleh mesin pengisar pemacu skru bebola XY menggunakan kaedah rekabentuk sistem kawalan dengan model daya geseran. Objektif penyelidikan adalah untuk memodel daya geseran, rekabentuk dan mengesahkan pencapaian pampasan geseran pada kelajuan rendah. Dua model daya geseran telah diperkenalkan; model Sigmoid-Like-Curve-Slip (SLCS) dan Pseudo-Like-Curve-Slip (PLCS). Pengurangan menggunakan kaedah model geseran dengan pengawal posisi berbeza seperti; pengawal 'proportional-integral-derivative' (PID),'cascade proportional/proportional integral' (P/PI) dan 'sliding mode control' (SMC). Keberkesanan kedua-dua model geseran ini dibandingkan dengan model terdahulu iaitu Generalized Maxwell Slip (GMS) – model yang dikenali berkesan untuk pampasan geseran semasa pra-gelangsar. Analisa numerik dan validasi eksperimen yang dilaksanakan menunjukkan peningkatan pencapaian dengan pengurangan ralat kontur. Model SLCS menghasilkan 99% pengurangan pada magnitud 'glic' sukuan apabila bergandingan dengan posisi pengawal cascade P/PI pada kelajuan rendah sebayak 2 mm/s. Untuk pengawal posisi yang sama, model PLCS menghasilkan pengurangan maksimum magnitud 'glic' sukuan sebanyak 70%. Perbandingannya, model GMS hanya berjaya menghasilkan maksimum peratusan pengurangan magnitud 'glic' sukuan sebanyak 40%. Di samping itu, model SLCS dan PLCS menunjukkan pencapaian pampasan geseran yang lebih baik apabila diaplikasikan bersama pengawal posisi cascade P/PI berbanding pengawal SMC. Manakala, pengawal PID mempunyai keterbatasan dalam memampas 'glic' sukuan walaupun dengan model geseren suap depan. Secara kesimpulan, tesis ini telah berjaya menunjukkan peningkatan ketara pada ketepatan sistem pemacu yang ada melalui pelaksanaan kedua-dua model tersebut bersama pengawal posisi cascade P/PI. Model-model tersebut berjaya menggambarkan secara tepat tingkah-laku geseran pada pra-gelangsar melalui transisi yang lancar antara rejim pra-gelangsar dan gelangsar. Walaubagaimanapun, kajian lanjut diperlukan untuk meningkatkan keupayaan pencapaian pampasan geseran yang bersifat adaptif dan teguh. Selain itu, analisa lanjut adalah perlu dalam kajian rekabentuk pengawal SMC untuk pampasan geseran yang lebih berkesan.

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