Jurnal KALAM *Jurnal Karya Asli Lorekan Ahli Matematik Vol. 10 No.1 (2017) Page 001-013*

Jurnal Karya Asli Lorekan Ahli Matematik

NUMERICAL SOLUTION ON MHD FLOW AND HEAT TRANSFER FOR THE UPPER-CONVECTED MAXWELL FLUID OVER A STRETCHING/SHRINKING SHEET

Nazila Ishak[a], Iskandar Waini[b], Muhammad Khairul Anuar Mohamed[c], Norhayati Rosli[c] and Mohd Zuki Salleh[c*]

[a]*Mathematics Department, MRSM Tun Abdul Razak, 26600 Pekan, Pahang, Malaysia*
[b]*Faculty of Engineering Technology, Universiti Teknikal Malaysia Melaka, 76100 Durian Tunggal, Melaka*
[c]*Faculty of Industrial Science & Technology, Universiti Malaysia Pahang, 26300 UMP Kuantan, Pahang*

Abstract : In this paper we will discuss the effect of magnetohydrodynamic (MHD) towards the flow and heat transfer for the Upper-Convected Maxwell (UCM) fluid over a stretching/shrinking sheet with prescribed wall temperature (PWT). A similarity transformation is used to reduce the governing equations to a set of ordinary differential equations (ODEs), which are then solved numerically using Shooting method. The effect of various parameter such as the Prandtl number, the magnetic parameter, the suction parameter, the stretching or shrinking parameter, and the Maxwell parameter are considered. The results obtained under the special cases are illustrated graphically and compared with previous results to support their validity.

Keywords: Magnetohydrodynamic (MHD); Upper-Convected Maxwell fluid (UCM); Heat transfer; Stretching/Shrinking Sheet.

1.0 Introduction

The concept of boundary layer flow on a continuous solid surface moving at constant speed was first investigated by Sakiadis [1]. Due to the entrainment of the ambient fluid, this boundary layer flow is quite different from the Blasius flow past a flat plate. Sakiadis's theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. [2]. The fluid dynamics to a stretching sheet is important in extrusion processes. The production of sheeting material arises in number of industrial manufacturing processes including both metal and polymer sheets like the cooling of an infinite metallic plate in a cooling bath, paper production, glass blowing and fiber spinning. Since the pioneering study by Crane [3], who presents an exact analytical solution for the steady two-dimensional flow due to a stretching surface in a quiescent fluid, many authors have considered various aspects of this problem and obtained similarity solutions. Gupta and Gupta [4] investigate the heat and mass transfer corresponding to the similarity solutions for the boundary layer over a stretching sheet subjected to suction or a blowing. Chen and Char [5] analyze the heat transfer characteristics over continuous stretching surface with variable surface temperature.

Researches towards steady two-dimensional flow due to a stretching surface in a quiescent fluid have been considered by many authors. It was investigated in different aspects by various authors and the results obtained show the similarity solutions of the aforementioned problem. Amongst them are Wang [6] and Gorla and Sidawi [7]. Ali [8, 9] studied the heat transfer characteristics of a continuous stretching surface and on thermal boundary layer on a power-law stretched surface with suction or injection. Elbashbeshy [10] has studied the heat transfer over a stretching surface with variable surface heat flux. Recently, analytical solution for suction and injection flow of a viscoplastic Casson fluid past a stretching surface in the presence of viscous dissipation is investigate by Hussanan et al. [11].

During the manufacturing of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The investigation of magnetohydrodynamic (MHD) flow is very important for industrial and other different areas of researches such as petroleum production and metallurgical processes [12]. The heat and mass transfer for a hydromagnetic flow over a stretching sheet has been well-studied by Liu [13]. Chakrabarti and Gupta [14] discovered that the temperature distribution in this flow when a uniform suction is applied at the stretching surface. It is also considered by Chiam [15] with a power-law velocity. Anjali and Thiyagarajan [16] studied the steady of nonlinear hydromagnetic flow and heat mass transfer over a stretching surface of variable temperature. Afify [17] discovered the similarity solution in MHD about the effects of thermal diffusion and diffusion thermo on free convection heat and mass transfer over a stretching surface by considering the suction or injection. Recent study regarding this topic include the works by Hussanan et al. [18,19] and Alavi et al. [20] who investigated the unsteady free and mixed convection flow in porous medium with Newtonian heating and stagnation point flow towards an exponentially stretching sheet.

Birds et al. [21] suggest that the viscoelastic fluid models used in previous works are simple models. These include second-order model and/or Watler's B model which are known to be suitable only for weakly elastic fluids subject to slow and slowly-varying flows. However, these two fluid models are known to violate certain rules of thermodynamics. Moreover, a non-Newtonian second grade is less applicable for highly elastic fluids (polymer melts) which occur at high Deborah numbers. Therefore, the significance of the results reported in the above works are limited, at least as far as polymer industry is concerned. Sarpkaya [22] was the first researcher to study the MHD flow of non-Newtonian fluid. Indeed, the viscoelastic fluid model has recently been used by Alizadeh-Pahlavan et al. [23] to study the flow of viscoelastic fluids above porous stretching sheets with no heat transfer effects involved. Sadeghy et al. [24], Hayat et al. [25] and Aliakbar et al. [26] have studied the UCM fluid by using numerical methods with no heat transfer also. Alizadeh-Pahlavan and Sadeghy [27] have done the work related to UCM fluid using homotopy analysis method (HAM). MHD stagnation point flow of upper-convected Maxwell fluid over a stretching sheet has been studied by Hayat et al. [28]. The boundary layer flow and heat transfer due to stretching or shrinking sheet for the upper-convected Maxwell fluid have been considered and extended by many researches (see [29] – [35]) in various ways.

The aim of this paper is to study the MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching/shrinking sheet. Thermal boundary condition is considered in this research which is prescribed wall temperature (PWT). The results obtained under special cases are then compared with those in the literatures to support the validity of our result. The governing of nonlinear partial differential equations is first transformed into ordinary differential equations and is solved using Shooting method.

2.0 The Problem Formulation

Consider a steady two-dimensional boundary layer flow of an upper-convected Maxwell fluid past a stretching/shrinking sheet at $y = 0$. Here, the magnetic field B_0 is applied in the momentum equation where it is imposed along the y-direction, produces magnetic effect in the x-direction. The physical model and coordinate system for stretching surface of this problem is shown in Figure 1. The induced magnetic field which is due to the motion of the electrically conducting fluid is negligible. We also assumed that the external electric field is zero and the polarization charges created by the electric field are negligible

[15]. The *x*-axis runs along the continuous surface in the direction of the motion and the *y*-axis is perpendicular to it.

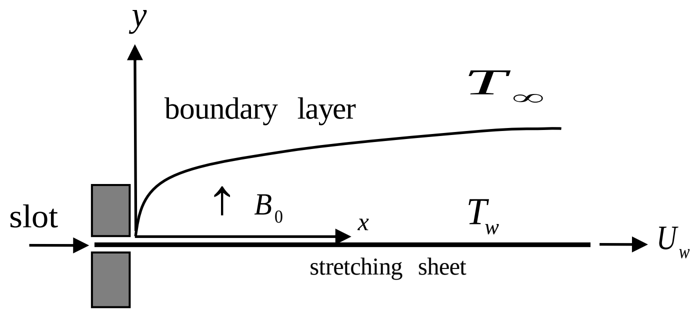

Figure 1: Physical model and coordinate system for stretching/shrinking surface.

Therefore, under these assumptions the governing equations for the upper-convected Maxwell fluid can be written as follows [26]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions:

$$\begin{aligned}&u=U_w(x)=\varepsilon ax\,,\qquad v=v_w(x)\,,\\&T=T_w(x)=T_\infty+bx^2\qquad at\quad y=0\\&u\to 0,\;T\to T_\infty\qquad as\quad y\to\infty\end{aligned} \tag{4}$$

where a is a constant rate of stretching/shrinking, *u* and *v* are the velocity components along the x and y axes, respectively. Meanwhile, T is the temperature of the fluid, T_∞ is the temperature of the ambient, ρ is the fluid density, σ is the electrical conductivity of the fluid, λ is the relaxation time parameter of the fluid, while ν is the kinematic viscosity of the fluid, c_p is the specific heat at a constant pressure, k is the thermal conductivity of the fluid, $v=v_w(x)$ is the wall mass suction velocity and $u=U_w(x)=\varepsilon ax$ is the stretching/ shrinking velocity of the sheet where $\varepsilon=1,-1$ is respectively for stretching and shrinking sheet.

Introduce the stream function, ψ for similarity solution is defined as

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x} \tag{5}$$

so that the continuity Equation (1) is automatically satisfied. Equations (2) and (3) can then be written as

$$\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial x\partial y}\right)-\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)+\lambda\left[\left(\frac{\partial\psi}{\partial y}\right)^2\left(\frac{\partial^3\psi}{\partial y\partial x^2}\right)+\left(\frac{\partial\psi}{\partial x}\right)^2\left(\frac{\partial^3\psi}{\partial y^3}\right)-2\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial^3\psi}{\partial x\partial y^2}\right)\right]$$
$$=\nu\left(\frac{\partial^3\psi}{\partial y^3}\right)-\frac{\sigma B_0^2}{\rho}\left(\frac{\partial\psi}{\partial y}\right) \qquad (6)$$

$$\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial T}{\partial x}\right)-\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial T}{\partial y}\right)=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (7)$$

The similarity solution is obtained by assuming

$$\eta=\left(\frac{a}{\nu}\right)^{1/2}y,\quad \psi=(a\nu)^{1/2}xf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\quad \text{(PWT)} \qquad (8)$$

On substituting (8) into Equations (6) and (7), we obtained the following nonlinear ordinary differential equations

$$f'''+ff''-f'^2-M^2f'+\beta(2ff'f''-f^2f''')=0 \qquad (9)$$

$$\frac{1}{\Pr}\theta''+f\theta'-2f'\theta=0 \qquad (10)$$

and the boundary conditions (4) become

$$f(0)=S,\quad f'(0)=\varepsilon,\quad \theta(0)=1\quad \text{(PWT)}\qquad at\quad \eta=0$$
$$f'(\eta)\to 0,\quad \theta(\eta)\to 0\quad as\quad \eta\to\infty \qquad (11)$$

where primes denote differentiation with respect to η. The dimensionless Maxwell parameter β, Magnetic parameter M, Prandtl number Pr, and Suction parameter S are given by

$$\beta=\lambda a,\quad M^2=\frac{\sigma B_0^2}{a\rho},\quad \Pr=\frac{\mu c_p}{k},\quad S=-\frac{v_w(x)}{(a\nu)^{1/2}} \qquad (12)$$

It is worth mentioning that the problem discussed by Abel et al. [12] can be retrieved when $\varepsilon=1$ which is for the stretching sheet problem.

The physical quantities of interest are the skin friction coefficient C_f and local Nusselt number Nu_x which are defined as

$$C_f=\frac{\tau_w}{\rho U_w^2(x)/2},\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)} \qquad (13)$$

where ρ is the fluid density, while the surface shear stress τ_w and the surface heat flux q_w are respectively given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\quad \text{and}\quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (14)$$

with μ and k being the dynamic viscosity and thermal conductivity. Using the similarity variables (8), we obtain

$$C_f\,\mathrm{Re}_x^{1/2}=f''(0), \qquad (15)$$

and

$$\frac{Nu_x}{\mathrm{Re}_x^{1/2}} = -\theta'(0) \tag{16}$$

Where $\mathrm{Re}_x = U_w(x)x/\nu$ is the local Reynolds number based on the stretching velocity $U_w(x) = \varepsilon ax$ whereas the skin friction and heat transfer coefficients are represent by $f''(0)$ and $-\theta'(0)$respectively.

3.0 Results and Discussion

The appropriate similarity transformation is adopted to transform the governing partial differential equations of flow and heat transfer into a system of non-linear ordinary differential equations. Equations (9) and (10) are subjected to the boundary conditions (11) are solved numerically by the Shooting method. In order to verify the accuracy of the present method and results are compared with those reported by Sadeghy et al. [24], Mamaloukas et al. [32], Abel et al. [12], Muhaimin et al. [34] and Bhattacharyya [35] as shown in Tables 1 and 2, respectively. It has been found that they are in good agreement.

TABLE 1 The comparison of the skin friction coefficient $f''(0)$ for a different values of Maxwell parameter β when $\Pr = 1$, $M = 0$ and $S = 0$ in the case of stretching sheet $\varepsilon = 1$.

	$f''(0)$			
β	Sadeghy et al. [24]	Mamaloukas et al. [32]	Abel et al. [12]	Present results
0	-1.0000	-0.999962	-0.999962	-1.000173
0.2	-1.0549	-	-1.051948	-1.051973
0.4	-1.10084	-1.101850	-1.101850	-1.101945
0.6	-1.0015016	-	-1.150163	-1.150158
0.8	-1.19872	-1.196692	-1.196692	-1.196722
1.2	-	-1.285257	-1.285257	-1.285367

TABLE 2 Values of the skin friction coefficient $f''(0)$ for a different values of Suction parameter s when $\Pr = 1$, $M^2 = 1$ and $\beta = 0$ in the case of shrinking sheet $\varepsilon = -1$.

	$f''(0)$		
s	Muhaimin et al. [34]	Bhattacharyya [35]	Present results
0	-	-	-
1	-	-	-
2	2.414214	2.414300	2.41421357
3	3.302776	3.302750	3.30277563
4	4.236068	4.236099	4.23606797

TABLE 3: Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for a different values of the Maxwell parameter β when $\Pr = 1$, $M^2 = 1$ and $S = 0.5$ in the case of stretching sheet $\varepsilon = 1$.

β	$f''(0)$	$-\theta'(0)$
0.2	-0.927337	1.654122
0.4	-1.251505	1.570546
0.8	-1.963845	1.402870
1.0	-2.371653	1.321075
1.2	-2.828126	1.242116
1.6	-3.944897	1.095081

The value of the skin friction coefficient $f''(0)$ and the heat transfer coefficient $-\theta'(0)$ for various values of β in Table 3. It is found that when β increases, both the values of $f''(0)$ and $-\theta'(0)$ decrease.

TABLE 4: Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for a different values of Suction parameter S when $\Pr = 1$, $M^2 = 1$ and $\beta = 0.8$ in the case of stretching sheet $\varepsilon = 1$.

S	$f''(0)$	$-\theta'(0)$
-0.5	-0.2473098	1.2371365
-0.2	-0.4916347	1.3194128
0	-0.7267158	1.3662627
0.2	-1.0620072	1.3999277
0.4	-1.5808100	1.4105613

From Table 4 it can be seen that the different value of suction parameter S for the skin friction coefficient $f''(0)$ and heat transfer $-\theta'(0)$. From this table, note that when the suction parameter S increases the $f''(0)$ decrease and $-\theta'(0)$ increase.

TABLE 5: Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for a different values of the stretching parameter ε when $\Pr = 1$, $M^2 = 1$, $S = 2$ and $\beta = 0.8$.

ε	$f''(0)$	$-\theta'(0)$
0.1	-0.4884634	1.0999593
0.2	-1.1892046	1.1331568
0.3	-2.0791961	1.1586620
0.5	-4.3962750	1.1961594

TABLE 6: Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for a different values of the Magnetic Parameter M when $\Pr = 1, S = 2$ and $\beta = 0.8$ in the case of stretching sheet $\varepsilon = 1$.

M	$f''(0)$	$-\theta'(0)$
0.1	-2.3980966	1.3242181
0.3	-2.3670331	1.3296220
0.6	-2.2577130	1.3489589
0.9	-2.0569792	1.3856136
1.2	-1.7532869	1.4315114
1.5	-1.3095711	1.4925915

From Table 5, when the stretching parameter ε increase the skin friction coefficient $f''(0)$ decrease but the heat transfer coefficient increase. The values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for a different values of the Magnetic Parameter M when $\Pr = 1, S = 2, \beta = 0.8$ and $\varepsilon = 1$ presented in Table 6. It is found when the magnetic parameters M increase the both values of $f''(0)$ and $-\theta'(0)$ increase.

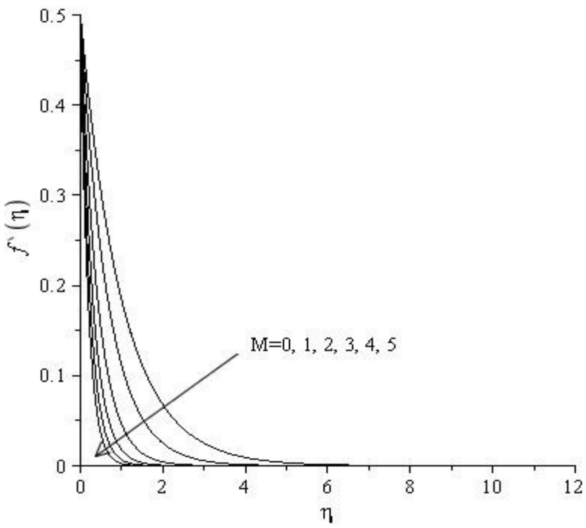

Figure 2: The effect of magnetic parameter M on the velocity profile $f'(\eta)$ for $\beta = 0$

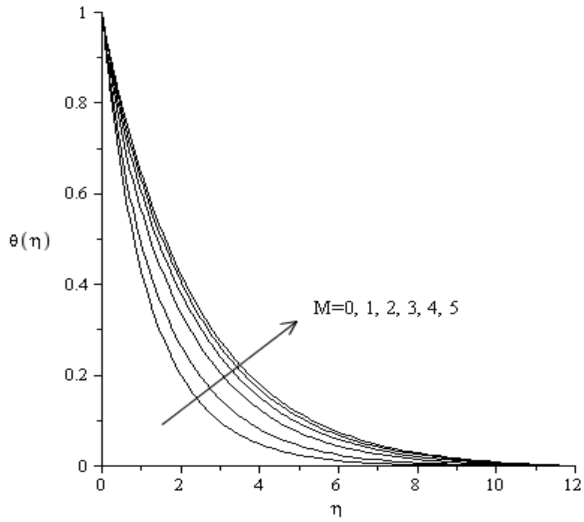

Figure 3: The effect of magnetic parameter M on the temperature profile $\theta(\eta)$ for $\beta = 0$

Figures 2, 3, 4 and 5, illustrates the effect of magnetic parameter, M on the velocity $f'(\eta)$ and temperature profile $\theta(\eta)$ for two value of Maxwell parameter $\beta = 0$ and $\beta \neq 0$ when $S = 0.5, \Pr = 0.7$ and $\varepsilon = 0.5$ respectively. It is found that as the value of Magnetic parameter increases the velocity profile decrease but the temperature profile increase, this due to a M change the profile to be less flatter to be near the edge of the boundary layer and less steeper near to the edge of the surface.

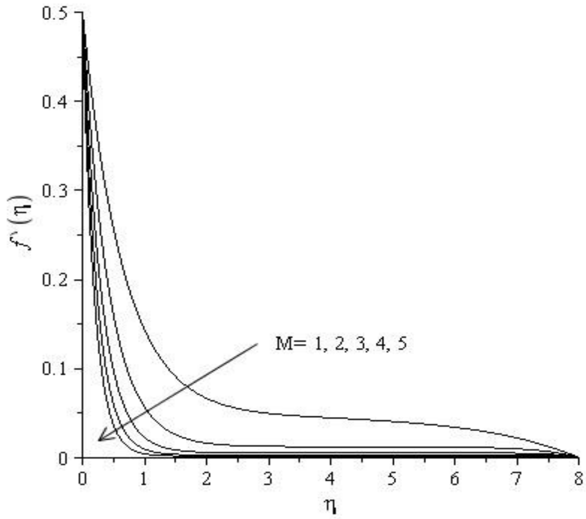

Figure 4: The effect of magnetic parameter M on the velocity profile $f'(\eta)$ for $\beta \neq 0$

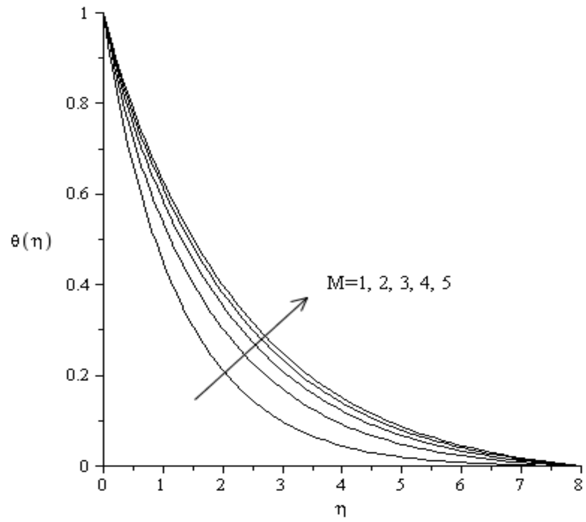

Figure 5: The effect of magnetic parameter M on temperature profile $\theta(\eta)$ for $\beta \neq 0$

Figures 6 and 7, show the effect of the suction parameter, S on velocity and temperature profile when $\Pr = 0.7, M = 0.2, \varepsilon = 0.5$ and $\beta = 0.5$ respectively. An increasing of suction parameter S, lead to decreasing on velocity, temperature profile and the thermal boundary layer thickness.

Figures 8 and 9, show the effect of Maxwell parameter, β on velocity and temperature profile when $\Pr = 0.7, M = 1, \varepsilon = 1$ and $S = 0.5$ respectively. From these figure show that when. From these figure show that when Maxwell parameter, β increase the value for velocity and temperature profile become decrease. Therefore, when the momentum boundary layer thickness decrease it can induces an increase in the absolute value of the velocity gradient at the surface. The value of the concentration gradient at the surface increases depend when the value Maxwell parameter, β also increase. Then, the mass of the transfer rate at the surface also increases.

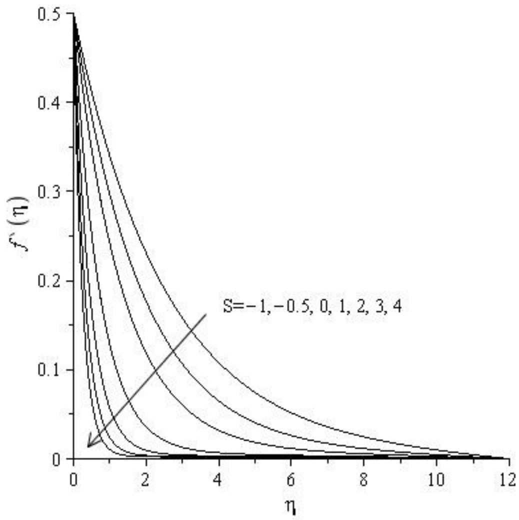

Figure 6: The effect of suction parameter S on the velocity profile $f'(\eta)$ for $\beta \neq 0$

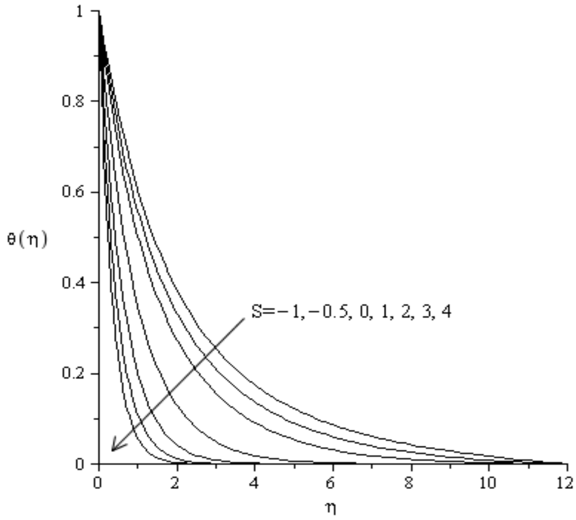

Figure 7: The effect of suction parameter S on the temperature profile $\theta(\eta)$ for $\beta \neq 0$

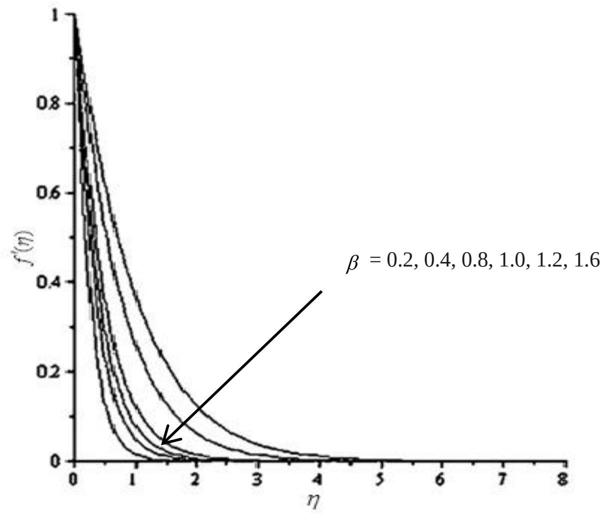

Figure 8: The effect of maxwell parameter β on the velocity profile $f'(\eta)$

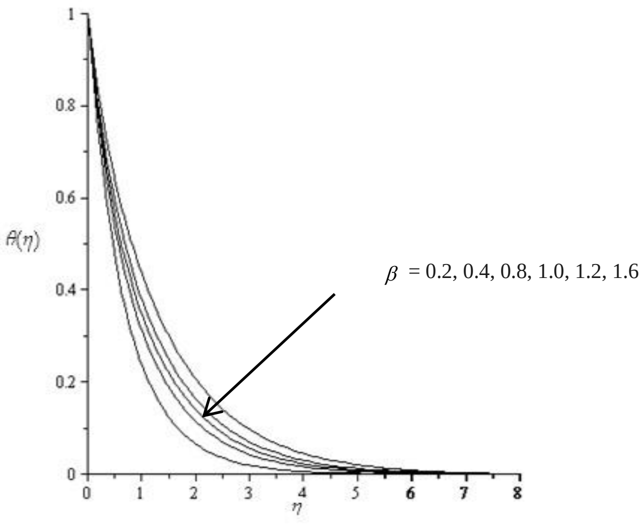

Figure 9: The effect of Maxwell parameter β on the temperature profile $\theta(\eta)$

Figures 10 and 11, show the effect of the stretching sheet ε when for $\Pr = 0.7, M = 1, S = 0.5$ and $\beta = 0.05$. respectively. From these figure, it is found that when stretching sheet ε increase, the velocity profile increase and the temperature profile decrease and the thermal boundary layer thickness also decrease.

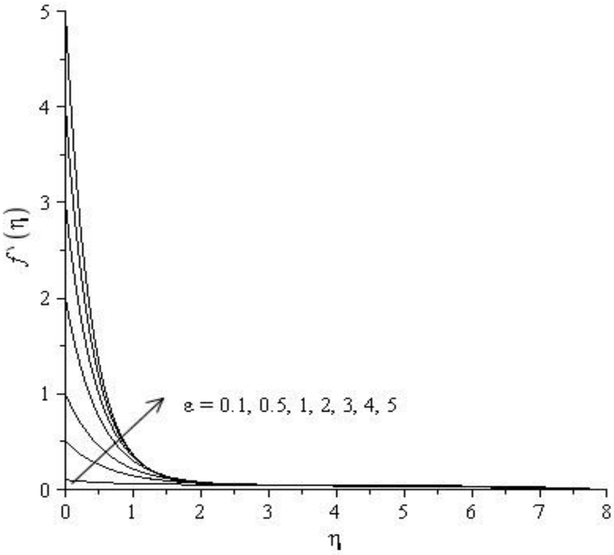

Figure 10: The effect of stretching sheet ε on the velocity profile $f'(\eta)$

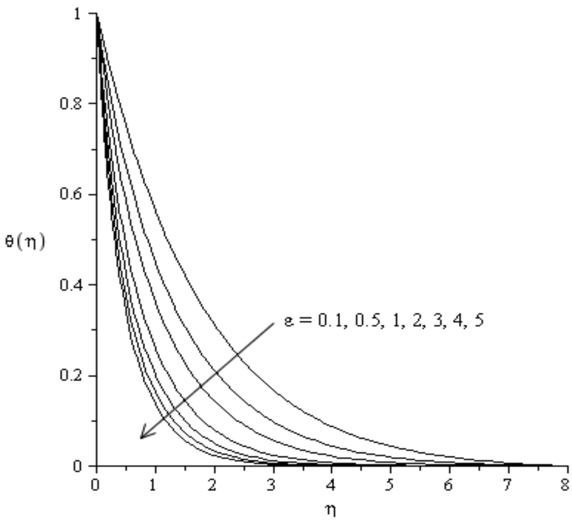

Figure 11: The effect of stretching sheet ε on the temperature profile $\theta(\eta)$

4.0 Conclusion

In this paper, we have numerically studied the problem of the MHD flow and heat transfer within a boundary layer of upper-convected Maxwell fluid over a stretching/ shrinking sheet with prescribed wall temperature using the Shooting method. It is shown how the magnetic parameter M, Maxwell parameter β, suction parameter S, stretching sheet ε affect the skin friction coefficient $f''(0)$, local heat transfer coefficient $-\theta'(0)$, velocity $f'(\eta)$ and the temperature profile $\theta(\eta)$. We can conclude that

1. an increase in the value magnetic parameter leads to increase of the skin friction coefficient, local heat transfer coefficient and temperature profile and decrease on the values of velocity profile.
2. an increase in the Maxwell and suction parameter leads to decreases of the skin friction coefficient, velocity and temperature profile and increase on the local heat transfer coefficient.
3. an increase in the value stretching sheet leads to increases of the local heat transfer coefficient and temperature profile and decreases skin friction coefficient and velocity profile.

Acknowledgement.

The author thanks to Universiti Malaysia Pahang for financial supports (RDU170358) and Ministry of Higher Education (RDU150101).

References

(1) Sakiadis, B.C., 1961. Boundary-layer behaviour on continuous solid surfaces: I. Boundary layer equations for two-dimensional and axisymmetric flow. *Am. Inst. Chem. Engineers J*. 7: 26-28.

(2) Tsou, F., Sparrow, E., and Goldstein, R., 1967. Flow and heat transfer in the boundary layer on a continuous moving surfaces. *Int. J. Heat Mass Transfer*. 10: 219-235.

(3) Crane, L. J., 1970. Flow past a stretching plate. Z. Angew. *Math.Phys*. 21: 645-647.

(4) Gupta, P. S., and Gupta, A. S., 1977. Heat and mass transfer on a stretching sheet with suction or blowing. *Can. J. Chem. Eng*. 55: 744-746.

(5) Chen, C. K., and Char, M., 1988. Heat transfer of a continuous, stretching surface with suction or blowing. *Journal of Mathematical Analysis and Applications*. 135: 568-580.

(6) Wang, C. Y., 1989. Free convection on a vertical stretching surface. *J. Appl. Math. Mech. (ZAMM)*. 69: 418-420.

(7) Gorla, R. S. R., and Sidawi, I., 1994. Free convection on a vertical stretching surface with suction and blowing. *Appl. Sci. Res*. 52: 247-257.

(8) Ali, M. E., 1994. Heat transfer characteristics of a continuous stretching surface. *Warme Stoffubertragung*. 29: 227-234.

(9) Ali, M. E., 1995. On thermal boundary layer on a power-law stretched surface with suction or injection. *International Journal of Heat and Fluid Flow*. 16: 280-290.

(10) Elbashbeshy, E. M. A., 1998. Heat transfer over a stretching surface with variable surface heat flux. *J. Phys. D: Appl. Phys*. 31: 1951-1954.

(11) Hussanan, A., Salleh, M. Z., Khan, I., and Shafie, S., 2016. Analytical solution for suction and injection flow of a viscoplastic Casson fluid past a stretching surface in the presence of viscous dissipation. Neural Computing and Applications, 1-9.

(12) Abel, M.S., Tawade, J.V, and Nandeppanavar, M.M., 2012. MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet. *Meccanica.* 47: 385-393.

(13) Liu, I. C., 2005. A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet. *Int. Commun. Heat Mass Transfer.* 32: 1075-1084.

(14) Chakrabarti, A. and Gupta, A. S., 1979. Hydromagnetic flow and heat transfer over a stretching sheet. *Quart. Appl. Math.* 37: 73-78.

(15) Chiam, T. C., 1995. Hydromagnetic flow over a surface stretching with a power-law velocity. *Int. J. Engng SCi*. 33: 429-435.

(16) Anjali Devi, S. P., and Thiyagarajan, M., 2006. Steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature. *Heat and Mass Transfer.* 42: 671-677.

(17) Afify, A. A., 2009. Similarity solution in MHD: Effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. *Commun Nonlinear Sci Numer Simulat.* 14: 2202-2214.

(18) Hussanan, A., Ismail, Z., Khan, I., Hussein, A. G., and Shafie, S., 2014. Unsteady boundary layer MHD free convection flow in a porous medium with constant mass diffusion and Newtonian heating. The European Physical Journal Plus, 129: 46.

(19) Hussanan, A., Salleh, M. Z., Khan, I., Tahar, R. M. and Ismail, Z., 2015. Soret effects on unsteady magnetohydrodynamic mixed-convection heat-and-mass-transfer flow in a porous medium with Newtonian heating. Maejo International Journal of Science and Technology. 9: 224-245.

(20) Alavi, S. Q., Hussanan, A., Kasim, A. R. M., Rosli, N., and Salleh, M. Z., 2017. MHD Stagnation Point flow Towards an Exponentially Stretching Sheet with Prescribed wall Temperature and Heat Flux. International Journal of Applied and Computational Mathematics, 1-13.

(21) Bird R. B., Armstrong R.C., and Hassager O., 1987. Dynamics of polymeric liquids, vol 1. Wiley, New York.

(22) Sarpkaya, T., 1961. Flow of non-Newtonian fluids in a magnetic field. *AIChE J.* 7: 324-328.

(23) Alizadeh-Pahlavan, A., Aliakbar, V., Vakili-Farahani, F., and Sadeghy, K., 2009. MHD flows of UCM fluids above porous stretching sheets using two-axillary-parameter homotopy analysis method. *Commun. Nonlinear. Sci. Numer. Simul.* 14: 473-488.

(24) Sadeghy, K., Hajibeygi, H., and Taghavi, S.M., 2006. Stagnation point flow of upper-convected Maxwell fluids. *Int. J Non-Linear Mech.* 41: 1242-1247.

(25) Hayat, T., Abbas, Z., and Sajid, M., 2006 Series solution for the upper-convected Maxwell fluid over a porous stretching plate. *Phys Lett A.* 358: 396-403.

(26) Aliakbar, V., Alizadeh-Pahlavan, A., and Sadeghy, K., 2009. The influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. *Commun. Nonlinear. Sci. Numer. Simul.* 14(3): 779-794.

(27) Alizadeh-Pahlavan, A., and Sadeghy, K., 2009. On the use of ho-motopy analysis method for solving unsteady MHD flow of Maxwellian fluids above impulsively stretching sheets. *Commun. Nonlinear. Sci. Numer. Simul*. 14(4): 1355-1365.

(28) Hayat, T., Abbas, Z., and Sajjid, M., 2009. MHD stagnation point flow of an upper convected Maxwell fluid over a stretching sheet. *Chaos Solitons Fractals*. 39: 840-848.

(29) Merkin, J.H., 1994. Natural-convection boundary-layer flow on a vertical surface with Newtonian heating. *International Journal of Heat and Fluid Flow.* 15(5): 392-398.

(30) Renardy, M., 1997. High Weissenberg number boundary layers for the upper convected Maxwell fluid. *J. Non-Newton. Fluid Mech.* 68: 125.

(31) Raftari, B. and Yildirim, A., 2010. The application of homotopy perturbation method for MHD flows of UCM fluids above porous stretching sheets. *Computers and Mathematics with Applications.* 59: 3328-3337.

(32) Mamaloukas, C., Abel, M.S., Tawade, J.V., and Mahabaleswar, U.S., 2011. On effects os a transverce magnetic field on an UCM fluid over a stretching sheet. *International Journal of Pure and Applied Mathematics*. 66(1): 1-9.

(33) Sadeghy, K., Najafi, A.H., and Saffaripour, M., 2005. Sakiadis flow of an upper convected Maxwell fluid. *Int J Non-Linear Mech.* 40: 1220-1228.

(34) Muhaimin, Kandasamy, R., and Khamis, A.B., 2008. Effects of heat and mass transfer on nonlinear MHD boundary layer flow over a shrinking sheet in the presence of suction. *Applied Mathematics and Mechanics*. 29 (10): 1309-1317.

(35) Krisnedu Bhattacharyya., 2011. Effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction. *Chemical Engineering Research Bulletin*. 15: 12-17.