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INVARIANCENESS OF HIGHER ORDER UNITED SCALED-INVARIANTS

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Abstract

This paper presents an alternative formulation of invariant moments using higher order united scaled-invariants for unequal scaling images. Moment invariants have been proposed as pattern sensitive features in classification and recognition applications. Hu was the first to introduce the concept of invariants based on combinations of regular moments using algebraic invariants. In this study, we consider unconstrained handwritten words of different presentations. Results of computer simulations for images of unequal scaling are also included verifying the validity of the method proposed.

1. Introduction

Recognition of visual patterns and characters independent of position, size and orientation in the visual field has been a goal of much recent research. To achieve maximum utility and flexibility, the methods used should be insensitive to variations in shape and should provide for

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improved performance with repeated trials. Hu's invariant moments which are based on regular moments and algebraic invariants are invariant under change of size, translation and orientation [3]. But in the case of unconstrained images of unequal scaling, Hu's invariants would generate different moments values for the same images of different orientations or scale [2, 5]. In this paper, we present higher order centralized united scaled-invariants for unconstrained handwritten words of unequal scaling in x and y directions. The rest of the paper is organized as follows. Section 2 gives review on feature extraction with regular moments, its basic theory and invarianceness, while Section 3 gives the proposed method and its invarianceness. Section 4 gives simulation results and Section 5 is a conclusion.

2. Feature Extraction with Regular Moments and its Basic Theory

Feature extraction is methodologies of extracting distinguish features from the images. In other words, the image is represented by a set of numerical features to remove redundancy from the data and reduce its dimension. These features can be categorized as good features if fulfilling the characteristics below:

(i) Small interclass invariance whereby slightly different shapes with similar general characteristics should have numerically close values.

(ii) Large interclass separation in which features from different classes should be quite different numerically.

In order to recognize many variations of the same character, features that are invariant to certain transformations on the character need to be used. Invariants are features which have approximately the same values for samples of the same character that are, for example, translated, scaled, rotated, stretched, skewed or mirrored.

Regular moments or Geometrical moments are defined with the basis set $\{x^p, y^q\}$. The $(p+q)$th order two-dimensional geometric moments are denoted by m_{pq}, and can be expressed as

$$m_{pq} = \int_{\zeta} \int x^p y^q f(x, y) dxdy, \quad p, q = 0, 1, 2, 3, \ldots, \tag{1}$$

where ζ is the region of the pixel space in which the image intensity function $f(x, y)$ is defined, and follows by the uniqueness theorem and the existence theorem [3].

Uniqueness Theorem. *Assuming that the intensity function* $f(x, y)$ *is a piecewise continuous and bounded in the region* ζ, *the moment sequence* $\{m_{pq}\}$ *is uniquely determined by the intensity function* $f(x, y)$ *and conversely.*

Existence Theorem. *Assuming that the intensity function* $f(x, y)$ *is piecewise continuous and bounded in the region* ζ, *the moments* m_{pq} *of all orders exist and are finite.*

The translation invariant central moments are obtained by placing origin at the centered of the image

$$\mu_{pq} = \sum_x \sum_y f(x, y)(x - x_0)^p (y - y_0)^q,$$

where

$$x_0 = \frac{m_{10}}{m_{00}} \text{ and } y_0 = \frac{m_{01}}{m_{00}}.$$

Consider the following linear transformation of the image coordinates from (x, y) to (x', y') :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \tag{2}$$

where a_{ij}'s are constants and $| a_{11}a_{12} - a_{12}a_{21} | \neq 0$.

The geometric moments m'_{pq} of the transformed image $f'(x, y)$ given by

$$\begin{aligned} m'_{pq} &= \int_\zeta \int (x')^p (y')^q f'(x', y') dx'dy' \\ &= \int\int (a_{11}x + a_{12}y)^p (a_{21}x + a_{22}y)^q f(x, y) \Delta dxdy,\ p, q = 0, 1, 2, 3, \ldots \end{aligned} \tag{3}$$

where $\Delta = a_{11}a_{22} - a_{21}a_{12}$ is the determinant of the transformation matrix. Equation (3) can be expanded into a series of moments m_{pq} of the initial image, to obtain the equation relating moments of the initial and transformed images. The moment equations thus derived for different orders, can be algebraically manipulated to eliminate the image transformation parameters a_{ij}'s to yield invariant expressions in moment functions.

A transformation of the image pixel coordinates by a uniform scale factor α is given by

$$x' = \alpha x, \quad y' = \alpha y. \tag{4}$$

The above transformation also leads to the following expression for the scaled area element

$$dx'dy' = \alpha^2 dxdy. \tag{5}$$

Substituting equation (5) into equation (3) gives the moments of the scaled image in terms of the moments of the original image as

$$\mu'_{pq} = \alpha^{p+q+2}\mu_{pq}. \tag{6}$$

Eliminating the unknown scale factor α from the above two equations as

$$\alpha = \left(\frac{\mu'_{00}}{\mu_{00}}\right)^{1/2}. \tag{7}$$

Substituting equation (7) into equation (6) and simplifying it, gives

$$\frac{\mu'_{pq}}{(\mu'_{00})^{(p+q+2)/2}} = \frac{\mu_{pq}}{(\mu_{00})^{(p+q+2)/2}}. \tag{8}$$

To show its invarianceness, let equation (8) be denoted as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\left(\frac{p+q}{2}+1\right)}}, \tag{9}$$

and

$$\eta'_{pq} = \frac{\mu'_{pq}}{\mu'^{\left(\frac{p+q}{2}+1\right)}_{00}}. \tag{10}$$

When $p = q = 0$, equation (6) becomes

$$\mu'_{00} = \alpha^2 \mu_{00}. \tag{11}$$

Substituting equation (6) and equation (11) into equation (10) gives

$$\eta'_{pq} = \frac{\alpha^{p+q+2}\mu_{pq}}{(\alpha^2)^{\frac{p+q}{2}+1}\mu_{00}^{\frac{p+q}{2}+1}}. \tag{12}$$

Simplifying equation (12) will give us,

$$\eta'_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\left(\frac{p+q}{2}+1\right)}} = \eta_{pq}. \tag{13}$$

Thus the invarianceness is shown using regular moment invariants.

If an image is transformed with unequal scale factors k_1, k_2 along x and y axes, respectively, then the transformed moments can be obtained as

$$m'_{pq} = (k_1)^{p+1}(k_2)^{q+1} m_{pq}. \tag{14}$$

Writing the explicit equations for the first few orders of the moments, and eliminating k_1, k_2 from these equations, the term becomes [4],

$$\eta_{pq} = \frac{(\mu_{00})^{(p+q+2/2)}}{(\mu_{20})^{(p+1)/2}(\mu_{02})^{(q+1)/2}}, \tag{15}$$

which is invariant with respect to non-uniform scaling of the image. These invariants are called *Aspect Ratio Invariants*.

3. Improved Scaled-invariants and its Invarianceness

In this section, we introduce higher order centralized scaled-invariants. For images with unequal scaling, consider the following linear transformation which performs different scales in x and y directions [2],

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}. \tag{16}$$

Every coefficient of $f(x, y)$ will be an algebraic invariant by the definition of invariants [6],

$$a'_{pq} = \alpha^p \beta^q a_{pq}. \tag{17}$$

Using the same approach as previous section, moment invariant for unequal scaling is given as

$$\mu'_{pq} = \alpha^{p+1} \beta^{q+1} \mu_{pq}. \tag{18}$$

Using higher order centralized invariants of the scale normalization yields

$$\begin{aligned} \mu'_{02} &= \alpha\beta^3\mu_{02}, \\ \mu'_{20} &= \alpha^3\beta\mu_{20}, \\ \mu'_{04} &= \alpha\beta^5\mu_{04}, \\ \mu'_{40} &= \alpha^5\beta\mu_{40}. \end{aligned} \tag{19}$$

To show its invarianceness, solving the above equation for α and β,

$$\alpha^2 = \left(\frac{\mu'_{40}\mu_{20}}{\mu_{40}\mu'_{20}}\right)^{p+1/2}, \quad \beta^2 = \left(\frac{\mu'_{02}\mu_{04}}{\mu_{02}\mu'_{04}}\right)^{q+1/2}, \tag{20}$$

gives the relation between μ'_{pq} and μ_{pq},

$$\mu'_{pq} = \left(\frac{(\mu'_{40})^{p+1/2}(\mu_{20})^{p+1/2}}{(\mu_{40})^{p+1/2}(\mu'_{20})^{p+1/2}}\right)\left(\frac{(\mu'_{04})^{q+1/2}(\mu_{02})^{q+1/2}}{(\mu_{04})^{q+1/2}(\mu'_{02})^{q+1/2}}\right). \tag{21}$$

Substituting equation (21) into equation (18) gives the moment invariants which are independent of the different scaling in the x and y directions,

$$\left(\frac{(\mu'_{20})^{p+1/2}(\mu'_{02})^{q+1/2}}{(\mu'_{40})^{p+1/2}(\mu'_{04})^{q+1/2}}\right)\mu'_{pq} = \left(\frac{(\mu_{20})^{p+1/2}(\mu_{02})^{q+1/2}}{(\mu_{40})^{p+1/2}(\mu_{04})^{q+1/2}}\right)\mu_{pq}. \tag{22}$$

Thus improved scale-invariants is given as:

$$\eta_{pq} = \left(\frac{\mu_{20}^{p+1/2}\mu_{02}^{q+1/2}}{\mu_{40}^{p+1/2}\mu_{04}^{q+1/2}}\right)\mu_{pq}. \tag{23}$$

4. Higher Order United Scaled-invariants and its Invarianceness

Searching for images using shape features has attracted much attention by many researchers. Shape is an important visual feature and it is one of the basic features used to describe image content [8]. However, to extract the features that represent and describe the shape precisely is a difficult task. A good shape descriptor should be able to find perceptually similar shape where it is usually means rotated, translated, scaled and affined transformed shapes. Furthermore, it can tolerate with human beings in comparing the image shapes. Yinan et al. [7] proposed united moment invariants where the rotation, translation and scaling can be discretely kept invariant to region, closed and unclosed boundary. The united moment invariants are good set of discriminant shape features and valid in discrete condition. United moment invariant can be related to geometric moment by Hu [3], which consider equation (13) and equation (15), and improved moment invariant by Chen [1] using equation (24) as below:

$$\eta'_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{p+q+1}}. \tag{24}$$

Each has the factor μ_{pq}. The united formula can be addressed whenever, these equations ignore the influence of μ_{00} and ρ. Yinan et al. [7] have presented eight formula of united moment invariants as below:

$$\theta_1 = \frac{\sqrt{\phi_2}}{\phi_1}, \quad \phi_2 = \frac{\phi_6}{\phi_1\phi_4}, \quad \phi_3 = \frac{\sqrt{\phi_5}}{\phi_4},$$

$$\phi_4 = \frac{\phi_5}{\phi_3\phi_4}, \quad \phi_5 = \frac{\phi_1\phi_6}{\phi_2\phi_3}, \quad \phi_6 = \frac{(\phi_1 + \sqrt{\phi_2})\phi_3}{\phi_6},$$

$$\phi_7 = \frac{\phi_1\phi_5}{\phi_3\phi_6}, \quad \phi_8 = \frac{(\phi_3 + \phi_4)}{\sqrt{\phi_5}},$$

where ϕ_i are Hu's moment invariants.

4.1. Derivation of higher order united scaled-invariants

United Moment Invariants are formulated using normalized central moments, and this can be written as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\frac{p+q+2}{2}}}. \tag{25}$$

In discrete form, the central and normalized higher order central invariants are given as:

$$\mu'_{pq} = \rho^{p+q}\mu_{pq},$$
$$\eta'_{pq} = \left(\frac{\mu_{20}^{p+1/2}\mu_{02}^{q+1/2}}{\mu_{40}^{p+1/2}\mu_{04}^{q+1/2}}\right)\mu_{pq}, \tag{26}$$

and boundary representation as:

$$\eta''_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{p+q+1}}. \tag{27}$$

We outline three phases that involve in formulation of United Moment Invariants integrated with higher order scaled-invariants from geometric moments.

1st Phase

We consider only $\theta_1 = \frac{\sqrt{\phi_2}}{\phi_1}$. From Hu, $\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$, substituting equation (25) into $\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$, we get

$$\phi_2 = \left(\frac{\mu_{20} - \mu_{02}}{\mu_{00}^2}\right)^2 + \frac{4\mu_{11}^2}{\mu_{00}^4}. \tag{28}$$

Substituting equation (25) into equation (28) gives us

$$\sqrt{\phi_2} = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{00}^2}, \tag{29}$$

and

$$\phi_1 = \eta_{20} + \eta_{02} = \frac{\mu_{20} + \mu_{02}}{\mu_{00}^2}.$$

Thus,

$$\frac{\sqrt{\phi_2}}{\phi_1} = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{20} + \mu_{02}} = \theta_1. \quad (30)$$

2nd Phase

Now consider θ_1'. Again $\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$ from Hu's formulation. From equation (26),

$$\phi_2 = \left(\frac{\rho^2\mu_{20}}{\mu_{00}^2} - \frac{\rho^2\mu_{02}}{\mu_{00}^2}\right)^2 + \frac{4\rho^4\mu_{11}^2}{\mu_{00}^4},$$

$$\phi_2 = \frac{\rho^4}{\mu_{00}^4} + \{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2\}.$$

Thus,

$$\sqrt{\phi_2} = \frac{\rho^2}{\mu_{00}^2}\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}.$$

Now $\phi_1 = (\eta_{20} + \eta_{02})$, and substitute into equation (26) gives,

$$\phi_1 = \frac{\rho^2\mu_{20}}{\mu_{00}^2} + \frac{\rho^2\mu_{02}}{\mu_{00}^2}.$$

Thus,

$$\frac{\sqrt{\phi_2}}{\phi_1} = \theta_1' = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{20} + \mu_{02}}.$$

3rd Phase

Using the same approach as phases 1 and 2, again we consider $\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$, (from Hu's formulation). Substituting into equation (27) gives

$$\phi_2 = \left(\frac{\mu_{20}}{\mu_{00}^3} - \frac{\mu_{02}}{\mu_{00}^3}\right)^2 + \frac{4\mu_{11}^2}{\mu_{00}^6},$$

$$\phi_2 = \frac{1}{\mu_{00}^6}\{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2\}.$$

Thus,

$$\sqrt{\phi_2} = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{00}^3}.$$

Now consider $\phi_1 = (\eta_{20} + \eta_{02})$, (from Hu's formulation), and substitute into equation (27) gives us

$$\phi_1 = \frac{\mu_{20}}{\mu_{00}^3} + \frac{\mu_{02}}{\mu_{00}^3}.$$

Thus,

$$\frac{\sqrt{\phi_2}}{\phi_1} = \theta_1'' = \frac{\sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{\mu_{20} + \mu_{02}}.$$

Finally, we conclude that

$$\theta_1 = \theta_1' = \theta_1''.$$

5. Simulation Results

Higher order united scaled-invariants are tested on unconstrained handwritten words, and the comparisons are made with united moment invariant, aspect invariant moments and geometric invariants. Unconstrained handwritten words are taken from different individuals with different styles of writings.

Tables 1-4 present moment invariants of word *the* of higher order centralized united scaled-invariants, united moment invariant, aspect invariants and geometric invariants for 10 writers. Mean error for the comparison of each image with image number one also is shown in the last column. Table 5 shows the mean errors as illustrated in Figure 1.

Table 1. Invariants of the unconstrained handwritten words (Higher Order United Scaled-invariants)

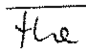
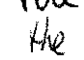
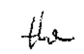
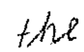
Word *the*	Feature1	Feature2	Feature3	Feature4	Feature5	Feature6	Feature7	Feature8	Mean Error
	2.998835	3.084430	0.135886	0.577700	2.063768	3.934337	2.506730	0.771034	-
	3.353034	3.384627	1.113550	0.811775	4.736764	1.969496	2.572852	1.129927	0.8661233
	2.634687	2.669928	0.315395	1.669424	1.560812	3.709568	1.000504	1.392048	0.613106
	2.913457	3.119042	0.532047	1.584268	2.187700	3.639744	1.534774	1.166790	0.4136195
	2.930257	3.100332	0.136408	0.259284	2.773715	3.087309	2.841048	0.430725	0.3293775
	2.942373	3.865733	0.022419	0.133687	1.840486	4.044754	3.732047	0.376983	0.418539
	2.936159	3.571382	0.246676	0.578810	1.228774	4.644046	2.992571	0.860792	0.3477287
	3.132164	3.822065	0.071038	0.303078	1.997109	4.267539	3.518987	0.507270	0.3607895
	3.358388	3.442320	0.200238	0.344322	4.019255	2.697712	3.786642	0.272109	0.7482652
	2.777551	3.567794	0.350458	0.707845	0.578548	4.977279	2.859950	1.074925	0.5293275

Table 2. Invariants of the unconstrained handwritten words (United Moment Invariants)

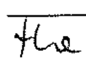
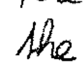
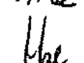
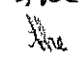
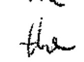
Word *the*	Feature1	Feature2	Feature3	Feature4	Feature5	Feature6	Feature7	Feature8	Mean Error
	0.241576	0.337478	0.068713	0.411716	0.403468	1.083447	0.074239	0.588451	-
	0.354808	0.598260	0.114556	0.016021	0.356490	0.512021	0.614281	0.310090	0.2815448
	1.177351	1.594657	0.084414	1.261690	0.332817	2.715469	0.332968	1.211000	0.5963787
	0.346014	0.499584	0.289525	1.130738	0.359246	1.212881	0.631154	0.948676	0.2871467
	0.813263	1.868338	0.313193	0.471987	0.087413	1.776041	1.396352	0.543849	0.5978327
	0.400397	0.403034	1.003696	1.605185	0.799966	0.146255	1.202151	1.148609	0.6718236
	1.277853	1.289823	0.246698	0.666729	0.105760	2.472268	0.623094	0.942470	0.6263778
	0.348874	0.363352	0.146101	0.091805	0.534794	0.323677	0.271547	0.358390	0.231117
	0.238783	0.303278	0.083613	0.002811	0.338704	0.336709	0.300467	0.310170	0.2220966
	0.379390	0.854537	0.368965	3.285536	2.643364	3.553653	2.430999	2.917800	1.6531445

Table 3. Invariants of the unconstrained handwritten words (Aspect Moment Invariants)

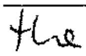
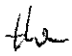
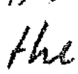
Word *the*	Feature1	Feature2	Feature3	Feature4	Feature5	Feature6	Feature7	Mean Error
	1.378416	3.330348	2.949914	3.799133	7.327741	5.771416	7.342160	-
	1.378345	4.122138	4.193430	2.779765	7.782701	4.852702	6.185584	0.7978564
	1.378469	2.519238	2.218042	3.256904	7.141300	4.532887	6.351607	0.6429695
	1.378309	4.147912	1.959868	2.479160	6.076452	4.800176	4.265392	1.2038552
	1.378167	3.306577	2.685310	2.672262	5.616238	4.332095	5.405377	0.9290145
	1.378883	3.131132	3.001449	3.179834	6.314957	5.412041	6.932878	0.3788511
	1.379270	3.255016	1.915080	2.986245	5.479281	5.113086	4.924116	0.9783917
	1.378513	3.508191	3.072298	3.517482	6.892869	5.876633	7.061254	0.2004242
	1.379202	4.293291	4.099594	3.354859	7.110530	5.512280	7.535612	0.4610688
	1.378795	2.800689	1.905141	3.313507	5.926298	5.323352	5.092858	0.8798922

Table 4. Invariants of the unconstrained handwritten words (Geometric Moment Invariants)

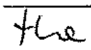
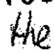
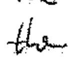
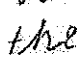
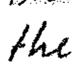
Word *the*	Feature1	Feature2	Feature3	Feature4	Feature5	Feature6	Feature7	Mean Error
	0.689803	1.862758	8.060365	8.609507	17.081588	9.636788	17.385284	-
	0.729140	2.167896	8.228204	7.983070	16.195252	9.310470	17.145600	0.3701555
	0.775449	3.905599	7.630235	8.723096	17.615020	11.093202	17.122960	0.7034822
	0.726918	2.145863	7.334965	7.886654	16.352358	9.113156	16.364512	0.5774438
	0.770915	3.168356	8.275756	8.121358	16.869100	10.760611	16.409028	0.6289738
	0.739427	2.279648	8.698134	8.295928	18.599248	9.438390	17.041442	0.4968231
	0.776605	4.108917	7.319694	8.479818	16.466241	10.546246	15.844305	0.8955864
	0.727845	2.153439	8.933858	8.733461	17.759125	9.824659	17.673009	0.3541861
	0.689208	1.855981	8.519663	8.355247	16.877720	9.347733	18.777193	0.3722517
	0.735155	2.229089	7.332707	9.880314	20.498557	11.470006	17.579072	1.1220175

Table 5. Mean errors for unconstrained handwritten words

Proposed Invariants	United Invariant	Aspect Invariants	Geometric Invariants
0.5140973	0.5741624	0.719147	0.6134355

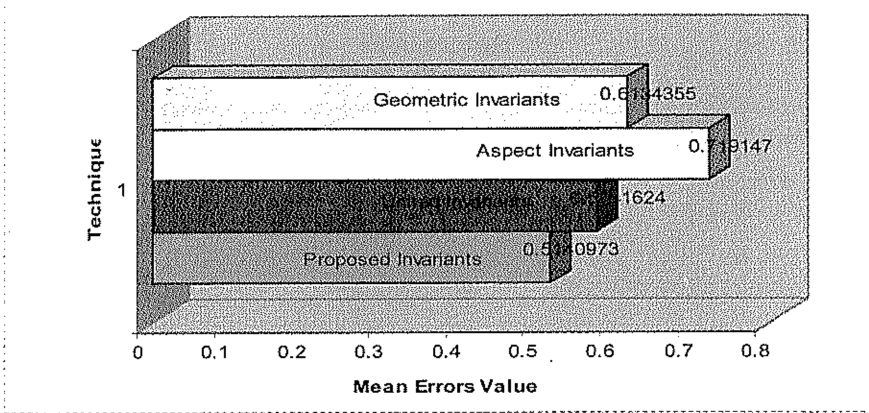


Figure 1. Mean errors for four techniques.

It is found that using the improved scale-invariants gives better invariants features compared to united invariants, aspect invariants and geometric invariants. This is due to the fact that using higher order invariants, the better the results in recognizing the features. It can be seen from the graphs of mean errors that using proposed method gives the lowest of mean error values compared to the other three methods.

5. Conclusion

This research proposed a method for constructing improved united moment scale-invariants for unconstrained handwritten words using higher order centralized invariants in the formulations. A proposed method is meant for images with unequal scaling. The results of the experiments have shown that the proposed method generates better invariants for unconstrained handwritten words within its class. Mean errors for the proposed method have shown the consistency of numerical values for the invariants of various shapes and style of writings. This is due to the fact that higher order moment contains finer details about the image. These invariants can be used as inputs to neural network or any classifier.

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