A Yaw Rate Tracking Control of Active Front Steering System Using Composite Nonlinear Feedback

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Abstract. In this paper, the composite nonlinear feedback (CNF) technique is applied for yaw tracking control of active front steering system with the objectives to improve the transient performance of yaw rate response. For lateral and yaw dynamics analysis, nonlinear and linear vehicle models are utilized as actual vehicle plant and for controller design respectively. The designed controller is evaluated using J-turn cornering manoeuvre condition in computer simulation. The simulation results demonstrate that the application of CNF for yaw rate tracking control improves the yaw stability and vehicle handling performances.

Keywords: composite nonlinear feedback, yaw rate control, active front steering, vehicle yaw stability.

1 Introduction

Yaw stability control system is one of the approaches utilized for lateral dynamics motion control of road vehicles. The vehicle yaw rate is an important variable to be controlled to ensure vehicle lateral stability. Thus, the main objective of yaw stability control is to ensure the proposed controller is able to track a desired yaw rate i.e. the actual response of yaw rate is close to a desired response that is generated by reference model. The yaw rate tracking control can be realized by implementing an active front steering (AFS) for vehicle handling improvement especially during low to midrange of lateral acceleration. In AFS system, the front wheel steer angle is a sum of steer angle commanded by the driver and corrective steer angle that is generated by the proposed controller.

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Various control strategies have been developed and implemented for vehicle yaw stability control based on AFS [1-8]. The yaw stability controller based on sliding mode and back stepping algorithm is designed to ensure the vehicle yaw rate follow its reference [1]. As implemented in [2], the model predictive control (MPC) technique is adapted for yaw stability control based on active differential braking to improve the vehicle stability during critical manoeuvre while in [3], the mixedsensitivity minimization technique is applied as feedback control loop to track the desired yaw rate as close as possible. To cater the uncertainties of vehicle parameters, the robust yaw control is designed based on second order sliding mode control (SOSM), internal mode control (IMC) and these control performances was compared each other as discussed in [4-6]. As reported in [7], the yaw rate control of active front steering is designed based on fuzzy logic control to improve the vehicle stability. Similarly, the yaw rate controller is developed based on fuzzy logic control in order to track the desired yaw rate as discussed in [8]. In point of view of tracking control, the transient response performance is vital. However, based on above review, the improvement of transient response performance in yaw rate tracking control is not well emphasized and an appropriate control technique should be proposed for this purpose.

The composite nonlinear feedback (CNF) control is one of nonlinear control technique that has been developed in last decade based on state feedback law. This technique was introduced in [9] for tracking control of 2nd order linear system had been improved for higher order MIMO linear system in [10]. It was further explored and extended for linear system with actuator nonlinearities in [11], general multivariable system with input saturation in [12], hard disk drive servo system and servo positioning system with disturbance in [13-16]. Recently, the CNF have been applied in vehicle dynamics control particularly for active suspension system in order to improve suspension deflection, velocity of car body, tire deflection, velocity of car wheel and body acceleration [17]. In principle, the use of CNF control could improve the performance of transient response based on variable damping ratio concept. The CNF control keep low damping ratio during transient and varied to high damping ratio as the output response close to the reference set point. To realize this concept, the CNF control that consists of linear and nonlinear feedback control law is designed in three important steps which will be discussed later. Therefore, based on previous studies and above discussion, an advantages of CNF control technique especially for improving transient response of tracking control is not yet been examined for vehicle yaw rate tracking control and should be further explored.

In this paper, the CNF control is introduced for vehicle yaw rate tracking control of AFS system. The lateral and yaw dynamics of nonlinear two track model is used as vehicle plant and linearized single track model is utilized for the controller analysis and design. To evaluate the performance of propose controller, the vehicle handling test of cornering manoeuvre are performed in computer simulation.

This paper is organized in 5 sections as the following: Section 1 provides an overview of yaw stability control, its existing control strategies and a review of the CNF control technique. In Section 2, the dynamics of a nonlinear vehicle model and a linear single track model are discussed. The theory of CNF control and design procedures are explained in Section 3. The simulation results and discussion are presented in Section 4. Finally, a conclusion and future works is presented in Section 5.

2 Vehicle Dynamic Models

In this section, dynamic equations of a nonlinear two track vehicle model and a linear single track model are presented and discussed. These two models are constructed for vehicle plant and controller design respectively, whose performance is analyzed using a computer simulation tool.

2.1 Nonlinear Two Track Model

A nonlinear two track model as shown in Figure 1 is used as the actual vehicle plant for controller evaluation. The nonlinear dynamics for lateral and yaw motion are describe as in equations (1) and (2) respectively;

$$\dot{\beta} = \frac{1}{mv} (\cos\beta \sum F_y - \sin\beta \sum F_y) - r$$
(1)

$$\dot{r} = \frac{1}{I_z} \Big[l_f (F_{y1} \cos \delta_f + F_{y2} \cos \delta_f + F_{x1} \sin \delta_f + F_{x2} \sin \delta_f) - l_r (F_{y3} + F_{y4}) + M_z \Big]$$
(2)

where the sum of longitudinal forces $\sum F_y$, sum of lateral forces $\sum F_x$ and yaw moment M_z in the above equations are given as follows;

$$\sum F_{y} = \cos \delta_{f} (F_{x1} + F_{x2}) - \sin \delta_{f} (F_{y1} + F_{y2})$$
(3)

$$\sum F_{y} = \sin \delta_{f} (F_{x1} + F_{x2}) + \cos \delta_{f} (F_{y1} + F_{y2})$$
(4)

$$M_{z} = \frac{d}{2} \left(F_{x1} \cos \delta_{f} - F_{x2} \cos \delta_{f} - F_{y1} \sin \delta_{f} + F_{y2} \sin \delta_{f} + F_{x3} - F_{x4} \right)$$
(5)

The vehicle parameters involved in equations (1) - (5) above are vehicle speed v, vehicle mass m, vehicle width track d, distance of front axle to center of gravity (CG) l_f and distance of rear axle to CG l_r . The front wheel steer angle δ_f is the input to the system while the nonlinear longitudinal tire forces F_{xi} and lateral tire forces F_{yi} can be described using Pacejka tire model. Notice that the vehicle speed v is always assumed constant when no braking and accelerating are involved.



Fig. 1. Two track model

2.2 Tire Model

In the equations (1) - (5) above, the longitudinal tire force F_{xi} and lateral tire force F_{yi} may exhibit nonlinear characteristics. The pure longitudinal and lateral tire forces during pure side slip can be described using the Pacejka tire model as described in the following equations (6) and (7) respectively

$$F_{xi} = D_{xi} \sin \left[C_{xi} \tan^{-1} (B_{xi} . \lambda_i - E_{xi} (B_{xi} . \lambda_i - \tan^{-1} (B_{xi} . \lambda_i))) \right]$$
(6)

$$F_{yi} = D_{yi} \sin \left[C_{yi} \tan^{-1}(B_{yi}.\alpha_f - E_{yi}(B_{yi}.\alpha_f - \tan^{-1}(B_{yi}.\alpha_f))) \right]$$
(7)

where the parameters $D_{xi}, D_{yi}, C_{xi}, C_{yi}, B_{xi}, B_{yi}, E_{xi}$ and E_{yi} are known as tire model parameters that depending on tire characteristics, road surface and vehicle conditions while λ_i and α_i are longitudinal wheel slip and tire sides slip angle respectively that given by the following equations

$$\lambda_i = \frac{R\omega_i - V_i}{V_i} \tag{8}$$

$$\alpha_1 = \alpha_2 = \delta_f - \tan^{-1} \left(\beta + \frac{l_f r}{v} \right)$$
(9)

$$\alpha_3 = \alpha_4 = \tan^{-1} \left(-\beta + \frac{l_r r}{v} \right) \tag{10}$$

where in equation (8), R is wheel radius, ω_i is wheel angular velocity and V_i is ground contact speed for each tire.

2.3 Linear Single Track Model

To design the controller for yaw rate tracking control of active front steering, a linear single track model as shown in Figure 2 is utilized. This model is linearized from the two track nonlinear vehicle model based on few main assumptions: tire force operates in linear region, very small of front steer angle δ_f and vehicle side slip β , vehicle

speed v is constant and two tires at front and rear axle are lumped into single tire at the centre line of vehicle [18].



Fig. 2. Single track model

The dynamics equation for the lateral and yaw motions are described as follows

$$mv(\beta + r) = F_{yf} + F_{yr} \tag{11}$$

$$I_z \dot{r} = l_f F_{yf} - l_r F_{yr} \tag{12}$$

As assumed above which tire forces are operates in linear region, front lateral tire force F_{yf} and rear lateral tire force F_{yr} are exhibit linear characteristics as described in the following equations

$$F_{yf} = C_f \alpha_f \tag{13}$$

$$F_{\rm vr} = C_r \alpha_r \tag{14}$$

where C_f and C_r are front and rear tire cornering stiffness respectively. For linear tire forces, sideslip angle of front and rear tire are given in the equation (15) and (16) respectively as follows

$$\alpha_f = \delta_f - \beta - \frac{l_f r}{v} \tag{15}$$

$$\alpha_r = -\beta + \frac{l_r r}{v} \tag{16}$$

By re-arrange and simplify the equations (11) - (16), the differential equations of sideslip and yaw rate variable can be simplified as linear state space model as shown in equation (17). Notice that the parameters are same as discussed in section 2.1 above.

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_f - C_r}{mv} & -1 + \frac{C_r l_r - C_f l_f}{mv^2} \\ \frac{C_r l_r - C_f l_f}{I_z} & \frac{-C_f l_f^2 - C_r l_r^2}{I_z v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_f}{mv} \\ \frac{C_f l_f}{I_z} \end{bmatrix} \delta_f$$

$$(17)$$

2.4 Yaw Rate Reference Model

The main objective in yaw rate tracking control of active front steering is to bring the actual response of vehicle yaw rate close to desired response. The desired yaw rate response is determined as a function of vehicle speed v and front wheel steer angle δ_f in steady state condition as follows

$$r_d = \frac{v}{l + k_u v^2} \cdot \delta_f \tag{18}$$

where k_u is known as cornering stability factor and define as follows

$$k_{u} = \frac{m(l_{r}C_{r} - l_{f}C_{f})}{(l_{f} + l_{r})C_{f}C_{r}}$$
(19)

However, due to lateral acceleration of the vehicle in g unit could not exceed the maximum road friction coefficient μ , the steady state value of yaw rate must be limited as express in the following equation

$$\left|r_{ss}\right| \le \frac{\mu g}{v} \tag{20}$$

3 Composite Nonlinear Feedback for Active Front Steering

In this section, the composite nonlinear feedback (CNF) control design procedures for yaw rate tracking control of active front steering (AFS) are presented. In general, The CNF control technique is applicable for systems with or without external disturbances. In this paper, the plant of vehicle dynamics is considered without an external disturbance and all states variable are assumed available for measurement. The following subsection will discuss the design procedures for CNF control which have been established in [11].

3.1 CNF Control Design

A linear time-invariant continuous system with input saturation is considered as follows

$$\dot{x} = Ax + Bsat(u) \quad x(0) = x_o$$

$$y = Cx$$
(21)

where $x \in \Re^n$, $u \in \Re$, $y \in \Re$ are the state variable, control input and controlled output respectively. *A*, *B* and *C* are constant system matrices with appropriate dimensions and *sat* : $\Re \to \Re$ represent actuator saturation defined as follows

$$sat(u) = \operatorname{sgn}(u)\min\{u_{\max}, |u|\}$$
(22)

with u_{max} is saturation level of the input. To design and apply the CNF control, the following assumptions of system matrices are considered;

- 1. (A,B) is controllable/stabilizable
- 2. (A,C) is observable/detectable
- 3. (A, B, C) is invertible and has no zero at s=0

To design the CNF for tracking control of step input without large overshoot and without adverse actuator saturations effects, the CNF control laws is design in three important steps. The details for step by step design process are discussed as follows

Step 1: design a linear feedback control law

$$u_L = Fx + Gr \tag{23}$$

where *r* is step input reference and *F* is feedback matric chosen such that A + BF is an asymptotically stable matrix and closed loop system $C(sI - A - BF)^{-1}B$ has certain desired properties such as small damping ratio. *G* is a scalar which given as follows

$$G = -[C(A + BF)^{-1}B]^{-1}$$
(24)

The selection of matric F is not unique where it can be determined using linear control design techniques such as pole placement assignment.

Step 2: design a nonlinear feedback control law

$$u_N = \rho(r, y)B'P(x - x_e) \tag{25}$$

where $\rho(r, y)$ is nonpositive function locally Lipschitz in *y* that used to change the damping ratio of closed loop system as output approaches the step command input. *P* > 0 is a solution of the following Lyapunov equation

$$(A+BF)'P+P(A+BF) = -W$$
(26)

for some given W > 0 and x_e is defined as

$$x_e \coloneqq G_e r \tag{27}$$

$$G_e \coloneqq -(A + BF)^{-1}BG \tag{28}$$

The nonlinear function $\rho(r, y)$ is not unique. To adapt the variation of tracking target, the nonlinear function $\rho(r, y)$ in [19,20] is utilized as follows

$$p(r, y) = -\gamma e^{-\varphi \varphi_o |y-r|}$$
⁽²⁹⁾

where

$$\varphi_o = \begin{cases} \frac{1}{|y_o - r|}, & y_o \neq r \\ 1, & y_o = r \end{cases}$$
(30)

Step 3: complete CNF control law

Both control laws in step 1 and 2 are combined to form the complete CNF control law as follows

$$u = u_{L} + u_{N} = Fx + Gr + \rho(r, y)B'P(x - x_{e})$$
(31)

3.2 Tuning Parameters of Nonlinear Function

The selection of design parameters of nonlinear function is essential for CNF control design so that the performance of closed loop system is improved as the controlled output approaches the reference set point. In this paper, the parameter γ and φ of nonlinear function are tuned according to the method as proposed in [20] as follows

- 1. choose the desired steady state damping ratio ξ_{ss}
- 2. determine γ by letting the steady state system has a damping ratio of ξ_{ss}
- 3. determine an optimal φ by solving minimization some appreciable criterions of integral of absolute error (IAE) and integral of time-multiplied absolute value of error (ITAE) which given by equations

$$\min_{\varphi} \int_{0}^{\infty} e^{t} dt \quad \text{or} \quad \min_{\varphi} \int_{0}^{\infty} t^{2} e^{t} dt \tag{32}$$

where e = y - r is tracking error.

3.3 Active Front Steering Based on CNF Control

The AFS based on the CNF control technique for yaw rate tracking control is illustrated in Figure 3. The front wheel steer angle is a sum of steer angle commanded by the driver, δ_f and corrective steer angle, δ_c that generated by the CNF control. Based on vehicle parameters and linear single track model as discussed previously, the parameters of CNF control are obtained as follows

$$F = \begin{bmatrix} 0.5 & -0.05 \end{bmatrix}, \quad G = 0.2321, \quad P = \begin{bmatrix} 0.8224 & 0.0562 \\ 0.0562 & 0.1535 \end{bmatrix}, \quad G_e = \begin{bmatrix} -0.1711 \\ 1 \end{bmatrix}, \quad \gamma = 0.2, \quad \varphi = 0.03$$



Fig. 3. Active front steering based CNF control

4 Simulation Results

The computer simulation of AFS based on the CNF control for yaw rate tracking control is conducted in Matlab/Simulink. The vehicle parameters used are taken from [21] as follows;

$$m = 1704.7kg \ , \ I_z = 3048.1kgm^2 \ , \ l_f = 1.035m \ , \ l_r = 1.655m \ , \ C_f = 105,800N \ ,$$

$$C_r = 79,000N \ , \ d = 1.54m$$

The road surface adhesion coefficient is assumed for dry road $\mu = 1$ and vehicle speed v = 100 km/h is assumed constant. To evaluate the performance of propose CNF control for AFS, a J-turn manoeuvre test is conducted in order to analyse the response of yaw rate tracking control. J-turn cornering manoeuvre which similar to a step input is a simple handling test to evaluate the transient and steady state behaviour of yaw stability control. The response obtained is compared to uncontrolled and classical PID controller. Figure 4 shows the input of 1° steer angle that generate 0.32g lateral acceleration which is considered an appropriate level for AFS. The response of yaw rate and yaw rate tracking error are shown in Figures 5 and 6 respectively.



Fig. 4. J-turn steer input at 1° of steer angle







Fig. 6. Yaw rate tracking error of J-turn at 1° steer angle

controllers	amplitude	% OS	t_r (t)	$t_{s}(t)$
CNF	Nil	0	0.0524	0.107
PID	7.57	7.10	0.137	0.46
uncontrolled	7.39	4.53	0.299	1.03

Table 1. Transient response parameters of J-turn manoeuvre

From the Figures 5 and 6, it is observed that the CNF control could track the yaw rate reference with fast transient response compared to PID controller and uncontrolled vehicle. The performances of transient response parameters of this valuation are tabulated in Table 1.

The simulation of AFS based on the CNF control technique has been presented. From Table 1, it is observed that the yaw rate response with the CNF control technique have no maximum peak of amplitude i.e. 0% overshoot compared to PID controller and uncontrolled response that obtained 7.1% and 4.53% overshoot respectively. In term of rise time t_r , the CNF control performed better with 0.0524s which is obviously fastest than PID controller i.e. 0.137s and 0.299s of uncontrolled response. The CNF control also achieves fastest sampling time t_s i.e. 0.107s compared to 0.46s of PID controller and 1.03s of uncontrolled response.

5 Conclusion

A new technique for vehicle yaw rate tracking control of active front steering is proposed based on the composite nonlinear feedback (CNF) control scheme. From the results obtained, it is shown that the CNF control is capable to achieve fast response of yaw rate tracking control where it is able to track the desired yaw rate with minimum overshoot and fast settling time for a J-turn cornering manoeuvre. As a conclusion, the CNF control is able to improve the transient response of vehicle yaw rate. For future works, the CNF control technique will be evaluated with other cornering manoeuvre. It will improve to cater external disturbances such as crosswind and uncertainties of vehicle parameters. Co-simulations with vehicle dynamics software such as CarSim will be conducted to validate the proposed controller.

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