Estimator and Controller Design for a Didactic Liquid Level system by System Identification Approach

M. Nizam Kamarudin, Sahazati Md. Rozali

Abstract— This manuscript is a piece of report on the identification of Didactic Liquid Level Systems. The term didactic is used as to enlighten the purpose of proto-typing liquid level system for educational use. A liquid level system with unknown mathematical characteristic is identified using ARX (Auto-Regressive with Exogenous Input) model. The determination of such system, on the basis of experimental data is conducted to obtain the discrete transfer function as well as state space representation of it. The identification process which exploits the advantage of linear parametric ARX gives beneficial information such as correlation, best fit and poles-zeros location. The Linear Quadratic Regulator (LQR) is tested for such liquid level system before the estimator is designed. Literal analysis of the estimator performance is conducted as a preliminary insight to the next investigation on the self tuning Proportional – Integral – derivative (PID) algorithm.

Keywords: System identification, Estimator, Liquid Level, PID

I. INTRODUCTION

IT is quite often when the actual systems are about to be designed, the mathematical model of such systems are unknown. As the mathematical model is a description of the behavior of a system using mathematics, the knowledge of it is vital when good controller design is to be considered [1, 2]. In the identification process, a mathematical relationship or the governing dynamics between the input and the output of the system should be known. The underlying principle and knowledge of the system should be investigated to comprehend the occurrence of nonlinearity in the system dynamics. Apart from nonlinearities, there may be a consequence of unknown parameters which hinders the objective to obtain a complete detail model of a process available for control purpose. There are factors that abstained researchers to use conventional control theory; where many systems are nonlinear and contain unknown parameters. Those parameters may not be estimated accurately if reliable experimental data is absent.

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II. IDENTIFICATION PROCESS

In this manuscript, a mathematical model of Liquid Level System shown in figure 1 is obtained by performing system identification process. The general model identification process is shown in figure 2. The terms u(k) and y(k) are the input and output signals respectively measured at discrete-time k. Through system identification process, all important information such as best fit, residual analysis, correlation, and pole-zero location are obtained [3]. As such, the mathematical model and estimated model of Liquid Level System is acquired. As the system suffer from chaotic behavior and bifurcation phenomena which come from the dynamics of the motor pump, solenoid valve and sensors, the use of identification tool in MATLAB is beneficial.
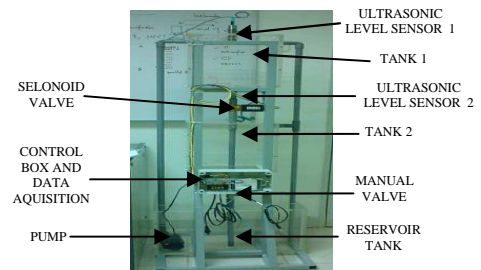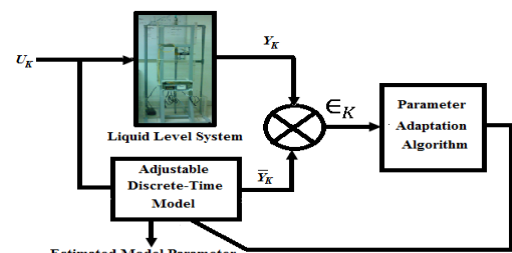

Figure 1: Liquid Level System.

Figure 2: Liquid Level system identification process.

A few structures of parametric model such as ARX model, ARMAX (Auto-Regressive Moving Average with Exogenous Input) model, OE (output-error) model and BJ (Box-Jenkins) model can be used as liquid level model structure. ARX model serves the basic structure as this structure ignores the moving average or the error dynamics of the system. As such, ARX model is used and the general model with appearance of error dynamics e(k) is represented by the difference equation (1). Lowercase 'd' is time delay which represent the difference between u(k) and y(k). The general ARX model is shown in figure 3.

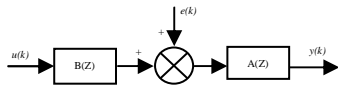$$A(Z^{-1})y(k) = B(Z^{-1})u(k-d) + e(k) \quad \cdots\cdots\cdots(1)$$

Figure 3: ARX Model Structure.

III. GENERATING INPUT SIGNAL

Liquid level systems are rich with non-linearities and disturbances. As such, building the dynamic model of such system is tedious without any known parameters presented. The used of system identification methods are worthwhile for the parameter estimation of such systems. The process begins with investigating the input-output relation of the system. Generating input signal is important and crucial for the good identification process. Good parameters identification requires the usage of input signal that are rich in frequencies. As such, many researchers proposed the sinusoidal signals which shown in the equation (2)

$$u(k) = V_{dc} + \sum_{i=1}^{p} a_i \cos\omega_i t_s k \quad \cdots\cdots\cdots(2)$$

In equation (2), a_i is amplitude; ω_i is frequency and t_s for sampling time. Note that P is determined by the number of system parameter, n that needs to be identified. The generated input signal u(k) with three different frequencies is shown in Figure 4. The input signal u(k) is finite power signal as it does not decay but it having infinite energy.
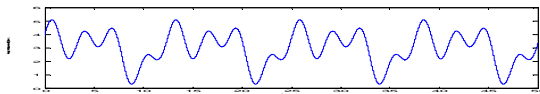

Figure 4: Generated input signal of three different frequencies.

Pseudo-Random Binary Sequences (PRBS) that generating the sequence of square pulses that modulated by different width can also be used. Paper [4] provides idea by exploiting the use of PRBS while studying the scheme to fault detection and isolation using adaptive ARX model. However, this method has drawback as it is bit difficult to be implemented as compared to the sinusoidal input.

IV. PLANT MODEL

The set of input-output data produced by the sinusoidal waveform of equation (2) is used and been evaluated and estimated by the identification tool of SIMULINK / MATLAB. The relationship of a measured and simulated input-output data is shown in figure 5. The model output yields 86.93 best fit. Figure 6 shows the plot of frequency response for the identified liquid level system. By looking at the plot, it is acceptable that the system is the best model of the identification process, plus the system is stable in nature. However, the plot shows that the stability need to be improved as the phase plot has not meet the required -180° when the magnitude plot intersect 0dB line. Step response in figure 7 confirms that the system suffers around 50% steady state error due to the step change. Whereas figure 8 depicts how the liquid level system may response to the sinusoidal input voltage. The previous figure 3 depicts the structure of ARX model in which the identification process of liquid

level system took place. The identification tool in MATLAB generates discrete time polynomial shown in equation (3). As ARX model ignore error dynamic of the plant, equation (4) can be simplified further. Equation (4) provides information on the mathematical model of the liquid level system in discrete time domain. Whereas equation (5) shows the transfer function of a liquid level system in continuous frequency domain. Equation (5) provides information on the dynamics of the system. The system is known as a minimum phase second order system.

$$A(Z^{-1})y(k) = B(Z^{-1})u(k) + e(k) \quad \cdots\cdots\cdots(3)$$
$$\therefore A(Z^{-1}) = 1 - 1.418Z^{-1} + 0.4948Z^{-2}$$
$$\therefore B(Z^{-1}) = 0.07887Z^{-1} + 0.03591Z^{-2}$$
$$G(Z^{-1}) = \frac{B(Z^{-1})}{A(Z^{-1})}$$
$$G(Z^{-1}) = \frac{0.07887Z^{-1} + 0.03591Z^{-2}}{1 - 1.418Z^{-1} + 0.4948Z^{-2}}$$
$$G(Z) = \frac{0.07887Z + 0.03591Z}{Z^2 - 1.418Z + 0.4948} \quad \cdots\cdots\cdots(4)$$
$$G(S) = \frac{104.8S + 4.032 \times 10^6}{S^2 + 3518S + 2.698 \times 10^6} \quad \cdots\cdots\cdots(5)$$
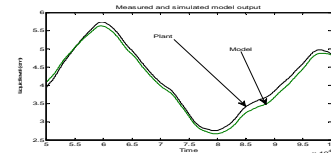

Figure 5: Measured and simulated model output yields best fit of 86.93
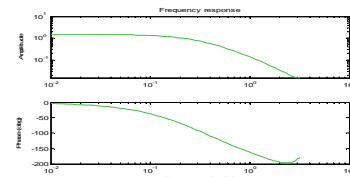

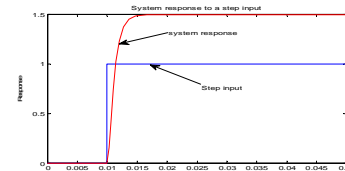Figure 6: Frequency response

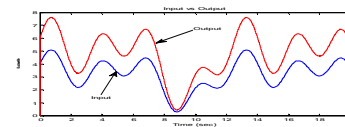

Figure 7: Response to a step change.

Figure 8: Response to a sinusoidal input

V. STATE FEEDBACK CONTROL

State space is a technique of using the states to describe the system and hence represent the state into a space. For instance, it is difficult to study theoretically (using geometric and / or analytic methods) the nth order system which represented by $X^{(n)} = f(x, x', x'', \ldots\ldots x^{(n-1)})$. As such, computer aided programs are normally been used as they are better with 1st order ordinary differential equation. For example, equation (6) represents any particular system.

248

The derived equation (7) which is in matrix form results from the manipulation of equation (6). Note that the liquid level system uses sensors; as such, equation (8) may result. In general, the linear time invariant system of the plant can be described as in equation (9) and (10). Note that X is an n x 1 state vector, U is an q x 1 input vector, Y is an p x 1 output vector, A is an n x n state matrix, B is an n x q input matrix, C is an p x n output matrix and D is an p x m feed forward matrix.

$$mx = F - Bx - kx \;\cdots\cdots (6)$$
$$where$$
$$x_1 = x$$
$$x_2 = \dot{x}$$

$$\left.\begin{array}{l} \dot{x}_1 = \dot{x} = x_2 \\ \dot{x}_2 = \ddot{x} = \frac{1}{m}(F - Bx_2 - kx_1) \end{array}\right\} \rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{B}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{F}{m} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{B}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}F$$

$$\dot{X} = AX + BU \;\cdots\cdots (7)$$

$$y = x \rightarrow [1 \;\; 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow Y = CX + DU \;\cdots\cdots (8)$$

$$U = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_q \end{bmatrix}, \qquad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_p \end{bmatrix}$$

$$\dot{X}(t) = AX(t) + BU(t) \;\cdots\cdots (9)$$

$$Y(t) = CX(t) + DU(t) \;\cdots\cdots (10)$$

The state vector describes the state of the system, which is a complete summary of the system at a particular point of time. If the current state of the system and the future input signals are known then it is possible to define the future states and outputs of the system. The uses of MATLAB command produce the matrix A, B, C and D of the liquid level system as in equation (11). The state space model of such system is shown in figure 9.

$$\left.\begin{array}{c} \therefore A = \begin{bmatrix} -3518 & -2.698e + 006 \\ 1 & 0 \end{bmatrix} \\ \therefore B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \therefore C = [104.8 \;\; 4.032e + 006 \;] \\ \therefore D = 0 \end{array}\right\} \;(11)$$

From matrix B of the liquid level system, it is apparent that only one state can be controlled. However, from the knowledge of matrix C, it is obvious that both states can be observed. However, test must be developed to find the controllability matrix (M$_C$) and observability matrix (M$_O$) of the system.

Observability matrix, $M_O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$, if the rank of this

matrix is less than n then the system is unobservable. Controllability matrix is defined as,

$$M_C = [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B \;],$$

If the rank of this matrix is less than n, then the system is uncontrollable. With the use of MATLAB command, the rank of controllability matrix and observability matrix are obtained as '2'. The liquid level system are both controllable and observable and therefore a minimal realization system. By denoted the systems dynamics as G(S), the state feedback control of the liquid level system can is introduced in figure 10. The derivation of closed loop eigenvalues A_{CL} = (A-BK) in equation (12) is then provided. By applying pole placement method, the speed and stability is then depending on the value of gain K.
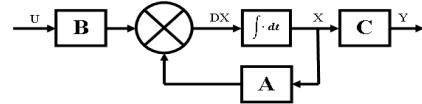

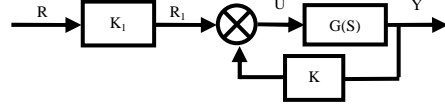Figure 9: General state space model of liquid level system.

Figure 10: State feedback control.

The input signal, $U(t) = K_1R(t) - KX(t)$

$$\dot{X}(t) = AX(t) + BU(t)$$
$$\dot{X}(t) = AX(t) + B(K_1R(t) - KX(t))$$
$$\dot{X}(t) = (A - BK)X(t) + BK_1R(t)$$
$$Y(t) = CX(t) + DU(t) = (C - DK)X(t) + DK_1R(t)$$
$$\left.\begin{array}{l} \dot{X}(t) = A_{CL}X(t) + B_{CL}R(t) \\ Y(t) = C_{CL}X(t) + D_{CL}U(t) \end{array}\right\} \cdots\cdots (12)$$

VI. ESTIMATOR DESIGN

Prior to the estimator design, the Linear Quadratic Regulator (LQR) is tested to the system beforehand. In practice, higher gain may be required to place poles of the liquid level system at the desired location. This may force the system to be a lot faster. However, the required control signal is increased by more than 100% which is impossible to be achieved by the control input of the liquid level pump. As such, a compromise between speed and energy must be taking into account while placing poles of the system. For the input *U=-KX*, LQR will minimize energy as stated in equation (13). *Q* and *R* are positive definite matrices where *Q* determines the important of the error and R determines the importance of the energy. The derivation of the *Reduced Riccati Equation* in equation (14) is then introduced. The design procedure for LQR controller begins by finding the optimum P and optimum *K*.

$$J = \int_0^\infty (X^TQX + U^TRU)\,dt \cdots\cdots (13)$$

$$J = \int_0^\infty (X^TQX + (-KX)^TR(-KX))\,dt$$

$$J = \int_0^\infty X^T(Q + K^TRK)X\,dt$$

$$\gg X^T(Q + K^TRK)X = -\frac{d}{dt}(X^TPK)$$

$$\gg \dot{X} = (A - BK)X \;\; and \;\; \dot{X}^T = X^T(A - BK)^T$$

$$Q + K^TRK = (A - BK)^TP + P(A - BK) \rightarrow for \; X = 1$$
$$For \; A - BK \; stable, K = R^{-1}B^TP,$$

$$\therefore A^TP + PA - PBR^{-1}B^TP + Q = 0 \cdots\cdots (14)$$

LQR function in matlab m-file can be used to determine the value of vector K which determines the feedback control law. This is done by choosing two parameter values, namely input *R=1* and matrix *Q = C' x C*. The controller can be tuned by changing the nonzero x and y elements in Q matrix which is done in m-file code. By computing in MATLAB, the values of matrix *K* are obtained as [1.0e-003*0.1421 1.0e-003*0.0002]. In order to reduce the steady state error of the system output, a constant gain should be added after

the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, and multiplied by the chosen gain K, and use a new value as the reference for computing the input. This constant gain can be found using the user-defined function which can be used in m-file code. The value of constant gain is -70.7107. The use of state feedback control and LQR is governed by assumption that all the states, X are measurable. Nevertheless, the actual plant only reveals the output Y and not X. As the perfect model of the liquid level system is available, the states and output of the system can be estimated, and denoted as $\tilde{X}$ and $\tilde{Y}$ respectively. As such, the estimator can be designed to estimate and observe the states. The basic estimator block diagram is shown in figure 11. Whereas figure 12 shows the estimating technique of a liquid level system. The error between the estimated and real state hence can be derived. The state space model of the estimator is shown in equation (16). The error dynamics hence is rewrite as in equation (17). The value of G can be obtained by placing the poles of the error dynamics at least 10 times greater than the poles of the feedback system. The value of G is recorded as [43.0229 0.0067].

$$E(t) = X(t) - \tilde{X}(t) \rightarrow \dot{E}(t) = \dot{X}(t) - \dot{\tilde{X}}(t)$$
$$AX(t) + BU(t) - A\tilde{X}(t) - BU(t)$$
$$\rightarrow \dot{E}(t) = AE(t) \cdots\cdots (15)$$
$$\dot{\tilde{X}}(t) = F\tilde{X}(t) + GY(t) + BU(t) \cdots\cdots (16)$$
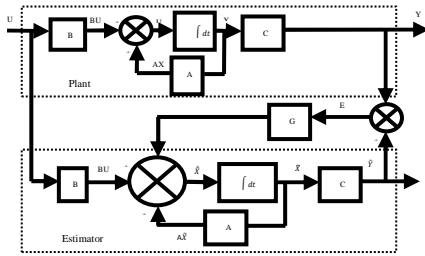$$\dot{E}(t) = (A - GC)X(t) - (A - GC)\tilde{X}(t) = (A - GC)E(t) \cdots\cdots (17)$$

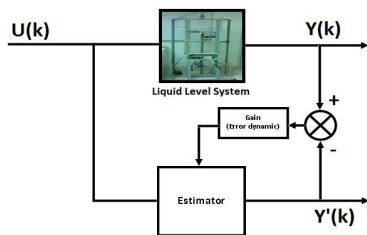Figure 11: Basic estimator block diagram.

Figure 12: Estimating technique of liquid level system.

VII. RESULT – ESTIMATOR AND PID

Figure 13 shows both the output of estimator and actual plant. There is a slight difference in the output of estimator and actual plant. This is due to the different initial condition. The estimator is considered good as it converge to zero very fast at particular point of initial condition. This is proved by the dynamics of error shown in figure 14. The liquid level system is then tested with the PID control. PID is the best-known controller in industries. This scheme offers simple structure as well as robust performance. In the design process, further increase of K_p will result in the closed loop system becoming unstable. Therefore, additional integral is to improve steady state accuracy without introducing

instability. Integrator gives the system one pole at origin, hence reducing steady state error as a steady-state error of a system depends upon the number of poles at the origin of the closed loop characteristic function [5]. The tuning of PID parameters called K_p, K_i and K_d are based on Open Loop Zigler-Nichols tuning algorithm. The dynamics of PID is tabulated in Table 1. The compensator is represented in equation (16). Figure 15 shows that the system has good transient and less steady state error due to the step change.
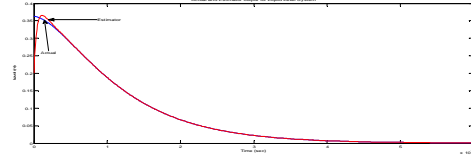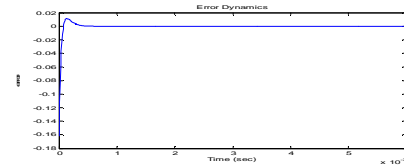

Figure 13: Estimator vs plant output.

Figure 14: Error dynamics of the estimator.

Table 1: Dynamics of PID

Type	Location	Damping	Frequency
Real Zero	-14809	1	14809
Real Zero	-547	1	547
Integrator	0	-1	0

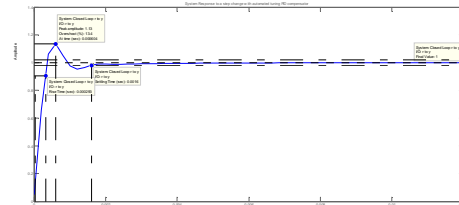$$C(S) = 4200.8 \times \frac{(1 + 6.8 \times 10^{-5} S)(1 + 0.0018S)}{S} \cdots (16)$$

Figure 15: Step response of the improved system with PID controller.

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