Markovian Approach Enhancement to Simplify

Optimal Mean Estimation

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Abstract

The determination of process mean is important in industries especially items that governed by laws and regulations on net content labeling. Thus, the economic selection of process targeting mainly the optimum process mean is critically significant since it will directly affected the quality characteristic of the item. Depending on the value of quality characteristic, an item can be reworked, scrapped or accepted by the system which is successfully transform to the finishing product by using the Markovian model. By assuming the quality characteristic is normally distributed, the optimum process mean is obtained via probability of the item being rework, scrap and accepted. In this paper, we present the analysis of selecting the process mean by referring to drink bottling production process. By varying the rework and scrap cost, the analysis shows the sensitivity of the Markov approach to determine process mean which maximizes the expected profit per item. Then an improvement of determining the optimal mean estimation is proposed to simplify the iteration process.

Keywords: Markovian approach; optimal mean

1 Introduction

Process targeting has received a considerable attention from researchers as well as practitioners. It is critically important to industries which are governed by laws and regulations on the net content labelling [1]. One of the ways to improve quality is to ensure that the product deviates little from the customer defined process target of quality characteristics. Setting the optimum target value (or process mean) is a classical problem in quality control. The improper selection of the process mean affects defect rate, material cost, scrap or rework cost and possible losses due to the deviation of the product performance from the customer's and/or producer's target [2]. Every product is inspected to determine whether its quality characteristic satisfies the specification limits [3].

One of the most important decision making problems encountered in a wide variety of industrial process is the determination of optimum process target (mean). The selection of optimum process mean will directly affect the process defective rate, production cost, rework cost, scrap cost and the cost of use. According to [4], if the process mean approaches the target value and the process standard deviation approaches zero, then the process is achieved an optimum control. Therefore, determination of optimal value mean is important to ensure the manufacturer will not facing loss due to penalty cost while the customer do not need to pay the excessive quantity of the product.

2 Literature Review

There are considerable attentions referring to the study of economic selection of process mean. The first real attempt to deal with this problem was in [5] which considered the problem of finding the optimal process mean for a canning process when both upper and lower control limits are specified. [6] studied the same problem as [5] except that only the lower limit was specified, and used a trial and error procedure to tabulate a set of values for the specified lower limit. Furthermore, he assumes undersized and oversized items are reprocessed at a fixed cost.

[4] presented the quadratic quality loss function for redefining the product quality. According to the author, product quality is the society's loss when the product is sold to the customer. [7] adopted the quadratic quality loss function for evaluating the quality cost of a product for two different markets and obtained the optimum process mean based on maximizing the expected profit per item. Most of the works have addressed the 100% screening in the filling or canning process. However, the 100% inspection policy cannot be executed in some situations due to several constraints that also faced by SME. Hence, one needs to consider the use of sampling plan for deciding the quality of a lot. Usually, the non-conforming items in the sample of accepted lot are replaced by conforming

ones. [8] considered the case of acceptance sampling where the rejection criterion was based on the sample mean. [9] developed an algorithm for finding optimum target values for two machines in series when sampling is used.

A great majority of process target models in the literature were derived assuming a single-stage production system, except for [10] and [11]. [10] proposed a model considering a manufacturing system with two stages in series to find the optimum target values with a lower specification limit and also application of 100% inspection policy. According to the author, the rework items are sent back to the first stage. [11] introduced a Markovian approach for formulating the model in multi-stage serial production system by assuming that the quality characteristic is normally distributed with both-sided specification limits is known. The normal distribution is usually used in describing the characteristic of industrial product [12]. [13] extended [11] model by considering the cutting can into the right diameter, thick and length. In this paper the Markovian approach will be implemented as the preliminary analysis for determining the optimum process mean which fulfil the maximum expected profit [11].

3 Model Formulation

A transition probability matrix P to describe transitions among three states is proposed for the model formulation [14]. State 1 indicates processing state at first stage production. State 2 indicates that the item has been processed successfully and transform into finishing product. While Stage 3 represents that the item has been scrapped. Figure 1 show a single-stage production system which is used as the preliminary study for this research.
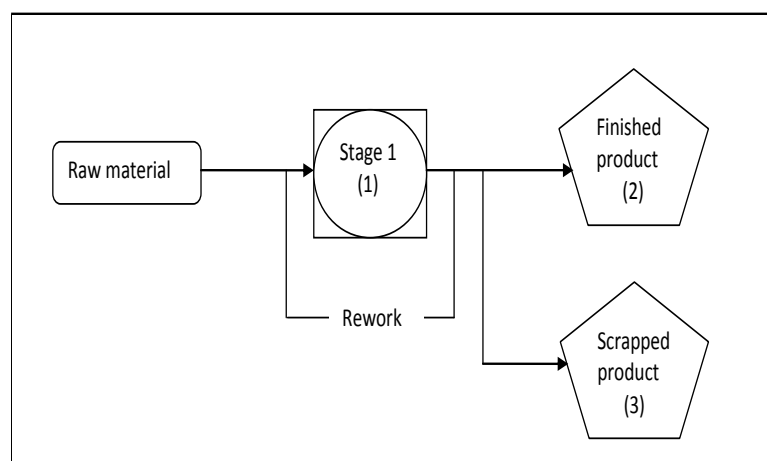

Figure 1. Single-stage production system process flow

The single-step transition probability matrix can be expressed in equation (1).

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

p_{12} is the probability of an item being accepted, and p_{13} is the probability of an item being scrapped. By assuming that the quality characteristic is normally distributed, these probabilities can be expressed as in Figure 2.
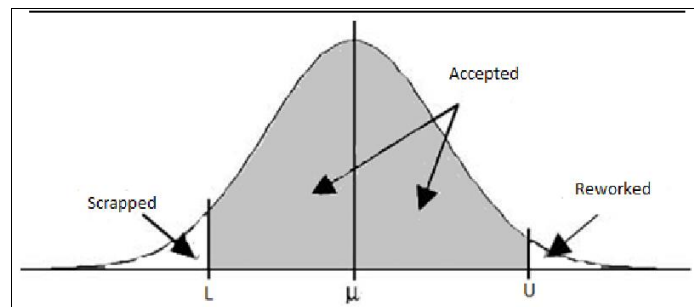

Figure 2. Normal Distribution graph for accepted, reworked and scrapped region

The details formulation of P_{11}, P_{12} and P_{13} is as follows.

$$p_{11} = \int_{U}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2}$$

$$p_{12} = \int_{L}^{U} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{3}$$

$$p_{13} = \int_{-\infty}^{L} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{4}$$

Therefore, the expected profit per item can be expressed as follows:

$$EPR = SP.f_{12} - PC - SC.f_{13} - RC.(m_{11} - 1) \tag{5}$$

Where SP is represent the selling price per item, PC is processing cost per item and SC and RC is scrapped cost and rework cost per item, respectively. f_{12} represent the long-term probability of an item has been processed successfully at Stage 1 and being accepted. While f_{13} represent the long-term probability of the item is being scrapped as the item is rejected from Stage 1. The element of m_{11} is represent the expected number of times in the long run that the transient state 1 is occupied before accepted or scrapped. The m_{11} element is obtained by following fundamental matrix M:

$$M = (I - p_{11})^{-1} = m_{11} = \frac{1}{1-p_{11}} \tag{6}$$

Where I is the identity matrix. The long-run probability matrix, F, can be expressed as follows:

$$F = M \times R = 1 \begin{bmatrix} 2 & 3 \\ \dfrac{p_{12}}{1-p_{11}} & \dfrac{p_{13}}{1-p_{11}} \end{bmatrix} \tag{7}$$

Substituting for f_{13} and m_{11}, the expected profit equation (5) can be written as follows:

$$EPR = SP\left(1 - \frac{p_{13}}{1-p_{11}}\right) - PC - SC\left(\frac{p_{13}}{1-p_{11}}\right) - RC\left(\frac{p_{11}}{1-p_{11}}\right) \tag{8}$$

From this equation, one would like to find the value of optimum mean that maximizes the expected profit.

4 Preliminary Analysis

To perform the preliminary analysis, one of the small and medium enterprise (SME) companies is selected as a case study. The selected parameter is as follows. The selling price per item, SP is 1.20, processing cost per item, PC=0.50, rework cost, RC=0.30, scrapped cost, SC=0.55, standard deviation is set to 1 while the lower and upper limit is set to 28 and 31, respectively.

Graph in Figure 3 show the expected profit was optimum at mean 30.09 with maximum EPR of 0.6134.
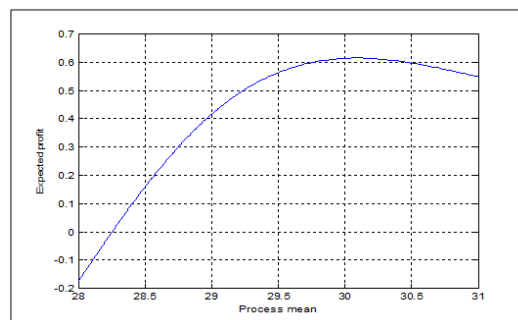

Figure 3. Expected profit versus process mean

The analysis is following the behaviour of the optimum process mean and the optimum expected profit with the variation of the scrap and rework costs for the production system [15]. The result is shown in Table 1.

Table 1: Optimum process mean and the optimum expected profit

Cost parameter	Case#	Parameter value	Optimum process mean	Optimum expected profit
Scrap Cost	1	0.45	30.05	0.6153
	2	0.50	30.06	0.6143
	3	0.55	30.09	0.6134
	4	0.60	30.10	0.6125
Rework Cost	5	0.20	30.22	0.6333
	6	0.25	30.15	0.6299
	7	0.30	30.09	0.6134
	8	0.35	30.04	0.6047

5 Simplification of Optimal Mean Estimation

According to above mentioned mean estimation, obviously the iteration process has taken a longer time to get the optimum process mean. In order to simplify the iteration process which aimed at reducing the time taken, we proposed a statistical based approach which using quartile method. The next section will discuss on the result of the approach.

6 Result

Table 2 shows the value of the upper limit to the lower limit where the lower limit is based on the difference from 1% to 10% of the upper limit. This is to observe the determination of the optimal value of trends in water filled in the bottles available in the market. Therefore, we can identify exactly where is the optimal value by using the statistical method in facilitating the search. With the trend, the time can be saved, where instead of searching from the lower limit to the upper limit, once the trend is identified, the search will be started somewhere in between the range. This will make the searching become faster and easier.

Table 2: Decomposition Value of Upper Limit to Lower Limit

Upper Limit	Lower Limit									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
100	99	98	97	96	95	94	93	92	91	90
125	123.75	122.5	121.25	120	118.75	117.5	116.25	115	113.75	112.5
280	277.2	274.4	271.6	268.8	266	263.2	260.4	257.6	254.8	252
310	306.9	303.8	300.7	297.6	294.5	291.4	288.3	285.2	282.1	279
500	495	490	485	480	475	470	465	460	455	450
1500	1485	1470	1455	1440	1425	1410	1395	1380	1365	1350
5500	5445	5390	5335	5280	5225	5170	5115	5060	5005	4950
6000	5940	5880	5820	5760	5700	5640	5580	5520	5460	5400

The normal distribution graph as shown in Figure 4 is aimed to differentiate the boundary for each statistical value.
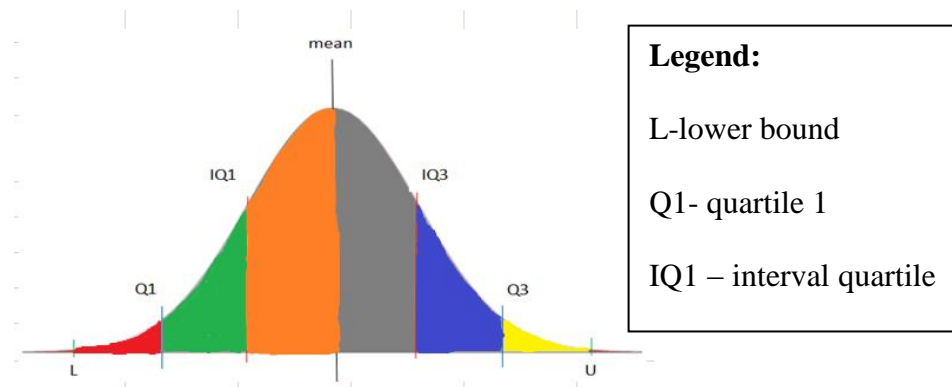

Figure 4: Normal distribution for each statistical boundary area

Table 3 shows the trend of optimal value using quartile based statistical methods, taking into account the 1% to 10% differences between upper limit and lower limit. From the optimal value, the trend of the optimal value is analyzed. Different colors in this table are based on the Figure 4.

Table 3: Trend of Optimal value according to quartile based statistical method

Upper Limit	Lower Limit									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
100	O=UB	O=UB	IQ3<O<Q3	O=IQ3	M<O<IQ3	M<O<IQ3	M<O<IQ3	IQ1<O<M	IQ1<O<M	IQ1<O<M
125	O=UB	Q3<O<UB	IQ3<O<Q3	M<O<IQ3	M<O<IQ3	M<O<IQ3	IQ1<O<M	IQ1<O<M	Q1<O<IQ1	Q1<O<IQ1
280	Q3<O<UB	M<O<IQ3	IQ1<O<M	Q1<O<IQ1	Q1<O<IQ1	Q1<O<IQ1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1
310	Q3<O<UB	M<O<IQ3	IQ1<O<M	Q1<O<IQ1	Q1<O<IQ1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1
500	M<O<IQ3	IQ1<O<M	Q1<O<IQ1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1
1500	Q1<O<IQ1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1
5500	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1
6000	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1	LB<O<Q1

From Table 3, it can be seen that:

1. For the upper limit of 125, 500, 1500, 5500 and 6000, the trend of the optimal values are following the statistical boundary which have been set according to the normal distribution graph. The order of the boundary start with the UB followed by Q3, IQ3, mean, IQ1, Q1 and LB.

2. Next, it can be identified that the optimal value is located between LB and Q1. However, for the upper limit of 100 and 125, it can be identified that the optimal value between LB and Q1 is above 10%. For the upper limit 100, the beginning of the optimal value between LB and Q1 is located at 17% difference between the upper limit and lower limit. While for 125, the beginning of the optimal boundary described above is started at 18% of the difference between the upper limit and lower limit.

3. For the upper limit of 100, 280 and 310, the optimal value is not in the range of UB and IQ3. In addition, the optimal value is located in all statistical boundary value that has been set.

According to the survey, most of the manufacturers will set 5% of differentiation between upper limit and lower limit. From the Table 2 and 3, based on the analysis from the difference, the determination of the optimal value by using statistical methods can be applied successfully. For the upper limit of 100 and 125, the optimal value is located between the mean and IQ3. While for 280 and 310,

the optimal value is between Q1 and IQ1. For the amount of bottle that larger than 310, the determination of the optimal value is between LB and IQ1. Thus, it can be concluded that the determination of the optimal value is simplified by using a statistical method.

7 Conclusion and Reccomendation

In this paper, we present the analysis by using the Markovian approach to find the optimum process mean that is maximizes the expected profit. From the analysis, it is observed that when either the scrap cost or rework cost is increase, the expected profit will be decrease. Due to this condition, the manufacturer will attempt to reduce the rework or scrap items. By determining the optimum process mean, the manufacturer also needs to produce items that are approach to target value so that they will gain maximum profit without ignoring the specification limit. In order to simplify the iteration process, the utilization of the statistical method has shown a very promising result to speed up in finding the optimal value estimation. For future work, we are considering to hybrid Artificial Intelligence (AI) techniques in order to find the optimal target value.

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