

## **Faculty of Mechanical Engineering**

# LOW FREQUENCY FRICTION INDUCED VIBRATION OF AUTOMOTIVE DISC BRAKE

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## LOW FREQUENCY FRICTION INDUCED VIBRATION OF AUTOMOTIVE DISC BRAKE

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A thesis submitted in fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

**Faculty of Mechanical Engineering** 

### UNIVERSITI TEKNIKAL MALAYSIA MELAKA

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C Universiti Teknikal Malaysia Melaka

## DECLARATION

I declare that this thesis entitle "Low Frequency Friction Induced Vibration of Automotive Disc Brake" is the result of my own research except as cited in the references. This thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature KunaReson. : KUMARESAN A/L MAGASWARAN Name 08/09/2014 Date

C Universiti Teknikal Malaysia Melaka

### APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Mechanical Engineering

Signature Supervisor Name : Dr. Muhammad 2ahir

Date

08 Sep' 14

## DEDICATION

To my beloved mother, father, friends, lecturers and God

### ABSTRACT

The automotive disc brake low frequency vibration has been a major concern in warranty issues and a challenging problem for many years. A variety of tools have been developed which include both experimental studies and numerical modeling technique to tackle the problem. The aim of this project is to develop a validated mathematical model considering the dynamic friction characteristics in the vibration of the disc brake. A key issue in the process is to investigate the structural deformation of the brake due to the friction distribution at the pad and disc contact during a typical braking cycle. A new methodology is introduced whereby the Green's equation is utilized to deduce the motion of the brake pad during braking. An experimental investigation using a brake dynamometer is also carried out to measure the vibration characteristics which are then used to validate the results predicted by the mathematical modeling. It is demonstrated that the mathematical model enhances the understanding of the time dependent non-linear contact behavior at the friction interface. The model was able to correspond to the frequencies of each modes obtained from the experiment with an average of 5 %. This, model in turn, demonstrated the fugitive nature of brake pad oscillation that appears and disappears as a function of friction distribution throughout the braking period. Parametric studies on the pressure and speed effects determine the contribution of each of these factors to brake pad leading end and trailing end behavior. The effects of these parameters were found to be modal depend, the decrease in speed during braking gave rise to vibration at frequencies between 500 Hz to 700 Hz where else braking at higher pressures subdued the vibration frequencies between 650 Hz to 850 Hz. Thus this approach can be used as a study tool to evaluate disc brake low frequency vibration using the mathematical model.



### ABSTRAK

Getaran frekuensi rendah cakera brek automotif telah menjadi kebimbangan utama dalam isu-isu jaminan dan masalah yang mencabar untuk beberapa tahun. Pelbagai alat telah ditubuhkan yang merangkumi kedua-dua kajian eksperimen dan teknik pemodelan berangka untuk menangani masalah tersebut. Tujuan projek ini adalah untuk membangunkan satu model matematik disahkan mempertimbangkan ciri-ciri geseran dinamik dalam getaran brek cakera. Isu utama dalam proses ini adalah untuk menyiasat ubah bentuk struktur brek disebabkan oleh pengagihan geseran pada pad dan kenalan cakera semasa kitaran brek biasa. Satu metodologi baru diperkenalkan di mana persamaan Green digunakan untuk menyimpulkan gerakan pad brek semasa membrek . Siasatan ujikaji menggunakan dinamometer brek juga dilakukan untuk mengukur ciri-ciri getaran yang kemudiannya digunakan untuk mengesahkan keputusan yang diramalkan oleh model matematik. Ia menunjukkan bahawa model matematik meningkatkan pemahaman kali bergantung tingkah laku sentuhan bukan linear pada antara muka geseran. Model ini dapat sesuai dengan kekerapan setiap mod pandangan yang diperoleh daripada satu eksperimen ini dengan purata sebanyak 5%. Ini, model pula menunjukkan ciri semulajadi buruan brek pad ayunan yang muncul dan hilang sebagai fungsi pengedaran geseran sepanjang tempoh brek. Kajian parametrik pada tekanan dan kelajuan kesan menentukan sumbangan setiap faktor-faktor ini berakhir terkemuka pad brek dan ketinggalan tingkah laku akhir . Kesan parameter ini telah didapati ragaman bergantung, penurunan dalam kelajuan semasa membrek menimbulkan getaran pada frekuensi antara 500 Hz 700 Hz mana lagi brek pada tekanan yang lebih tinggi rendah frekuensi getaran antara 650 Hz untuk 850 Hz. Oleh itu pendekatan ini boleh digunakan sebagai alat kajian untuk menilai cakera brek getaran frekuensi rendah menggunakan model matematik.

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|           | $A_{cs}$         | -   | Pad cross-section area                          |
|-----------|------------------|-----|---|
|           | $A_p$            | -   | Piston area,                                    |
|           | $F_T$            | -   | Total force acting on the brake pad             |
|           | $F_j(\omega)$    | -   | The excitation (input) in the frequency domain, |
|           | $H_{ij}(\omega)$ | ) - | The frequency response function,                |
|           | K <sub>G</sub>   | -   | pad sheer modulus                               |
|           | $X_i(\omega)$    | -   | The response (output) in the frequency domain.  |
|           | C ds             | -   | Pad sheer damping coefficient                   |
|           | c <sub>d</sub>   | -   | Pad damping coefficient                         |
|           | $k_p$            | -   | Pad stiffness coeficient                        |
|           | w <sub>b</sub>   | -   | Brake pad motion                                |
| $x_a$ and | x <sub>b</sub>   | -   | Points where piston force is applied            |
|           | μ                | -   | Brake pad coefficient of friction               |
|           | E                | -   | Pad young modulus                               |

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| G(x) | ,u)- | green's equation       |
|------|------|------------------------|
| 1    | -    | Pad moment of inertia  |
| L    | -    | Length of brake pad    |
| p    | -    | Brake pressure         |
| t    | -    | Time                   |
| и    | -    | Arbitrary point        |
| x    | -    | Brake pad x-coordinate |
| δ    | -    | Delta Dirac function   |
| ρ    | -    | Pad density            |

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## LIST OF ABBREVIATIONS

| EMA | - | Experimental modal analysis     |
|-----|---|---------------------------------|
| NVH | - | Noise, vibration and harshness  |
| SAE | - | Society of Automotive Engineers |
| SM  | - | Semi metallic                   |
| NAO | - | Non-asbestos organic            |
| FRF | - | Frequency response function     |
| FFT | - | Fast Fourier Transformation     |
| DAQ | - | Data acquisition system         |
| OEM | - | Original equipment manufacturer |
| FEA | - | Finite element analysis         |

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XV

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## LIST OF PUBLICATIONS

#### Journals

Magaswaran, K., Phuman Singh, A. S., & Hassan, M. Z., 2012. High Speed Experimental Study of Friction Induced Vibration in Disc Brakes. *Applied Mechanics and Materials*, 229, 747-749.

Magaswaran, K & Hassan, M. Z.,2013. A new method in the identification of noise and vibration characteristics of automotive disc brakes in the low frequency domain. *International Journal of Vehicle Noise and Vibration*. (Revised- waiting decision)

**Magaswaran, K** & Hassan, M. Z.,2013. Prediction of brake friction pair vibration in the low frequency domain using theoretical model. *International Journal of Vehicle Systems Modelling and Testing.* (Accepted-in press)

#### **Technical Paper**

Magaswaran, K., Phuman Singh, A., and Hassan, M., 2013. "An Analytical Model to Identify Brake System Vibration within the Low Frequency Domain," *SAE Technical Paper* 2013-01-2033, doi:10.4271/2013-01-2033.

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**Kumaresan Magaswaran**, Amrik Singh Phuman Singh and Muhammad Zahir Hassan, 2012. High Speed Experimental Study of Friction Induced Vibration in Disc Brakes *International Conference of Mechanical and Electronics Technology (ICMET 2012).* 24<sup>th</sup> – 26<sup>th</sup> July. Kuala Lumpur, Malaysia.

Kumaresan Magaswaran, Amrik Singh Phuman Singh and Muhammad Zahir Hassan, 2012. A parallel study of vibration analysis and acoustic analysis in low frequency brake noise *4thINTERNATIONAL CONFERENCE ON NOISE, VIBRATION AND COMFORT (NVC) 2012.* 26<sup>th</sup> - 29<sup>th</sup> November. Kuala Lumpur, Malaysia.

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#### CHAPTER ONE

#### INTRODUCTION

#### **1.1 Introduction**

Disc brake noise is a major issue in the current automotive industry. Investment in efforts and cost is still a major expenditure despite recent developments in the race to reduce the noise and vibration in the automotive brake system (Akay, 2002). The brake systems have been subjected to experiments and modeling since the invention of the disc brake system. Recent development in vehicle noise, vibration and harshness (NVH) has made the passenger vehicle operate in a quieter manner thus making the brake vibration to become the center of research focus.

Eliminating the low frequency brake vibration receives serious attention from car makers as it causes discomfort to the passengers inside the vehicle. Brake vibrations rarely compromise the performance of braking but it is subjected to warranty claims and customer complaints as the vibration is related with the quality of brake system (Akay, 2002).

The understanding of the vibration phenomena and the effort to control the brake vibration is a continuous challenge in the automotive braking industry. Generally the brake vibration is perceived as a friction-induced forced vibration (Rhee *et al.*, 1989). The characteristics of a braking event are complicated as the input to the brake system varies with

unlimited number of combinations and the brake assembly itself is a set of multiple components connected with complex interfaces.

The brake vibration is categorized into two categories based on its occurring frequencies. One is the low frequency domain which is the vibrations below the 1 kHz, and the other is the high frequency domains which are the vibration above 1 kHz. Vibration types such as groan, moan, hum and judder falls in the low frequency domain where else the high frequency domain contains the squeal type noise (Jacobsson, 2003). The low frequency noise types are generally caused by brake pad excited by the brake rotor at the contact and it is coupled with other vehicle components (Dunlap *et al.*, 1999). Where else the squeal noise is said to be a friction induced vibration combined with thermal and structural effects. (Hassan, 2007)

### **1.2 Problem Statement**

For the study of the mechanisms which produced the noise types in the low frequency domain the analytical method is suitable as the analytical model simulates the motion of the brake components relative to each other. The properties of the components can be altered in ease to deduce the response vibration. Achieving this experimentally involves high cost and increases the development process (Yumoto and Okamura, 2006).

The low frequency brake vibrations mentioned earlier are well studied and the analytical models were developed to explain the mechanisms behind each vibration types. Mainly these studies revolve around the stick and slip mechanism, mode couplings and dynamic system instability (Ammar *et al*, 2011, Hetzler and Seemann, 2006, Sinou and