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Variable Speed Wind Turbine with External Stiffness and Rotor Deviation Observer

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Abstract. Often in prominent literature, the appearance of external stiffness in wind turbine dynamical model has been neglected. The ignorance of external stiffness eliminates the presence of rotor-side angular deviation in the system dynamic. In order to give more practical look of the variable speed control system structure, we develop a linear observer to estimate the rotor angular deviation. We use Linear Quadratic Regulator (LQR) to design the observer gain, as well as the estimation error gain. To facilitate observer design, the system is linearized around its origin by using Jacobian matrix. By using the estimated rotor angular deviation, we design a variable speed control via Lyapunov and Arstein to enhance power output from the turbine.

Introduction

In recent research, a standard two-mass wind turbine model appeared to be a one-dimensional dynamical system [1-5]. Most of the research relax some fundamental aspect in the system dynamic. For instance, Beltran [1] and Boukhezzar [2] neglect the lumped external stiffness of the turbine rotor and generator. The reason of relaxing the external stiffness is for the ease of controller design. The appearance of external stiffness in wind turbine dynamical system increases the level of difficulty in term of control design and stabilization. This is because that the external stiffness incurs an integrator to the wind turbine dynamic, which is difficult to handle (see [1-2] for stiffness ignorance approach). The integrator comes from the rotor angular deviation in the system dynamic. As such, stabilizing wind turbine system with external stiffness require high mathematical complexity. The fact is that relaxing the external stiffness is not a valid assumption, at least, for the sack of practicality. In this paper, we design an observer in order to estimate the angular deviation of the rotor. We search the feedback gain as well as the estimation error gain by using LQR. By using the estimated angular deviation, the variable speed wind turbine structure can be realized. Motivated by Artstein [6], we design a variable speed control approach to obtain a maximum power generated by the turbine. To generalize the case, let an autonomous and affine nonlinear system $\dot{x} = f(x) + g(x)u$ with $x \in \mathbb{R}$ is the system state, $f(\cdot)$ and $g(\cdot)$ is analytic and smooth vector fields, and u is the control command. Thus, *Definition 1*, *Lemma 2* and *Theorem 3* are useful in the sequel.

Definition 1: *For nonlinear system $\dot{x} = f(x) + g(x)u$, if there exists a feedback law $u = k(x)$ for $\dot{x}$, then the closed loop system $\dot{x} = f(x) + g(x)k(x)$ is completely controllable linear system.*
Lemma 2: *[7]:Let $A = J_f(0)$ be the Jacobian matrix of $f(\cdot)$ at the origin ,then $B = g(0)$. If nonlinear system $\dot{x} = f(x) + g(x)u$ is linearizable, then (A, B) is completely controllable.*
Theorem 3: *Lyapunov's linearization method [8]:* ♦ *If the linearized system is strictly stable, then the equilibrium point for the actual nonlinear system is asymptotically stable* ♦ *If the linearized system is unstable, then the equilibrium for the actual nonlinear system is unstable* ♦ *If the linearized system is marginally stable, then one cannot conclude anything from the linear approximation.*

In wind turbine dynamical model, the appearance of aerodynamic power is necessary to functioning the turbine. The aerodynamic power is highly depends on the power coefficient of the turbine, air density of which the turbine to be installed, the blade radius, tip-speed-ratio and the wind speed profile. The power coefficient enlightens the optimum tip-speed-ratio the turbine should operate with, in order to guarantee a maximum power coefficient thorough out the operation. Often, the power coefficient curve can be obtained via the look-up table supplied by the wind turbine manufacturer. As such, obtaining maximum power coefficient and optimum tip-speed-ratio is a must for maximum power generation. Because of that reason, Ozbay in [4] assume the power coefficient expression as a constant positive value. For practicality purpose, in our model, we supersede Ozbay's approach by exploiting the empirical power coefficient expression proposed in [9-10].

In what follows, we present a derivation of a two-mass wind turbine system. Afterward, we realize the observer design and feedback control design for a variable speed wind turbine system. Next, we represent numerical analysis for the variable speed wind turbine control and tabulates the results. Lastly, we concluded the findings.

Wind Turbine Model

This research paper focuses on the mechanical power control side which deals with turbine dynamic up to generator front only. Table 1 tabulates wind turbine parameters.

Table 1: Nomenclature

Symbols	Definition	Symbols	Definition
R	Rotor blade radius (m)	K_g	Generator external damping $(N.m.rad^{-1}.s^{-1})$
v	Wind speed $(m.s^{-1})$	K_r	Generator external damping $(N.m.rad^{-1}.s^{-1})$
ρ	Air density $(Kg.m^{-3})$	B_r	Rotor stiffness $(N.m.rad^{-1})$
$C_p(\lambda,\beta)$	Power coefficient	B_g	Generator stiffness $(N.m.rad^{-1})$
λ	Tip speed ratio	T_m	Aerodynamic torque $(N.m)$
β	Pitch angle (deg)	T_g	Generator torque/Electromagnetic torque $(N.m)$
γ	Gearing ratio	T_{hs}	High-speed shaft torque $(N.m)$
ω_r	Rotor speed $(rad.s^{-1})$	T_{ls}	Low-speed shaft torque $(N.m)$
ω_g	Generator speed $(rad.s^{-1})$	θ_g	Generator-side angular deviation (rad)
J_r	Rotor inertia $(Kg.m^2)$	θ_r	Rotor-side angular deviation (rad)
J_g	Generator inertia $(Kg.m^2)$		

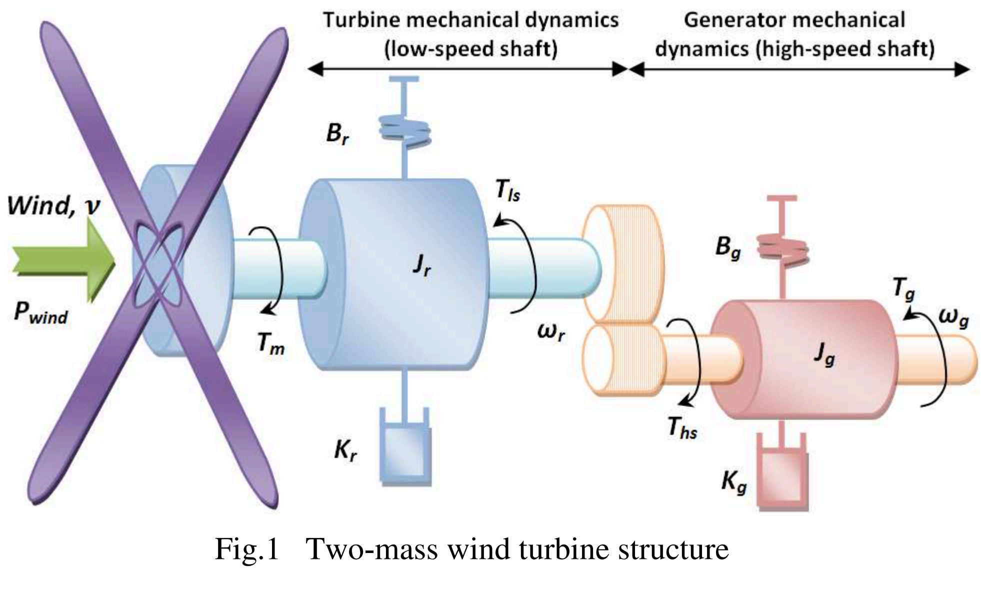

Fig.1 Two-mass wind turbine structure

Consider a two-mass wind turbine system in Fig. 1. The torque produced by the turbine can be represented as $T_m - T_{ls} = J_r\dot{\omega}_r + K_r\omega_r + B_r\theta_r$. The transmission output torque can be represented as $T_{hs} - T_g = J_g\dot{\omega}_g + K_g\omega_g + B_g\theta_g$. With gearing ratio $\gamma = \frac{\omega_g}{\omega_r} = \frac{T_{ls}}{T_{hs}}$, the dynamic of the turbine system can be represented as:

$$J\dot{\omega}_r + K\omega_r + B\theta_r = T_m - \gamma T_g \tag{1}$$

where the lumped inertia is defined as $J = J_r + \gamma^2 J_g$, the lumped external stiffness $B = B_r + \gamma^2 B_g$ and the lumped external damping $K = K_r + \gamma^2 K_g$. The aerodynamic power produced by the turbine can be expressed as $P_m = P_{wind}C_p(\lambda, \beta)$ (see [1-5, 11-12]). $P_{wind} = \frac{1}{2}\rho\pi R^2 v^3$ is the instantaneous power produced by the wind, and $\lambda = \frac{R\omega_r}{v}$ is the tip-speed-ratio. Power coefficient C_p is a nonlinear function where it values can be obtained from look-up table provided by turbine manufacturers. Other than look-up table, the empirical expression for power coefficient can be found in [9-10]. Knowing that $P_m = T_m\omega_r$, the wind turbine dynamic can be written as:

$$\dot{\omega}_r = -\frac{K}{J}\omega_r - \frac{B}{J}\theta_r + \frac{\rho\pi R^5 C_P}{2\lambda^3}\omega_r^2 - \frac{\gamma}{J}T_g \tag{2}$$

Estimator and Variable Speed Control Design

Note that ω_r^2-term is the only nonlinearity for the wind turbine dynamic in Eq. (2). In [1-2], the external damping B has been neglected. In our case, we consider B in order to gain practical validity. Although Eq. (2) approximates the dynamical of the wind turbine more accurately than the one proposed in [1-2], it is however more difficult to determine its stabilizing function for the turbine speed error. The appearance of θ_r replenishes an integrator in the system model when the mapping of θ_r towards ω_r is being considered. Moreover, the exact value for θ_r is immeasurable. Therefore, in this research paper, we develop observer for θ_r. The so-called rotor-side angular deviation observer estimates the real time value for θ_r such that the variable speed control for the turbine can be realized. Lets map the system Eq. (2) into two-dimensional nonlinear system:

$$\begin{bmatrix}\dot{\theta}_r \\ \dot{\omega}_r\end{bmatrix} = \begin{bmatrix} f_{\theta_r}(\theta_r, \omega_r) \\ f_{\omega_r}(\theta_r, \omega_r)\end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\gamma/J}{J}\end{bmatrix} T_g \tag{3}$$

where $f_{\theta_r}(\theta_r, \omega_r) = \omega_r$ and $f_{\omega_r}(\theta_r, \omega_r) = -\frac{K}{J}\omega_r - \frac{B}{J}\theta_r + \frac{\rho\pi R^5 C_P}{2\lambda^3}\omega_r^2$. By *Lemma 2*, we prove the controllability of a two-dimensional wind turbine system in Eq. (3). By observing the gradient of the system Eq. (2) with respect to $[\theta_r \quad \omega_r]^T$, we obtain the Jacobian matrix at origin $A = \begin{bmatrix}\nabla_{\theta_r} f_{\theta_r} & \nabla_{\omega_r} f_{\theta_r} \\ \nabla_{\theta_r} f_{\omega_r} & \nabla_{\omega_r} f_{\omega_r}\end{bmatrix}$, implies $A|_{(0,0)} = \begin{bmatrix} 0 & 1 \\ -\frac{B}{J} & -\frac{K}{J}\end{bmatrix}$. This yields the linearized wind turbine system:

$$\begin{matrix} \begin{bmatrix}\dot{\theta}_r \\ \dot{\omega}_r\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{B}{J} & -\frac{K}{J}\end{bmatrix}\begin{bmatrix}\theta_r \\ \omega_r\end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\gamma}{J}\end{bmatrix} T_g \\ y = [1 \quad 0]\begin{bmatrix}\theta_r \\ \omega_r\end{bmatrix} \end{matrix} \triangleq \begin{matrix} \dot{\mathcal{X}} = \mathcal{A}\mathcal{X} + \mathcal{B}\mathcal{U} \\ \mathcal{Y} = \mathcal{C}\mathcal{X} \end{matrix} \tag{4}$$

where $\mathcal{A} = \begin{bmatrix} 0 & 1 \\ -\frac{B}{J} & -\frac{K}{J}\end{bmatrix}$ is the system matrix, $\mathcal{B} = \begin{bmatrix} 0 & -\frac{\gamma}{J}\end{bmatrix}^T$, is the input matrix, $\mathcal{C} = [1 \quad 0]$ is the output matrix, $\mathcal{U} \equiv T_g$ is the control input and $\mathcal{X} = [\theta_r \quad \omega_r]^T$ is the system states. Let LQR-based observer for the wind turbine system Eq. (4) be in the form:

$$\left.\begin{array}{c}\dot{\hat{\mathcal{X}}}(t) = \mathcal{A}\hat{\mathcal{X}} + \mathcal{B}\mathcal{U} + \mathcal{G}(\mathcal{Y} - \hat{\mathcal{Y}}) \\ \hat{\mathcal{Y}} = \mathcal{C}\hat{\mathcal{X}}\end{array}\right\} \tag{5}$$

where $\hat{\mathcal{Y}} := \theta_{r(est)}$ is the estimated rotor angular deviation, and $\mathcal{G} \in \mathcal{R}^{nxp}$ is the gain for estimation error. Further, we prove that the observer fulfill the minimization of the linear quadratic performance index in Eq. (6):

$$J = \int_0^\infty (\hat{\mathcal{X}}^T \mathcal{Q} \hat{\mathcal{X}} + \mathcal{U}^T \mathcal{R} \mathcal{U}) dt \quad , \quad \forall\, \mathcal{X} \neq 0 \tag{6}$$

with $\hat{\mathcal{X}}$ is the estimated states, $\mathcal{Q}$ and $\mathcal{R}$ are square positive definite matrices. Term $\hat{\mathcal{X}}^T \mathcal{Q} \hat{\mathcal{X}}$ solves the state regulation problem. Term $\mathcal{U}^T \mathcal{R} \mathcal{U}$ minimizes the energy of the control signal $\mathcal{U}(t)$ so that the control input is bounded between $\mathcal{U}_{min} \leq \mathcal{U}(t) \leq \mathcal{U}_{max}$. Via $\mathcal{U}^T \mathcal{R} \mathcal{U}$-term and symmetric $\mathcal{P} \in \mathbb{R}^{2x2}$, we can prove that a feedback gain $\mathcal{K} = \mathcal{R}^{-1} \mathcal{B}^T \mathcal{P}$ produces minimum energy $\mathcal{U} = -\mathcal{K}\hat{\mathcal{X}}$ in order to drive the estimated states toward origin. Hence reducing estimation error and fulfill the output regulation of the actual system in Eq. (4). We can further write Eq. (6) as:

$$J = \int_0^\infty (\hat{\mathcal{X}}^T \mathcal{Q} \hat{\mathcal{X}} + \left(-\mathcal{K}\hat{\mathcal{X}}\right)^T \mathcal{R}(-\mathcal{K}\hat{\mathcal{X}})) dt = \int_0^\infty \hat{\mathcal{X}}^T (\mathcal{Q} + \mathcal{K}^T \mathcal{R} \mathcal{K}) \hat{\mathcal{X}} dt \tag{7}$$

Proposition 1. *Linear quadratic performance index in (7) quadratically stabilize the observer (5) for a linearized wind turbine system in (4).).* To prove the *Proposition 1*, let consider a candidate Lyapunov function $V = \hat{\mathcal{X}}^T \mathcal{P} \hat{\mathcal{X}}$. Taking the derivative of V along $\hat{\mathcal{X}}$ yields the embedded *Reduced Riccati Equation* in the negative definite function of $\dot{V}$, as follows:

$$\begin{aligned}\dot{V} &= 2\hat{\mathcal{X}}^T \mathcal{P}\left(\mathcal{A}\hat{\mathcal{X}} + \mathcal{B}\mathcal{U}\right) = \hat{\mathcal{X}}^T \mathcal{P}(\mathcal{A} - \mathcal{B}\mathcal{K})\hat{\mathcal{X}} + \hat{\mathcal{X}}^T (\mathcal{A} - \mathcal{B}\mathcal{K})^T \mathcal{P} \hat{\mathcal{X}} \\ &\Rightarrow \hat{\mathcal{X}}^T [\mathcal{A}^T \mathcal{P} + \mathcal{P}\mathcal{A} - \mathcal{Q} - (\mathcal{R}^{-1}\mathcal{B}^T\mathcal{P})^T \mathcal{R}(\mathcal{R}^{-1}\mathcal{B}^T\mathcal{P}) \equiv 0]\hat{\mathcal{X}} \\ &\Rightarrow \hat{\mathcal{X}}^T [\mathcal{A}^T \mathcal{P} + \mathcal{P}\mathcal{A} - \mathcal{P}\mathcal{B}\mathcal{R}^{-1}\mathcal{B}^T\mathcal{P} - \mathcal{Q}]\hat{\mathcal{X}} < 0 \quad (Proven)\end{aligned} \tag{8}$$

Proposition 2. *For $\mathcal{K} \in \mathbb{R}^{mxn}$, the control feedback law $\mathcal{U} = -\mathcal{K}\hat{\mathcal{X}}$ and observer gain $\mathcal{G} \in \mathbb{R}^{nxp}$ retain the stability of the closed loop system (4) and (5) with $m = p = 1$ and $n = 2$.* To prove *Proposition 2*, let consider the estimation error dynamics $\dot{e} = \dot{\hat{\mathcal{X}}} - \dot{x}$. Then the closed loop system:

$$\left.\begin{array}{c}\dot{e}(t) = (\mathcal{A} - \mathcal{G}\mathcal{C})e \\ \dot{e}_y = \mathcal{C}e_y\end{array}\right\} \tag{9}$$

With $\mathcal{U} = -\mathcal{K}\hat{\mathcal{X}}$, the closed loop system can be written in $[x \quad e]$ coordinate as:

$$\begin{bmatrix}\dot{x} \\ \dot{e}\end{bmatrix} = \begin{bmatrix}\mathcal{A} - \mathcal{B}\mathcal{K} & -\mathcal{B}\mathcal{K} \\ 0 & \mathcal{A} - \mathcal{G}\mathcal{C}\end{bmatrix}\begin{bmatrix}x \\ e\end{bmatrix} \tag{10}$$

By judicious choice of $\mathcal{G}$ and $\mathcal{K}$, the stability of the closed loop wind turbine with observer is guaranteed by the stability of both matrices $[\mathcal{A} - \mathcal{B}\mathcal{K}]$ and $[\mathcal{A} - \mathcal{G}\mathcal{C}]$. This complete the proof.

Motivated by Arstein [6], and by using the knowledge of estimated rotor angular deviation $\hat{\mathcal{Y}} := \theta_{r(est)}$, we design a variable speed controller. Let consider speed tracking error $e_\omega = \omega_r - \omega_r^*$, where ω_r^* is the demanded rotor speed for a particular wind speed profile. For a maximum power generation, the demanded rotor speed must be $\omega_r^* = \frac{\lambda_{opt} v}{R}$. Let consider a smooth positive definite function $V_{e_\omega} = \frac{1}{2} e_\omega^2$, then its derivative along speed tracking error yields $\dot{V}_{e_\omega} = e\left(a_1\omega_r + a_2\theta_r + a_3\omega_r^2 + b_1 T_g - \dot{\omega}_r^*\right)$. For there exists $C > 0$, then the variable speed control law in Eq. (11) renders the derivative of V_e negative definite (i.e. $\dot{V}_{e_\omega} = -Ce_\omega^2$), in order to satisfy the asymptotic tracking of the rotor speed.

$$T_g = \frac{1}{b_1}\left[-Ce_\omega - a_1\omega_r - a_2\theta_r - a_3\omega_r^2 + \omega_r^*\right] \tag{11}$$

Numerical Analysis

To observe the linearized and nonlinear system, let define wind turbine parameters $B = 5.2\ N.m.rad^{-1}$, $J = 37.65\ Kg.m^2$, $K = 27.56\ N.m.rad^{-1}s^{-1}$, $R = 21.65m$ and $\gamma = 43.165$. With 5 ms^{-1} wind speed, the rotor deviate anti-counterclockwise around 0.018 radian (see Fig. 2). The initial rotor speed to deviate the rotor by 0.018 radian is shown in Fig. 3.

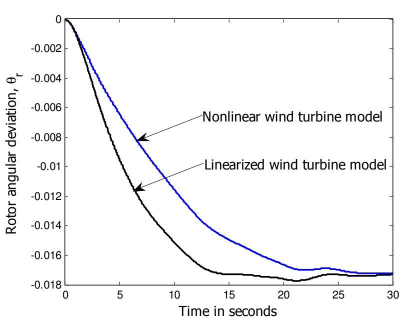

Fig.2 Rotor angular deviation

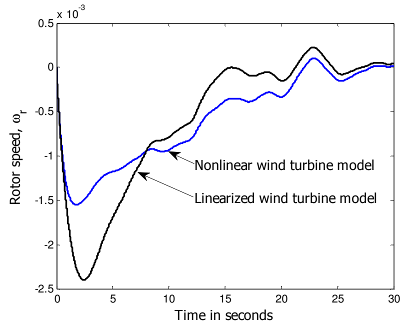

Fig.3 Rotor speed

Results

Linear observer is designed to estimate $\theta_{r(est)}$ which normally ignored due to the neglection of external stiffness (see [1-2]). With LQR, we obtain the estimation error gain $\mathcal{G} = [48.5349 \quad 686.0653]^T$ and the observer feedback gain $\mathcal{K} = [-0.8868 \quad -1.0804]$. Fig.4 and Fig.5 show the observer performance. Fig.6 and Fig.7 show the variable speed control performance for 5 ms^{-1} wind speed. Fig.6 and Fig.7 show that the actual rotor speed asymptotically track the demanded rotor speed with decaying tracking error at 0.06 sec. The power output and its distribution are shown in Fig.8 and Fig.9 respectively.

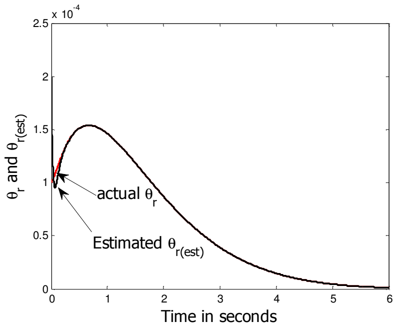

Fig.4 Testing the observer performance - rotor angular deviation

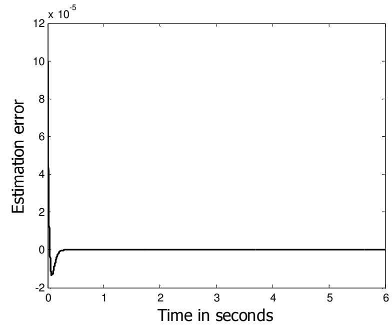

Fig.5 Testing the observer performance - The estimation error

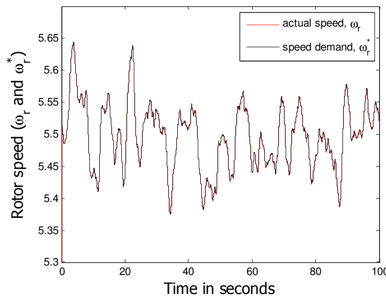

Fig.6 Comparison between the actual rotor speed and the demanded rotor speed

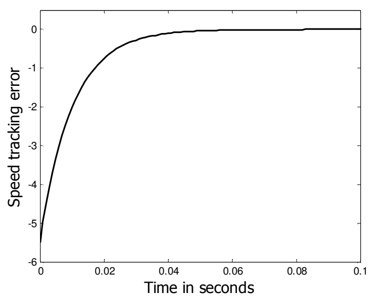

Fig.7 A speed tracking error

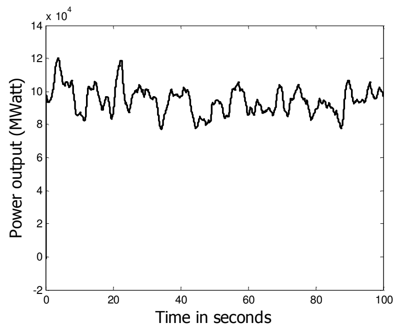

Fig.8 Generated power output

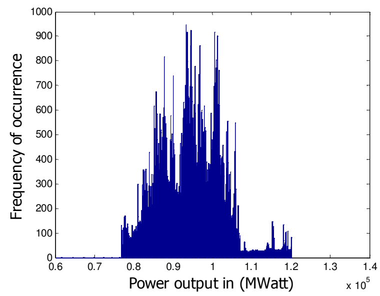

Fig.9 Power output distribution

Conclusion

In this paper, a linear observer has been designed to estimate the rotor angular deviation of a nonlinear wind turbine model. The observer is designed based on the Jacobian matrix at the system equilibrium. The estimated rotor angular deviation is then utilized by the variable speed control design, where the asymptotic tracking of the rotor speed is guaranteed by a Lyapunov stability criteria. However, there is a shortcoming of the proposed approach, that is the linearization is valid only for a small neighborhood about the fixed equilibrium point. As such, the initial rotor angular deviation should starts inside the linear range such that the observer estimation error is kept small. Otherwise, the variable speed control approach is only marginally stable. This phenomena give some idea for further research in variable speed wind turbine with consideration of an external stiffness and rotor angular deviation. In the future, when dealing with inevitable dynamic such as rotor angular deviation, we highly recommend the use of a direct nonlinear control approaches such as backstepping, Lyapunov redesign and variable structure control.

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