



Faculty of Electrical Engineering

**MODELLING AND CONTROL OF INVERTED PENDULUM ON THE
ROTATING DISC**

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Master of Science in Electrical Engineering

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ROTATING DISC**

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**A thesis submitted
in fulfillment of the requirements for the degree of Master of Science
in Electrical Engineering**

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DECLARATION

I declare that this thesis entitled “Modelling and Control of Inverted Pendulum on the Rotating Disc” is the result of my own research work except as cited clearly in the references.

Signature :

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Date : JULY 14, 2014

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Electrical Engineering.

Signature :

Supervisor Name: DATUK PROF. DR. MOHD RUDDIN BIN AB. GHANI

Date : JULY 14, 2014

DEDICATION

To my beloved mother and three brothers

ABSTRACT

This research work studies about modelling and control of inverted pendulum on the rotating disc by using classical, modern, and intelligent control techniques. In this study, the classical control techniques use proportional-plus-derivative (PD) and proportional-plus-integral (PI) controllers, the modern control techniques that use Linear Quadratic Regulator controller (LQR), and intelligent control technique that use Fuzzy Logic (FL) controller. The main goal of this study is to model and control the dynamic modelling of the inverted pendulum on the rotating by using the above-mentioned control techniques. Among the problems identified for this project are balancing inverted pendulum on the rotating disc with the presence disturbance and establishing stability for a dynamical inverted pendulum on the rotating disc. The practical results in controlling the inverted pendulum and eliminating the disturbance are obtained via the following techniques: the MATLAB root locus for the PD and PI controllers; the optimal control for LQR controller; and the fuzzification and the defuzzification for the FL controller as a perspicuous view of its transient response stability. In the transient response, the balancing and stability of the inverted pendulum on the rotating disc are affected by the presence of the disturbances. The presence of the disturbance that is controlled by LQR controller shows the condition to the inverted pendulum on the rotating disc. Moreover, from the results obtained, it is found to be asymptotically stable by Lyapunov's stability analysis. The mathematical model of the inverted pendulum on the rotating disc has been developed and the output response with disturbances show LQR controller have achieved good performance compared to PD, PI and FL controllers. This study applicable for the robot cycling transportation that delivers goods for customers.

ABSTRAK

Penyelidikan ini adalah suatu kajian tentang permodelan dan kawalan bandul terbalik di atas mekanisme cakera berputar dengan menggunakan teknik-teknik klasik, moden, dan pintar (intelligent). Untuk kajian ini, teknik-teknik kawalan klasik menggunakan pengawal-pengawal 'proportional-plus-derivative (PD)' dan 'proportional-plus-integral (PI)', teknik kawalan moden menggunakan pengawal-pengawal 'Linear Quadratic Regulator (LQR)', dan teknik kawalan pintar menggunakan pengawal-pengawal 'Fuzzy Logic (FL)'. Tujuan utama projek ini adalah untuk memodelkan dan mengawal bandul terbalik di atas cakera berputar dengan menggunakan teknik-teknik kawalan system. Masalah yang dikenal pasti untuk kajian ini adalah mengimbangi bandul terbalik pada cakera berputar dengan kehadiran gangguan dan mewujudkan kestabilan bagi bandul dinamik terbalik pada cakera berputar itu. Keputusan praktikal dalam mengawal bandul terbalik dan menghapuskan gangguan yang diperolehi melalui teknik-teknik berikut: MATLAB londar punca untuk pengawal 'proportional-plus-derivative (PD)' dan 'proportional-plus-integral (PI)', kawalan optimum untuk pengawal 'Linear Quadratic Regulator (LQR)' dan, fuzifikasi dan nyahfuzifikasi untuk pengawal 'Fuzzy Logic (FL)' sebagai pemerhatian jelas terhadap kestabilan sambutan fana. Kehadiran gangguan yang dikawal oleh pengawal LQR menunjukkan keadaan bandul terbalik pada cakera berputar. Selain itu, daripada keputusan ini, didapati kestabilan asimptot diperolehi melalui analisis kestabilan Lyapunov. Model matematik bandul terbalik pada cakera berputar telah dibangunkan dan tindak balas pengeluaran dengan gangguan menunjukkan pengawal LQR mencapai prestasi yang baik berbanding dengan pengawal-pengawal PD, PI, dan FL. Kajian ini sesuai digunakan sebagai pengangkutan robot berbasikal yang menghantar barang-barang untuk pelanggan.

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LIST OF ABBREVIATIONS

- PD - proportional-plus-derivative
- PI - proportional-plus-integral
- LQR - Linear Quadratic Regulator
- FL - Fuzzy Logic
- DC - Direct Current
- DAQ - Data Acquisition

LIST OF SYMBOLS

T_m	-	Torque for dc motor
T_d	-	Torque for disc
T_p	-	Torque for pendulum
J_m	-	Moment of inertia for the dc motor
J_d	-	Moment of inertia for the disc
J_p	-	Moment of inertia for the pendulum
K_1	-	Spring constant for the disc
K_2	-	Spring constant for the pendulum
F_s	-	Static force
F_c	-	Coulomb friction force
r_1	-	Radius for the disc
r_2	-	Radius for the pendulum
a_d	-	Disc acceleration
a_p	-	Pendulum acceleration
θ_m	-	Motor position in radian
θ_d	-	Disc position in radian
θ_p	-	Pendulum position in radian

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2. M.A. Nur Huda, H.A. Kasdirin and A.G. Mohd Ruddin, 2012. Development of PD Controller for Comparison Stability Study in Multiple Difference Disturbances. *In: The 3rd International Conference on Engineering and ICT (ICEI 2012)*. 4-5 April 2012, Melaka, Malaysia.
3. N.H. Mohd Amin, H.A. Kasdirin, M.R. Md. Nawawi and M.R. Ab. Ghani, 2011. Development of PI Controller for Disc Speed. *In: Proceedings of the 2011 IEEE Symposium on Industrial Electronics and Applications (ISIEA 2011)*. 25-28 September 2011, Langkawi, Malaysia.

CHAPTER 1

INTRODUCTION

Control engineering has been evolving rapidly in robotic field, all types of vehicles, and buildings that need a control system to organize, monitor, and stabilize movements. The control system approach is based on control system stability analysis such as a degree of stability, a steady-state performance and a transient response as cited by Nise (2008). The stability of control system attracts attention of researchers to develop more control system in robotic and automation field.

1.1 Background

Control system in this study includes classical, modern and intelligent controls. The classical control is based on Laplace Transform with the design of the transfer function for the Proportional plus Derivative (PD) and Proportional plus Integral (PI) controllers. For the input or output that equal or more than two, the modern control is used based on the state space equations model in matrix form with the design of an optimization method for the Linear Quadratic Regulator (LQR) controller. Moreover, from the two systems, this study is improved using intelligent control that based on Mamdani with design of the fuzzification and defuzzification for the Fuzzy Logic controller. This thesis has discussed each control to model and control of the inverted pendulum on the rotating disc.

Generally, the inverted pendulum is a dynamic system that reloads by the unstable inverse mechanism. Therefore, the classical, modern and intelligent control methods are implemented to control the inverted pendulum system. These methods were chosen for stabilization of the inverted pendulum, with or without disturbances (henceforth PD, PI, LQR and, FL controllers).

In fact, the stability of the inverted pendulum motion depends on the constraints at its pivot point, cart, or oscillatory base. For this study, the inverted pendulum motion depends on the constraint at oscillatory base. The constraint has a relationship to parameters like torque, gravity, force, inertia, mass, position, and speed.

From the parameters, the inverted pendulum motion has been modelled for good stabilization by employing PD, PI, LQR, or FL in closed-loop system. The closed-loop system was interrupted with pulse disturbance. Therefore, the stability of the system requires guarantees of the convergence of the system state and of the boundedness of the error in the approximator parameter vector that cited by Samad (2001). Nevertheless, in control system stability, studies on unstable and non-linear system of rotational disk is an interesting and challenging topic.

Moreover, this study aims to develop an inverted pendulum mechanical structure and controller designs. It also aims to stabilize inverted pendulum using real-time MATLAB software with one or more control systems. Besides, the set up of the inverted pendulum has included plant, electronic components, mechanical hardware, and controller which are synchronized with real-time MATLAB.

Physically, rotational inverted pendulum was mounted at the end of the disc on an encoder shaft. The physical mechanism of the rotational inverted pendulum caused problems to the motor performance and input directions. The main problem is to solve the performance of motor by controlling the disc in order to make the pendulum remains vertical. The velocity causes the positions of the input change directions when the pendulum stops and begins to swing in the opposite directions and cited by Sukontanakarn and Parnichkun (2009).

The configurations of inverted pendulum on rotating disc is set-up based on arm-driven inverted pendulum concept as cited by Ray (2007) and the configurations is shown in Figure 1.1. The hardware set up for the system consist a DC servomotor and other electronic components, and mechanical parts were employed in building the inverted pendulum plant. Additionally, the power driver was integrated with the MATLAB software and plant for the development of the control algorithms. The sensors in the plant were the feedback to the main controller at which, the control systems of the feedback control mechanism were designed and developed for this study.

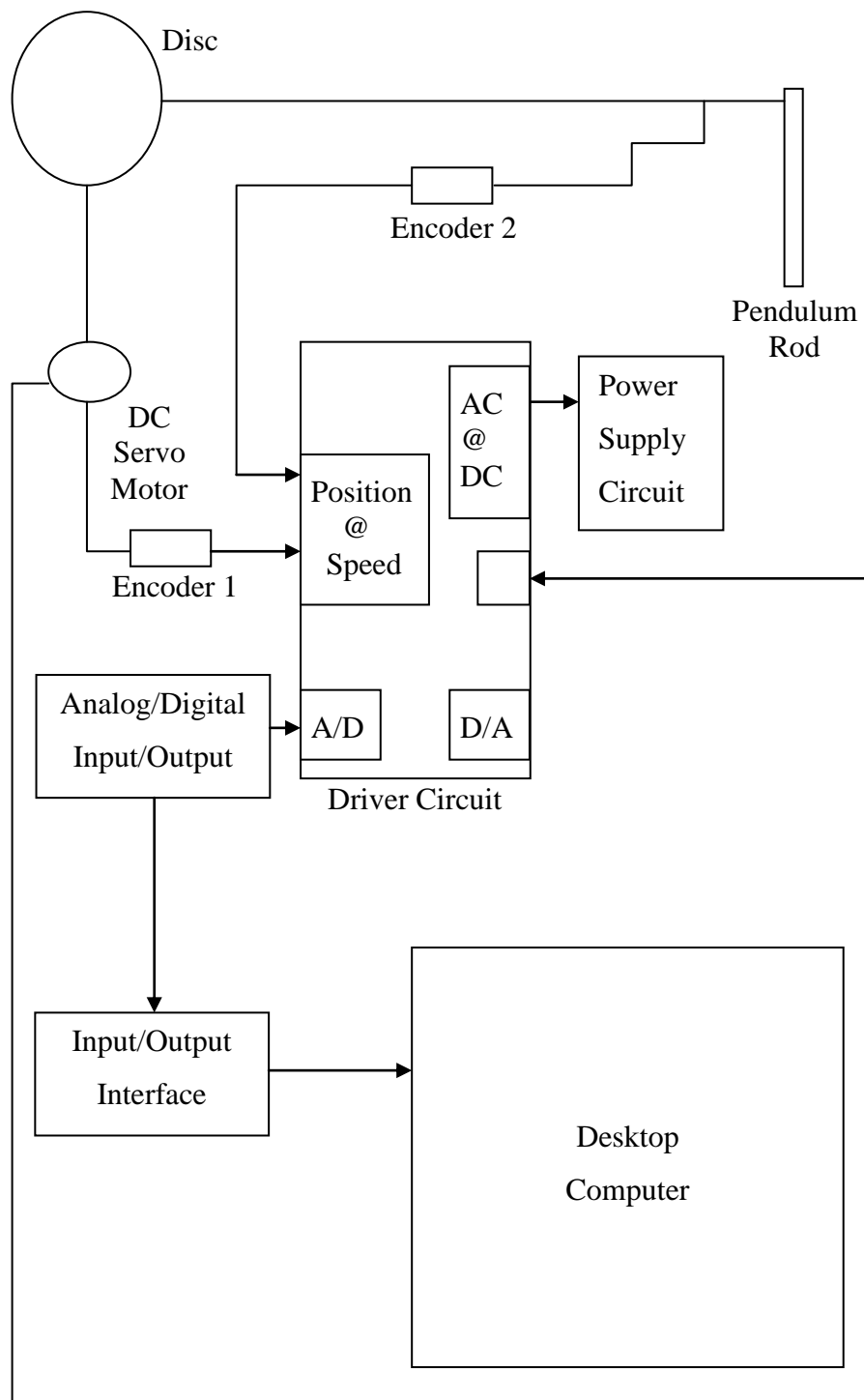


Figure 1.1 Hardware configurations