UNIVERSITI TEKNIKAL MALAYSIA MELAKA

Faculty of Manufacturing Engineering

AN ASSESSMENT ON THE EFFECT OF DIFFERENT CHARACTERISTICS OF SYMMETRIC MATRICES TO PRIMAL AND DUAL SOLUTIONS IN LINEAR PROGRAMMING PROBLEMS

Ikhsan Romli

Master of Science in Manufacturing Engineering

2013

AN ASSESSMENT ON THE EFFECT OF DIFFERENT CHARACTERISTICS OF SYMMETRIC MATRICES TO PRIMAL AND DUAL SOLUTIONS IN LINEAR PROGRAMMING PROBLEMS

IKHSAN ROMLI

A thesis submitted
In fulfillment of the requirements for the degree of Master of Science
In Manufacturing Engineering

Faculty of Manufacturing Engineering

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

2013

ABSTRACT

A linear optimization model or linear programming (LP) problem involves the optimization of linear function subject to linear constraints, where every constraints form of linear equation. In particular this research is focused on an interpretation on the effect of five different characteristics of symmetric matrices i.e. positive definite (PD), negative definite (ND), positive semi-definite (PSD), negative semi-definite (NSD), and indefinite (ID) to primal-dual solutions (duality) in LP problems. Various sizes of matrices order (5x5, 10x10, 20x20, 30x30, 50x50) were simulated, each size simulated for 30 times. There were 150 simulations of matrices generated for one type of matrix. In total, 750 simulated of LP problems were solved.

Based on simulations result, it was demonstrated that PD, ND, and some ID symmetric matrices were non-singular, whereas PSD, NSD and some ID matrices have been shown to be singular. An optimal solutions for a Primal and Dual of the PD, the PSD, and some ID matrices have been found with no duality gap. These were due to the matrices form a convex set of the LP problem. However, there was no optimal solution for the ND, the NSD, some ID matrices, provided that some ID matrices were concave functions. If the coefficient matrix of the LP problem was non-singular matrix then the LP solution did not necessarily had the optimal solution. In other words there was no correlation between singularity of the matrices and optimality of the Duality LP solutions. Experimental results by the simulation in MATLAB and EXCEL Solver show that the optimal solutions of both the primal and the dual solutions were same in the PD and the PSD matrices. This research provided new findings about the effects of the different characteristics of five types of the symmetric matrices in the LP duality problem which may contribute to an advancement of the LP theory and practice.

ABSTRAK

Satu model pengoptimuman linear atau masalah pengaturcaraan linear (LP) melibatkan pengoptimuman fungsi linear tertakluk kepada kekangan linear, di mana setiap kekangan membentuk persamaan linear. Secara khususnya kajian ini tertumpu kepada tafsiran tentang kesan lima ciri-ciri yang berbeza matriks simetri iaitu positive definite (PD), negative definite (ND), positive semi-definite (PSD), negative semi-definite (NSD), dan indefinite (ID) kepada penyelesaian dual primal (dualiti) dalam masalah LP. Pelbagai saiz matriks (5x5, 10x10, 20x20, 30x30, 50x50) telah disimulasi, setiap saiz disimulasi untuk 30 kali. Terdapat 150 simulasi matriks yang dijana setiap jenis matriks. Secara keseluruhan, 750 simulasi masalah LP telah diselesaikan.

Berdasarkan simulasi, ia telah menunjukkan bahawa PD, ND, dan beberapa matriks simetri ID tak tunggal (nonsingular), manakala PSD, NSD dan beberapa matriks ID telah ditunjukkan untuk menjadi tunggal (singular). Penyelesaian yang optimum untuk primal dan dual PD, PSD, dan beberapa matriks ID telah didapati dengan tiada jurang dualiti (duality gap). Ini adalah disebabkan oleh matriks membentuk satu set cembung (convex) dalam masalah LP. Walau bagaimanapun, terdapat tiada penyelesaian optimum untuk ND, NSD, matriks ID, dengan syarat bahawa beberapa matriks ID adalah fungsi cekung (concave). Jika koefisien matriks masalah LP adalah matriks tak tunggal (nonsingular), maka penyelesaian LP tidak semestinya mempunyai penyelesaian optimum. Dalam erti kata lain tidak ada korelasi antara ketunggalan (singularity) matriks dan optimality dalam penyelesaian LP dualiti. Keputusan eksperimen oleh simulasi dalam MATLAB dan EXCEL Solver menunjukkan bahawa penyelesaian yang optimum penyelesaian primal dan dual adalah sama di PD dan PSD. Kajian ini menyediakan penemuan baru tentang kesan ciri-ciri yang berbeza lima jenis matriks simetri dalam masalah dualiti LP yang boleh menyumbang kepada kemajuan teori dan amalan LP.

ACKNOWLEDGEMENT

Alhamdulillah, I thank to Allah the Almighty for His blessings. I would like to express my sincere gratitude to my supervisor Assoc. Prof. Dr. Adi Saptari for giving constructive advice to my master thesis, my co-supervisors Cik Muzalna bt. Mohd Jusoh and Pak Taufik for all their guidance and support throughout my study. At the same token, I would like also to thank to The Ministry of Higher Education Malaysia for giving financial support under Fund Research Grant Scheme (FRGS).

I thank to all lecturers of Department of Manufacturing Management, staffs and technicians who help me to solve technical problem during finishing my master degree. I also, thank to all colleagues of Faculty of Manufacturing Engineering postgraduate students for their helps in providing advices, support and togetherness. In addition, I thank to members of Indonesian Student Association who have provided material and spiritual support until this thesis completion.

I express my sincere appreciation to my mother for her unlimited supports during my study, ibu Sri Rahayu, and my lovely sisters, namely, mbak Roza, mbak Dwi and mbak Ana. At the end I would like to express my deep gratitude to someone who to be a part of my life, Annisa Wulandari, in which she has helped me in English and giving encouragement throughout the study. I really owe her a big share of this thesis. Hopefully Allah SWT would give His blessing and gathering us into the goodness. Aaamiiin.

DECLARATION

I declare that this thesis entitle "**AN ASSESSMENT ON THE EFFECT OF DIFFERENT CHARACTERISTICS OF SYMMETRIC MATRICES TO PRIMAL AND DUAL SOLUTIONS IN LINEAR PROGRAMMING PROBLEMS**" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ....

Name : Ikhsan Romli

Date : 18 July 2012

TABLE OF CONTENT

LIST OF FIGURES

LIST OF ABREVIATIONS

LP	-	Linear Programming
MP	-	Mathematical Programming
IPM	-	Interior Point Method
PD	-	Positive Definite
ND	-	Negative Definite
PSD	-	Positive Semi-Definite
NSD	-	Negative Semi-Definite
ID	-	Indefinite
RHS	-	Right Hand Side
OFC	-	Objective Function Coefficient
Var	-	Decision Variable

CHAPTER 1

INTRODUCTION

1.1 Background

Nowadays global competition among manufacturing industries is intensifying. In order to survive in this competition many companies need to implement a continuous improvement in terms of performance in many sectors or areas such as quality, cost, delivery, flexibility, and environment standard (Vachon and Klassen, 2008). These aspects are related to managing operations. Basically, manufacturing operations have three types of activities, i.e. supply chain, production, and delivery, in which issues often come up while these activities are being undertaken such as in production scheduling, mix product, blending problems, transportation, assignment problems and inventory applications. These problems need to be handled by using solution techniques which can provide the optimal solution.

One of the techniques which are applied in such improvement is an optimization or in other term sometimes called Mathematical Programming solution techniques. The term optimization has been used in engineering, operations research, and operations management for decades. Finding the most efficient utilization for resources need to be conducted in the optimization of a manufacturing system as an effort to find the optimal solutions. An initiative adopts the mathematical techniques to arrive at the best solution, considers what is being optimized (e.g. profit, cost, quality, market share, sales, or time).

In most of engineering problems, the decision makers need a method to find out the optimum solution of the problem particularly in the planning of manufacturing or the production engineering common problems, to find minimum cost or maximum profit with limited resources. In order to find out the optimum condition, one needs to formulate it into a mathematical model. The model can be formulated from a relation of entries variables involved. Once the formulation of the mathematical model is identified, this requires a validation to ensure that it represents the problem encountered. Assuming the validation has been performed, then mathematics procedure can be applied to find the optimum solution (McCarl and Spreen, 1996).

Depending on the problems, the optimization techniques are helpful on finding the minimum of a function of several variables under a determined set of constraints. The models of the techniques such as the Linear Programming, Quadratic Programming, Geometric Programming, Nonlinear Programming, Dynamic Programming, Integer Programming, Stochastic Programming, Multi-objective Programming, etc. One of the models widely implemented in the manufacturing operation is the Linear Programming, in which the objective function is a linear function of the proposed variables and the constraints are linear equations or inequalities (Ravindran *et al.* 2006).

Firstly, the Linear Programming model was introduced by Dantzig (1951) is often abbreviated to LP. Even though this model was founded at a half-century of history, it is still an active field area for research and important method for modelling and solving problems in the 21^{st} century (Dowman and Wilson, 2002). The LP models involve the optimization of linear function, which assumes relationship among the linearly variables. There are three requirements in the LP, namely an objective function, independent variables and subject to linear constraints which may be either equalities or inequalities.

Associated with every LP problem, called primal or original problem, there is another LP problem called the dual that basically is a mirror of the primal problem. It can be applied to get original program solution and exceed advantageous knowledge of the optimal solution to the primal problem (Bazaraa *et al.* 2009). These two problems have very interesting and closely related properties. If the optimal solution to one of the problems is known, the optimal solution to another can be easily obtained. Because of these properties, the solution of the LP problem can be obtained by solving either the primal or the dual, whichever is easier.

A significance of the optimal solution has drawn a lot of researchers' interest to consider and develop the primal and the dual solutions or the duality in the LP. The researchers have worked through the improvement of the LP duality, i.e. a constructively evidence of main theorem for the LP by an elementary inductive argument which directly establishes the duality results, a large number of transposition theorems, including Farkas Lemma, and derived by Balinski and Tucker (1969), the evidence of properties of the duality relationships has been proven utilized by Corley (1983), a weak and a strong dualities in an infinite dimensional (Romeijn, *et al.* 1992). Duality theory was frequently helpful in providing a certificate of the optimality (Robert, 2007).

Development of the techniques in solving the LP problem continues to grow over time. Commonly, there are two methods that are frequently improved by researchers in the

LP problem: Simplex Method discovered by George Dantzig (1951), Interior Point Method revealed by Kamarkar (1984). Recently, Todd (2002) stated that effectiveness of excellent performances of the simplex methods and the interior point methods are alike for the practiced applications of the LP. Nevertheless, for particular types of the LP problems, it may be that one type of the methods is better than another type. The Simplex method is usually used in a small or a medium-scale problem properly within measure of the coefficient matrix with order of 50 by 100, whereas the interior point method is more efficient for some large-scale problems. Thus, the position of the interior point method is not totally reasonable while the Simplex method is still able to overcome the interior point methods on many real life models.

1.2 Problem Statements

Relying on a collection of related publications with respect to the symmetric matrices and the duality in the LP, the major problems associated with the LP have been identified as follows:

1. Solving the LP problem normally used the Simplex methods in which can be terminated, by returning a feasible solution or a non-feasible one. However, it will not be able to know whether the feasible solution is optimal. Therefore, a concept of the LP duality is used to prove that the LP is the optimal solution. In particular, a post-optimality analysis or a sensitivity analysis depends on almost entirely on an understanding of the duality.
2. LP equations system can be concisely written as $A_{mxn}X = b$, where A_{mxn} is the coefficient matrices that may have many different structures. One of the structures is the symmetric matrix A_{nxn}. These matrices have five properties they are Positive Definite (PD), Negative Definite (ND), Positive Semi-definite (PSD), Negative Semi-definite (NSD), and Indefinite (ID). The study on the effect of different characteristics of symmetric matrices in the primal problem has not been explored by researchers.
3. Furthermore, there are many researches about dual problems which have been observed, yet the previous research in regards to the characteristics of the symmetric matrices in the dual problem has not been performed yet. So this research will explore about the effect of the symmetric matrices in LP (PD, ND, PSD, NSD, ID matrices).

1.3 Research Questions

Once the problem statement has been elaborated, research questions can be generated based on the problem described above are as follows:

1. How to build the randomly symmetric matrices that belong to five categories of its characteristics (the PD, the ND, the PSD, the NSD, the ID)?
2. What are implications of the different characteristics of the randomly symmetric matrices that belong to five categories of their characteristics to the primal and the dual solutions?
3. What are the effect of number of the variables and the constraints (the symmetric matrices) in the primal-dual solution?

1.4 Objectives

The goal of this research work is to develop duality theory in the LP for the various characteristics matrices. The specific objectives in this study are described in the following sentences:

1. To simulate the LP problems with the various number of the variables and the constraints based on the different characteristics of the randomly symmetric matrices.
2. To investigate the effect of different characteristics of the matrices on the primal solution of LP problems.
3. To investigate the effect of the different characteristics of the matrices on the dual solution of the LP problems.

1.5 Scopes of the Study

Scope of this research covers the following:

1. The Simulation of problem of the LP with the sizes of n variables and m constraints of a random number generation is used to create the problem simulation based on the character of each matrix i.e. the PD, the ND, the PSD, the NSD, and the ID.
2. A verification of the symmetric matrices are required, the procedure of checking is imposed.
3. A number of problems based on the different sizes of the matrices which represent the number of the variables and the constraints are tested by the difference of the matrices.
4. The MATLAB Software and the EXCEL Solver are used to solve the LP problems created.

1.6 Outline of Chapters

This thesis is structured in five chapters as follows:

Chapter 1 introduces background of the optimization development and a reason of the linear optimization. The problem statements and the research questions are stated to show the explanation of the real problem in this thesis area. The objective and the scope are also stated to achieve target and limit of this thesis. The chapter is ended by presenting outline chapters that explain the general overview of chapter by chapter.

Chapter 2 elaborates on brief literature review about the theory of the Linear Programming. It covers some previous related works, namely, the concept and development of Linear Algebra, the symmetric matrices as well as convexity.

Chapter 3 describes method of this research in detail to give the step by step in performing this research. It starts with generation of some random parameters of the Linear Programming. The verifications of the symmetric matrices based on its properties are depicted briefly in this section. The important issues in solving the Duality Linear Programming model on the MATLAB are then illustrated in detail. The validation is summarized at the end of this chapter.

Chapter 4 gathers and identifies results of the simulation of the LP model. The first section includes a comparison between the simulation results using the different characteristics of the symmetric matrices in the primal and the dual LP problems. The analyses of generated symmetric matrices are covered in the second section. The third section takes account of an evaluation of the simulation approach. The analysis of the simulation results of the study is at the last section of this chapter.

Chapter 5 presents a conclusion of the research and suggests further work related to this area.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter discusses some basic theories of the LP and the current development of the LP algorithm. This chapter also presents the theory of the linear algebra related to the LP, the symmetric matrices, the concavity and the convexity.

2.2 Theory of Linear Programming

Applied decision theory is a name given to a science in developing solutions to the real world problems. A part of such a theory is the mathematical programming. If the objective function and constraints are bounded to the linear functions, then a scope of the mathematical programming narrowed down to the Linear Programming (Armstrong, 1973). Meanwhile, in an opinion of Rao (2009) stated that LP was meant as an optimization method usable for problems solution, which the objective function and the constraints organized as the linear functions of decision variables. Luo *et al.* (1990) contended that LP was the essential solution procedure for the optimization problem, which were mostly concerned with the efficient utilization or an allocation of the limited resources in order to meet the desired objectives. This model was one of the powerful optimization technique and an important subject in the areas of engineering, science, and business (Kawaguchi and Nagel, 2008). Numerous preceding researches have discussed regarding the applications of the LP in the area of the engineering, namely, in a systems and a control (Hovelaque *et al.* 1995), an automation (Cogill and Hindi, 2007), a computer modeling (Alejandre *et al.* 2000).

The optimal production plan in a manufacturing company uses the LP model as a tool in order to maximize the profits or minimize the costs with limited resources. The company may have a variety of choices by considering sales of a firm rises and falls. It is able to construct a stock of manufactured goods to take it through period of peak transactions, but this engaged a stock holding cost. These may also give overtime rates to achieve higher production along the periods of higher order. Finally, firm does not need to meet the additional sales demand during the peak sales period, thus miss the potential profit. The LP model can take into account various cost and lost factors and reach at the most profitable production plan (Ravindran, 2008).

The LP model is now well established as the important tool to apply in many industrial areas and very active branch of applied mathematics that it is focused on the allocation or the powerful use of the limited resources to obtain the desired objectives. The wide-spread applicability of the LP models and the rich mathematical theory underlying these models and the methods developed to solve them have been driving forces behind a rapid and continuing evolution of the subject. Some of the major application areas to which the LP can be applied are namely, blending, production planning, oil refinery management, distribution, financial and economic planning, manpower planning, blast furnace burdening, farm planning (Kawaguchi and Nagel, 2008).

The Linear Programming problem has a structure in form of:

Scalar Form

Maximize/Minimize $f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots\dots + c_nx_n$ (2.1a)

Subject to the constraints

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n &\geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n &\geq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_n &\geq b_m \end{aligned}\right\} \quad (2.2a)$$

$$\begin{aligned} x_1 &\geq 0 \\ x_2 &\geq 0 \\ &\vdots \\ x_n &\geq 0 \end{aligned} \quad (2.3a)$$

where c_j, b_j, and a_{ij} $(i = 1, 2, \dots, m;\ j = 1, 2, \dots, n)$ as the matrices or vectors constants, and x_j are the decision variables.

Matrix Form

Max/Min $f(X) = c^T X$ (2.1b)

Subject to $A_{mxn}X \geq b$ (2.2b)

$X \geq 0$ (2.3b)

where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix},$$

$$A_{mxn} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Attributes of the LP problem, stated in a standard form, are:

1. The objective function is of the maximization or minimization type, at equation (2.1).
2. All the constraints are of the inequality type, at equation (2.2).
3. All the decision variables are nonnegative, at equation (2.3).

It is revealed that any LP problem may be expressed in the standard form by using following transformations.

a. The maximization of the function $f(x_1, x_2, \ldots, x_n)$ is alike to the minimization of the negative of the same function. For example, the objective function

   Minimize $\quad f = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$

   is equivalent to

   Maximize $f' = -f = -c_1x_1 - c_2x_2 - \ldots\ldots\ldots\ldots - c_nx_n$

b. Almost of the engineering optimization problems, the decision variables stand for some physical dimensions, and thereby the variables x_j are nonnegative ($x_j \geq 0$).

c. If the constraint shows in the form a "less than or equal to ($\leq$)" type of the inequality as

$$a_{k1}x_1 + a_{k2}x_2 + \ldots\ldots + a_{kn}x_n \leq b_k$$

   It is able to be changed into the equality form by adding a nonnegative slack variable x_{n+1} as follows:

$$a_{k1}x_1 + a_{k2}x_2 + \ldots\ldots + a_{kn}x_n + x_{n+1} = b_k$$

   When the constraint is in the form of inequality ($\leq$), the right-hand side generally represents the restriction of available resources, while the left-hand side represented the use of these limited resources by the variety of different activities (variable) of the model. In terms of this, the slack variable represented the amount of the un-used resources.

   Likewise, if the constraint is in the form of a "greater than or equal to ($\geq$)" type of the inequality as

$$a_{k1}x_1 + a_{k2}x_2 + \ldots\ldots + a_{kn}x_n \geq b_k$$

It is able to be changed into the equality form by subtracting a variable as

$$a_{k1}x_1 + a_{k2}x_2 + \ldots\ldots + a_{kn}x_n - x_{n+1} = b_k$$

where $-x_{n+1}$ is the negative variable known as the surplus variable. The constraint is in form of ($\geqslant$) commonly determines requirement of minimum specification, in which the surplus variable represented the amount of the excess of the limited resources that was compared with the minimum specification.

It indicates that there are m equations in n decision variables or unknowns in the LP problem. Assuming that $m < n$; for if $m > n$, there will be $m - n$ redundant equations in which can be reduced. There is a unique solution that meets equations (2.2) and (2.3) (where in case there can be no optimization) or no solution, in which case the constraints are inconsistent. The case $m < n$ corresponded to an adjusted format of the linear equations that whether this problem has one solution or one infinite number of solution. The problem of the LP is to get one of these solutions that meet equations (2.2) and (2.3) and the produces the optimal solution of f.

A review of basic definitions exerts the standard form of the LP problem as follow:

i. An extreme point or a Vertex. This is a point in the convex set which does not lie on a line segment joining two other points of the set.

ii. The feasible solution is a non-negative vector x meets the constraints $A_{mxn}X = b$.

iii. A feasible region, denoted by S, is the set of all of the feasible solutions. The feasible region for any LP is the convex set. If an LP has an optimal solution, there is an extreme (or corner) point of the feasible region that is an optimal solution to the LP. In other word, the LP problem is said to be feasible if there exists at least one feasible point, and infeasible otherwise. If the feasible set S is blank, then the linear program is said to be infeasible. Mathematically, $S = \{X | A_{mxn}X = b, X \geq 0\}$.

iv. A Basic solution is one in which the number of the equations subtracted to the number of the decision variables are set equal to zero $(n - m = 0)$. The basic solution can be obtained by setting $n - m$ variable to zero and solving the constraint equation (2.2) simultaneously.

v. Basis is a collection of the variables that not set equal to zero to obtain the basic solution.

vi. A basic feasible solution is that meets the non-negativity conditions of the equation (2.3).

vii. A non-degenerate basic feasible solution. This is the basic feasible solution that has got exactly m positive x_i.

viii. The optimal solution is the vector x° such that it is feasible, and its value of the objective function (cx°) is greater than that of any other feasible solution. Mathematically, x° is optimal if and only if $x^\circ \in S$, and $cx^\circ \geq cx$ for all $x \in S$.

ix. The optimal value of the LP is the value of the objective function according to the optimal solution. If Z° is the optimal value, then $Z^\circ = cx^\circ$.

x. An optimal basic solution. This is the basic feasible solution for which the objective function is optimal.

xi. As LP has more than one optimal solution, it is said to have alternative optimal solutions. In this case, there exists more than one feasible solution possessing the same optimal value (Z°) of the objective function.

xii. The optimal solution of the LP is said to be unique when only one optimal solution exists.

xiii. As the LP does not has a finite optimum (i.e., max $Z \rightarrow +\infty$ or $Z \rightarrow -\infty$), it is called as an unbounded optimum.

The LP problem can be accomplished both geometrically and algebraically. The two technical approaches produce final answer identically, but one or another may be easier for answering a certain question on the linear program. A perspective of a geometric is built on the geometry of the feasible region, for example as using the concept of the convexity to observe the linear program. The theories in the LP are simpler to understand by applying this approach, specifically for two-dimensional problems in which the feasible region can be depicted because it can be described in terms of insightful ideas such as moving along a border of the feasible region.

On the other hand, an algebraic point of view is built on writing the LP in a particular way, called as the standard form. Then, utilizing the instruments of the linear algebra made the coefficient matrix of the constraints of the LP can be analyzed. One of the algebraic approaches is used in the Simplex method (Griva *et. al.* 2009). In this study, this presents the theory, the development, and the applications of the Simplex method for solving the LP problem.

In many practical applications, elements of LP parameters (A_{mxn}, b, c) are not known constants and will best be described by the random variables. For the manufacturing example; a product-mix problem, the elements of $(c_1, c_2, \ldots, c_n)$ represent the contributions on a per unit basis and are very seldom known with a certainty. Losses, spoilage, an inventory control, and uncertain demands require the elements of c to be expressed as random variations for the next period. Similarly, the available resources

$(b_1, b_2, \ldots, b_m)$ and the resource requirements for each product $(a_{11}, a_{12}, \ldots, a_{mn})$ vary due to machine breakdown, quality control, technology, and other unforeseen circumstances (Armstrong, 1973).

In the manufacturing company problem, the constraints that are peculiar often affect the job processing, in which these have many distinctive characteristics. Some of the most common processing characteristics and constraints are namely, precedence constraints, machine eligibility constraints, workforce constraints, routing constraints, material handling constraints, storage space and waiting time constraints, and transportation constraints (Pinedo, 2009). One of the parameters that affect the solution of the LP problem is the constraints. The LP solution can be analyzed by part of its constraints, that is, the shape of its coefficient matrix. Recently, several researchers (Qiao and Wang, 1993; Qiao *et al.* 1994) have many considerations to the issue of combining fuzziness and randomness in the constraint for the LP problems.

2.2.1 Linear Programming Modeling

A process of representing the real world problem through the system of mathematical formulas and expression based on key assumptions, estimations, or statistical analysis is called modelling (Ravindran, 2008), in which the problem can be words, pictures, etc. One of the popular problems that can be formulated to a mathematical statement is the LP problem. Commonly, formulating the LP Model needs to be structured in five steps which are; first step is selection of a time horizon, second step is selection of the decision variables and parameters, third step is definition of the constraints, fourth step is selection of the objective function, the last step is solution and analysis (Bradley *et. al.* 1977). Recently, the processing steps of the whole LP model formulation, solution and result investigations by Ballesteros *et al.* (2002) as recognized in Figure 2.1 is illustrated in this situation.