DEVELOPMENT OF OUTPUT FEEDBACK SLIDING MODE CONTROL FOR NONLINEAR SYSTEM WITH DISTURBANCE

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bstract- **This paper discusses about the development of output eedback Sliding Mode Control (SMC) and demonstrates that his controller is able to reject disturbance in a nonlinear system. he designed SMC controller works well to ensure system tability and eliminates disturbance in linear and nonlinear ynamics systems. The approach to develop output feedback ontroller is by implementing High Gain Observer and the utput performance from this controller is compared with the utput response from the state feedback SMC. Both designed ontrollers are constructed in simulation by using MATLAB IMULINK tool. The required gains involve in the designed ontrol laws are tuned heuristically by trial and error approach. he state response results obtained from the simulations are nalyzed. Based on the observation and analysis to the results it hows that the output feedback SMC illustrates smooth state esponse and reduces the effect of disturbance as provided by the tate feedback (SMC) without disturbance.**

eywords: SMC controller, High Gain Observer, control law, ATLAB/SIMULINK, Simulation.

I. INTRODUCTION

Sliding Mode Control (SMC) is a technique where the nitial state of the dynamic system is forced to follow the esired structure which is called as sliding surface and then the tate will follow this surface behaviour afterward. This pproach has been reported as a powerful approach for solving arious applications in the nonlinear control systems.

The development of SMC is derived based on the concept f Variable Structure Control (VSC) that mainly about witching the nature of the dynamical system to another ifferent system structure [1]. There are two most important rocedures involve in SMC technique. First, an effective witching surface is designed to ensure that the system motion vill meet the required specifications during on the sliding tructure. Secondly, it is essential to deploy a sufficient control aw that will make the system's state attracted to the switching urface [2].

A number of researches have been done in exploring the eneficial for using VSC as well as SMC. Historically, the arliest work was started by Flügge-Lotz in 1950's. Then the esearch was continued in 1960's by Russian researchers named Emel'yanov and Barbashin in 1960's [3]. In mid 1970 the theory of VSC and SMC started to spread into the world after a book authored by Itkis in 1976 and a survey paper by Utkin were completely published in English [2]. After 50 years the development of this technique has been evolved and has been applied in many applications such as aircraft controller, DC Motor, furnace temperature control and etc.

SMC becomes popular in control application because of its ability that will robust the system performance by absolutely rejecting the presence of disturbance and insensitive to parameter variations [1]. Furthermore, SMC is a useful controller that able to works very well with a linear and nonlinear dynamic system.

Diong [10] had developed sliding mode controller to stabilize a nonlinear system model. The controller demonstrates reasonable disturbance rejection and obtains good settling time behavior from any initial conditions. To control the nonlinear speed control for a nonlinear DC Shunt Motor, SMC controller is applied and as the result close loop system performance improved and torque disturbance was eliminated much better [11].

The Sliding Mode Observer has been applied for disturbance rejection and servo-tracking in the nonlinear process control, specifically in the chemical reactors. The norms of estimation errors are analyzed to construct modelling errors for ensuring robustness in the process control [16].

The scope of this research study is to implement a SMC controller into a strongly nonlinear system. There are two different SMC controllers developed in this study which focus on the state feedback SMC and High Gain Observer SMC, (HGO-SMC). These approaches are demonstrated in 3-rd order nonlinear disturbed system model. During the SMC analysis, the dynamic of the nonlinear model is considered single input single output (SISO) system and the disturbance is assumed bounded and matched. The designed controllers are simulated in MATLAB software via SIMULINK tool. The nonlinear system model is taken from [21] and it is tested with those two approaches. Practically this project is aimed to ensure the nonlinear system become stable and the presence of disturbances and any uncertainties could be completely rejected by implementing SMC either following the state feedback or output feedback principles.

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II. CONTROLLER DESIGNS

Two different design procedures are developed in this esearch paper. The first procedure is conventional SMC based n state feedback controller technique and the second rocedure is HGO-SMC. The development of SMC is derived ased on the concept of Variable Structure Control (VSC) that ainly about switching the nature of the dynamical system to nother different system structure [1]. There are two important ey points in developing SMC technique. First, an effective iding surface is designed to ensure that the system motion ill meet the required specifications during on the sliding ructure. Secondly, it is essential to construct a sufficient ontrol law that will make the system's state attracted to the witching surface [2]. The development of conventional SMC d the HGO-SMC are different according on how the sliding rface and the control law for each SMC controllers to be eveloped.

. *Nonlinear System with Disturbance*

The 3^{-rd} order nonlinear system is described in this research udy is described as below [21].

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_1 + x_2 + 2.5x_3 - x_1^{\ 2} + 5\sin(\pi t) - \cos(2\pi t) + u(t) \qquad (1)$$

The nonlinear terms in this system is

$$f(x) = -6x_1 + x_2 + 2.5x_3 - x_1^{\ 2} \qquad (2)$$

The disturbance $d(t)$ presence in this system is set to be a nusoidal function.

$$d(t) = 5\sin(\pi t) - \cos(2\pi t) \qquad (3)$$

d the disturbance is assumed bounded with some kind of nction.

$$|d(t)| \leq K \qquad (4)$$

The initial states of the state variables are set based on the tting value decided by [21].

$$[x_1 \ \ x_2 \ \ x_3] = [0.5 \ \text{-}0.8 \ \ 0.25] \qquad (5)$$

B. *State Feedbak SMC Controller*

In the state feedback SMC all state variables in the nonlinear system is considered feasible to be measured. The stable sliding surface is constructed as:

$$s = x_3 + C_2x_2 + C_1x_1 \qquad (6)$$

The control input to be applied in the conventional SMC is decided to be:

$$u(t) = -f(x) - C_2x_3 - C_1x_2 - K\,sgn(s) \qquad (7)$$

To ensure the stability of the dynamic system is achieved, a Lyapunov function candidate is performed and the mathematical analysis is shown as below.

$$V = \frac{1}{2}s^2$$

$$\dot{V} = s\dot{s} \leq 0 \qquad (8)$$

At the reaching condition

$$\dot{V} = s\dot{s} = s[f(x) + u(t) + d(t) + C_2x_3 + C_1x_2] \leq 0 \qquad (9)$$

The $\dot{V}$ will always become negative semidefinite, $\dot{V} \leq 0$ if the control input $u(t)$ is chosen as in (7).

Therefore,

$$\dot{V} = s\dot{s} = s\big[f(x) + d(t) + bx_3 + ax_2 - f(x) - bx_3 - ax_2 - Ksgn(s)\big] \leq 0$$

$$\dot{V} = s\dot{s} = s\big[d(t) - Ksgn(s)\big] \leq 0$$

$$\dot{V} = s\dot{s} = |s|\big(d(t) - K\big)$$

$$\dot{V} = s\dot{s} = -|s|\big(K - d(t)\big) = -K|s| \leq 0 \qquad (10)$$

This method required to tune several gains that are C_1, C_2 and K. These gains are tuned heuristically.

C. *High Gain Observer SMC*

In practical control application not all state variables are able to be measured. There might be a part or only one state vector are measureable. Therefore it is convenient to develop Sliding Mode Control based on output feedback form.

One possible solution is by reconstruct the output state variable based on estimating state variables and this is called observer. Observer can work well with the linear and nonlinear dynamic system but the designed structures for both cases will be different. In this study the nonlinear SMC observer is designed by integrating High Gain Observer principle based on theoretical described in [6] into the SMC structure to construct HGO-SMC.

To implement output feedback control using only measurement of the output, y the high gain observer model is developed as below:

$$\dot{\hat{x}}_1=\hat{x}_2+\left(\frac{h_1}{\varepsilon}\right)(y-\hat{x}_1)$$

$$\dot{\hat{x}}_2=\hat{x}_3+\left(\frac{h_2}{\varepsilon^2}\right)(y-\hat{x}_1)$$

$$\dot{\hat{x}}_3=f_o(\hat{x})+u(\hat{x},t)+\left(\frac{h_3}{\varepsilon^3}\right)(y-\hat{x}_1)$$

$$f_o(\hat{x})=-6\hat{x}_1-\hat{x}_1^2 \qquad (11)$$

where $[\dot{\hat{x}}_1\ \dot{\hat{x}}_2\ \dot{\hat{x}}_3]^T$ are the state vectors of the model bserver, and $[\hat{x}_1\ \hat{x}_2\ \hat{x}_3]^T$ are estimated state variables. The otation ε is a real positive constant to be specified.

The function $f_o(\hat{x})$ is a nominal model of $f(x)$ which is equired to be considered as locally Lipshitz in its arguments ver the dominant of interest and globally bounded in x [6]. urthermore the initial condition for the function $f_o(\hat{x})$ is set to e all zero; $f_o(0,0,0) = 0$.

The stable sliding surface is also designed base on the stimated state variable and it is defined as:

$$s=\hat{x}_3+C_2\hat{x}_2+C_1\hat{x}_1 \qquad (12)$$

All the actual states are changed with the estimated state ariables. The control input becomes:

$$\hat{u}=\beta(\hat{x})-Ksgn(\hat{s})$$

$$u(\hat{x},t)=-\{-6\hat{x}_1-\hat{x}_1^2+C_2\hat{x}_3+C_1\hat{x}_2+Ksgn(\hat{s})\} \qquad (13)$$

The tuning procedures for gains C_1, C_2 and K are similarly ned as in state feedback SMC technique.

III. RESULTS AND DISCUSSION

State Feedback SMC

Fig.1 shows the behavior of state variables for 3^{-rd} order onlinear system in (1) when no disturbances are introduced. ne state variables of the dynamic system named as x_1, x_2 nd x_3 . All states move from their initial condition to a stable ondition at the 0 level after 2 seconds.

State x_1 is the fastest state converges to 0 and also very uickly remains stable after 2.5 seconds. State x_2 and x_3 ecome stable at 0 after 3 seconds. State x_3 has an overshoot bout 1.13 at 0.2 second and then decreases to converge on the able condition. All states become asymptotically stable at 0 ter 3 seconds.

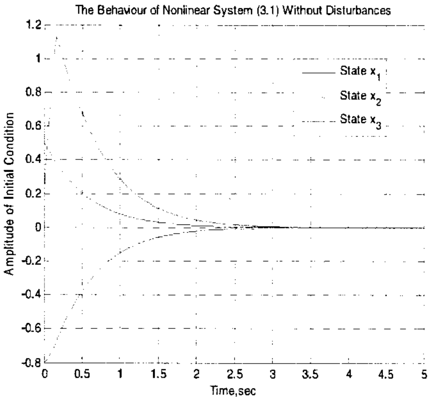

Figure 1. Fig.1: The behavior of 3rd order nonlinear system in (1) without disturbance.

Now the nonlinear model in (1) is simulated by considering the presence of sinusoidal disturbances described in (3). At this stage how the conventional SMC approach eliminates disturbances and provides stability are observed. The output response with the presence disturbance is shown in Fig.2

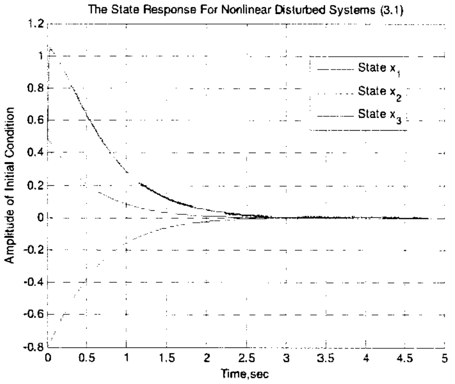

Figure 2. Fig.2: The behavior of 3rd order nonlinear system in with disturbance.

At this time the SMC gain K must be chosen properly because the SMC gain K not only provide stability, it also ensured that the disturbances are eliminated.

From the trial and error solution, the sufficient value for control gains C_1, C_2 and K have been selected as 6, 5 and 70 respectively. The simulation results of this conventional SMC procedure are shown in Fig.3. A saturation function also is added in the control law to reduce the chattering effect in the output system response.

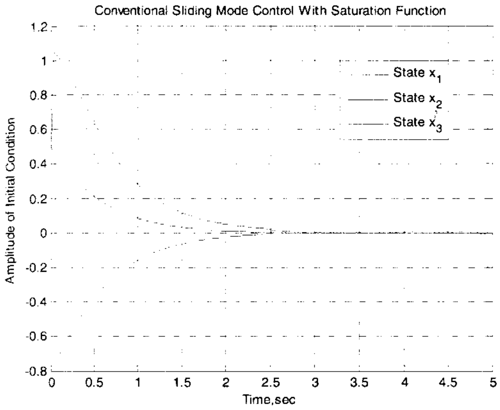

Figure 3. Fig.3: The output response of 3rd order nonlinear system after eliminating disturbance.

All state variables become asymptotically stable after 3 econds. The trajectories for all states look smoother. The vershoot in the state x_3 has been reduced from 1.13 to 1.05 nd the small oscillation has been eliminated. With the new MC gain value, all the state response behaviour has been nproved and the disturbance is successfully rejected.

High Gain Observer SMC

In development of HGO-SMC, only state variable x_1 is vailable can be measured and it is considered as the output of e dynamic system. To estimate the output state variable and iminate the effect of disturbance several gains are required to e tuned. The suggested gains to implemented result from trial nd error procedure are shown in Table 1.

TABLE I. THE VALUE OF HGO-SMC GAINS

Parameter Gain	Value
h_1	8
h_2	7
h_3	7
ε	0.03
C_1	25
C_2	20
K	150

Fig.4 shows the output state response for measured and timated state x_1. The measured state x_1 and estimated state are converging to a stable condition at 0 magnitudes after 3 conds. At early 0.5 seconds the measured state x_1 decreases ntil -0.2 magnitudes then rises to achieve the stable condition. or the observer model, the state $\hat{x}_1$ starts from 0 magnitude en suddenly increases to 0.5 to follow the actual state x_1 and ter that gradually decreases as the actual state x_1 decreasing uring the early 0.5 seconds.

The stability of measured state variable has been ccessfully stabilized by implementing the high gain observer the sliding mode control design. The actual state variable has been successfully constructed through the estimated state variable.

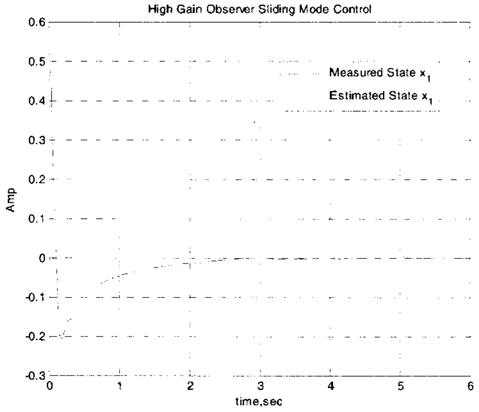

Figure 4. Fig.4: The comparison between the measured state variables x_1 and the estimated state variable $\hat{x}_1$

Fig.4 also shows how good the estimated variable follows the actual state variable. The blue line is referred to the measured state variable x_1 and the red line shows the estimated state variable $\hat{x}_1$. At 0 second, both states start with different initial value but when the high gain observer is applied the estimated state has been forced very quickly to follow the measured state. The consistency of both trajectories shows that the disturbance term presence in the actual plan has been eliminated by incorporating the high gain observer into sliding mode control.

The designed structure of the sliding surface is quite similar between conventional SMC and high gain observer SMC. The only different is, in the conventional SMC the actual state variables are used to design the stable sliding surface but in the high gain observer SMC estimated state variables are applied in the designed sliding surface.

The sliding surface response for both structures will be different at early few seconds depending on the structure of the control input and the initial condition of state variable. The time taken for each sliding surface response become stable is also different. In the conventional SMC, the sliding surface response becomes stable after 0.2 seconds but the sliding surface response for high gain observer SMC is a bit late at about 0.5 seconds. There are small oscillations along the stable condition in the conventional SMC sliding surface but in the high gain observer, the sliding surface seems smoother along the stable condition.

IV. CONCLUSION

This study has been centre upon the implementation of sliding mode control as a convenient tool to eliminate disturbances and establish the stability analysis for the nonlinear disturbed dynamic system. It is concluded that the stability of the nonlinear disturbed system is depending significantly with the sufficient value of controller gains. For state feedback SMC design the value for gain K is set to be 70,

$_1$=6 and C_2=5 in order the stability can be established and isturbance is eliminated. The parameters in the high gain bserver SMC are more than previous designed SMC. Those arameters are $h_1, h_2, h_3, C_1, C_2, K, \varepsilon$ and their values are 8, 7, 7, 5, 20, 150 and 0.03 respectively. Since the stability analysis in MC principle is derived based on Lyapunov function, all arameters involve in the SMC design must be real positive efinite.

From observation all states in the nonlinear disturbed ystem converge to the stable condition after 3 seconds. This esult proves that those two different designed SMC able to ring all the state variables into asymptotically stable. In ddition it also shows that the sinusoidal disturbances affecting e state variable's trajectories have been completely iminated. However there is chattering problem affected the ontrol input in state feedback SMC. A further research study required to improve and reduced the issue of chartering roblem.

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