STABILITY STUDY OF PD AND PI CONTROLLERS IN MULTIPLE DIFFERENCE DISTURBANCES

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Stability Study of PD and PI Controllers in Multiple Difference Disturbances

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bstract—**This paper discusses the stability study of PD and PI ontrollers in multiple difference disturbances. The multiple ifference disturbances in this paper are added to the inverted endulum model that based on robotic leg application such as endubot. By applying the pendubot model via ATLAB/Simulink block diagram, the performances between he model and disturbances are compared for stability in the imulation results. The simulation results showed that the PD ontroller could reduce and eliminate disturbances more ffective than PI controller in the pendubot model. Overall, the imulation results are based on stability analysis for the degree f stability, steady state performance and transient response.**

Keywords-inverted pendulum; pendubot; PD controller; PI controller; disturbance

I. INTRODUCTION

Technology development growth fast onward through ver the world including Malaysia. Each part of engineering ools is used to build and design the technology such as obot, all types vehicle, building and others. These echnology especially robot application, need a control ystem approach to organize, monitor and stabilize any novement. Within this control system approach, the robot vould operate smoothly. In case, the stability in control ystem is studied for the robotic leg application.

The robotic leg application gives advantage in the endubot at which also called the arm-driven that do tasks ike manipulating, moving, and painting. The tasks are sually found in industrial automation, architecture and rtistic. For the industrial automation as an example, the igh quality products are a primary goal to achieve. To chieve the goal, the arm-driven robots are developed more an a workers to maintain the products quality. This quality ased on the stability [1] in control system that is being xplained by James Clerk Maxwell, Edward John Routh, Villiam Kingdon Clifford, Adam Prize, and Alexandr Aichailovich Lyapunov in the latter half of the 19th century. he stability of the robotic leg application in this paper is e main problem for the study purposes with the PD type ontroller in the multiple difference disturbances.

The PD type controller refers to the familiarity terms of the Proportional (P), Integral (I) and Derivative (D) in the PID controller that are well-known as a conventional controller since the last few years ago. These terms are quite importance in define the gains P, I and D in the root-locus techniques. The root-locus techniques are based on the development of the advanced control systems from the state-space model. The state-space model refers to the inverted pendulum as apply in a robotic leg application such as the pendubot.

In a robotic leg application, the PID controller is not depending by self. Looking through from the previous research, the adapting idea of the PID controller [2], the researcher is preferred to use the PD than the PID controller as the PD controller is based on human naturally acts. Considerably from other authors paper: the study of the development of PI controller for disc speed [3], shows that PI controller is also a quite useful for eliminating and reducing disturbance.

Sometimes, the researcher looking for a wide view of research study, as posted in the article of observer control improves motion [4] by Kristin, found that the observer control enable to eliminate ringing and overshoot, and also solve the problem of the PID loops disable to do. By the way, this paper is focusing on the PD and the PI controllers for comparison stability in the multiple difference disturbances. Based on a few review of the introduction, the minority contents of the paper included the model, the controller design in general view, simulation results, conclusion, acknowledgement and references as following.

II. MODEL

The pendubot of the plant model in this paper is referring to the one of the underactuated model with the only 2encoders. Both encoders measure the speed and position for the DC servomotor and pendulum each. The plant model for a single diagram is showed as following:

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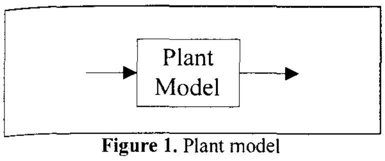

Figure 1. Plant model

Based on the plant model in Fig. 1, the input of the plant nodel is connected from the output sums of the controller nd the disturbance, and the output of the plant model is onnected from the output of pendulum. This connection is howed in Fig. 2 as following:

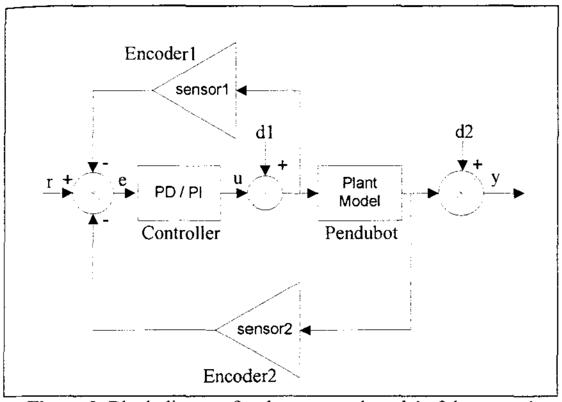

Figure 2. Block diagram for the proposed model of the control system

By the way, the block diagram in Fig. 2 is related to the roposed physical model in Fig. 3 from which is figured as ie subsystem in the simulation results. This proposed hysical model in Fig. 3 produces the algebra equation of nverted pendulum proposed model by applying the net orque of the DC motor T_m, the torque of disc T_d and the orque of the pendulum T_p at which is expanded into the iree separate algebra equations as following:

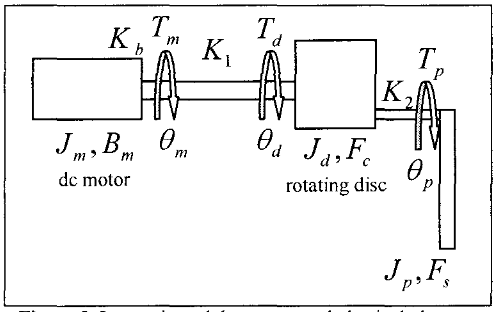

Figure 3. Inverted pendulum proposed physical plant on rotating disc

he torque of the DC motor T_m is derived as:

$$T_m = J_m\ddot{\theta}_m + B_m\dot{\theta}_m + K_1[\theta_m - \theta_d] + d1 + u$$

The torque of the disc T_d is derived as:

$$T_d = J_d\ddot{\theta}_d - K_1[\theta_m - \theta_d] - d1 - u + F_c\frac{\dot{\theta}_d}{|\dot{\theta}_d|}$$

The torque of pendulum T_p is derived as:

$$T_p = J_p\ddot{\theta}_p + K_2[\theta_d - \theta_p] \pm (F_s)|_{\dot{\theta}=0} + d2$$

These derivations are compiled in the state-space equation as following:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-a&a&0&-b&0&0\\c&-e&0&0&0&0\\0&d&f&0&0&0\end{bmatrix}\begin{bmatrix}x_1&x_2&x_3&x_4&x_5&x_6\end{bmatrix}^T + \begin{bmatrix}0\\0\\0\\g\\h\\i\end{bmatrix}^T$$

$$\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix}\begin{bmatrix}x_1&x_2&x_3&x_4&x_5&x_6\end{bmatrix}^T$$

Generally, the internal and external disturbances (d_1 or d_2) are placed at the plant input or output, or both points. These disturbances [5, 6] effected to the results output (y) at which considered the closed and the open loop in the control system.

In control system, the disturbances consist like the rejection power sources such as sine, pulse generator and step, environmental source such as wind, machinery source such as vibration, and others. Sometimes, the disturbances that come from the rejection power, environmental and machinery sources are all affected to the one system only.

This paper is focusing to the timer and step disturbances only at which used the PD or the PI controller for the reduction and elimination. By using the timer disturbance, the time period of the system affects with disturbances are identified from the response of the output result. Based on the response of the output result for the timer disturbance, this paper shows that the disturbance is happened for a certain period. Other than the timer disturbance, the step disturbance in this paper presents the final value of the response till infinity, except there is another disturbance coming through the system.

Overall, the sources of disturbances place in the proposed model, with the cascaded [7] loop in control system, give high performance to the results output. The performance of the results output is based on controller (u) design as following section:

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III. Controller Design

This paper proposes the PI and the PD controllers to liminate or reduce the multiple disturbances. Both ontrollers are designed by applying the controller transfer unction to get the gain values as following:

'or the PD type controller,

$$u_1 = K_p + K_d s$$

nd for the PI type controller,

$$u_2 = K_p + \frac{K_i}{s}$$

By the way, these controller transfer function is ompared the similarity with the compensator of the SISO ›ol from root locus in MATLAB application. Even though ıe root locus method quite accurate compared to Ziegler-!ichols Tuning Rule and other methods, but this root locus ıethod is from the figure viewing. This figure viewing ives an advantage to the plant model with multiple isturbances. The plant model is figured as the subsystem ›r the simulation results as following:

IV. Simulation Results

Via the model and the controller design, the rejection ower sources or disturbances are added in Fig.4, Fig.5, ig.6, Fig.7, Fig.8, Fig.9, Fig.10, Fig.11, and Fig.12 to ompare the simulation results output. These disturbances re placed at the plant input or output, or both. For the limination or reduction of disturbances, either the PD or PI ontroller is placed before the plant in the open loop or losed loop model as following.

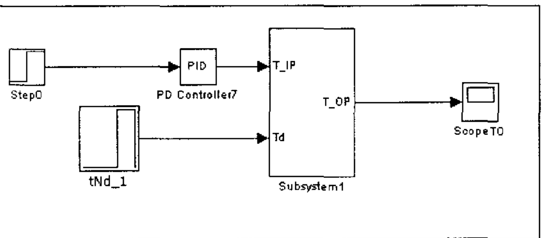

Figure 4. The open loop model with PD controller

By referring to the open loop model with the PD ›ntroller in Fig. 4, the disturbance (tnd_1) is placed at the lant input. The plant input is determined the disturbance ›r the torque of DC motor. Based on control theory, the pen loop system does not correct the errors of the model. he errors of the model that are not measured might affect ıe results output even though the disturbances are liminated by the PD controller.

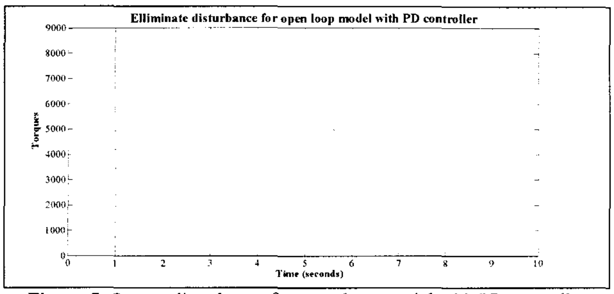

Figure 5. Output disturbance for open loop model with PD controller

The results output in Fig.5 shows that there is no disturbance at all, that gives between 4 to 6 seconds. By the way, there is the highest overshoot of the torque. In control system, the overshoot is not available to use for this model. This is because the highest or maximum torque [8] only available for a wide speed range.

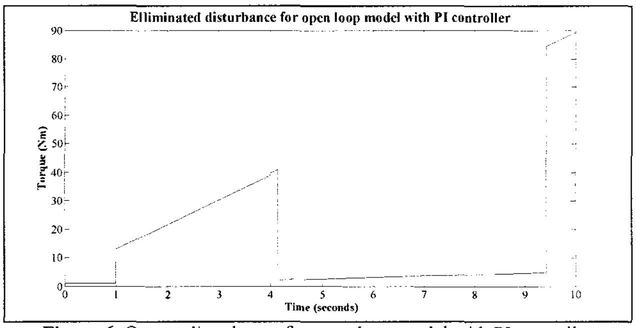

Figure 6. Output disturbance for open loop model with PI controller

In Fig.6 the output response shows a quite difference from Fig.5. The PI controller reduced the maximum overshoot value less than the PD controller for the open loop model with a longer time period after a second to 4 seconds. This response continually shows a gain after 9 to 10 second for at least 90 percent overshoot.

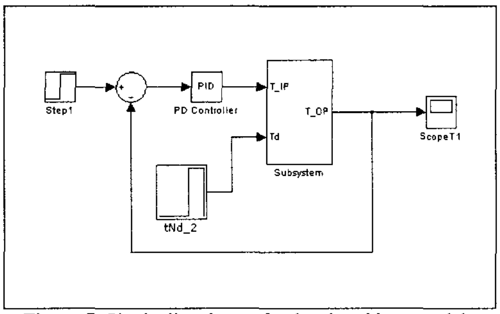

Figure 7. Single disturbance for the closed loop model with PD controller

The single disturbance (tNd_2) of the closed loop model in Fig.7 referred to the timer disturbance. The PD controller for the closed loop model is tuned with a few gains P and D. From the gains tuning, the output response is produced in Fig.8.

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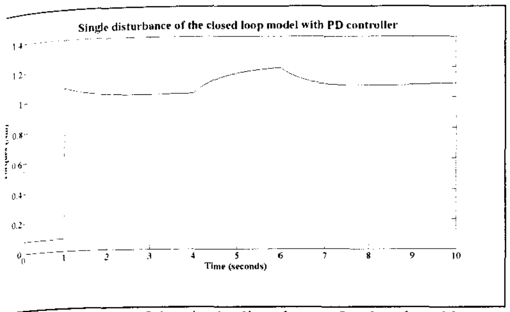

'igure 8. Output of the single disturbance for the closed loop model with PD controller

Based on the single disturbance of the closed loop model ı Fig.7, Fig.8 shows that the disturbance presented in etween 4 and 6 seconds with the torque range for a zero nd one point two. Within the closed loop or also well nown as feedback model, the error is corrected. From this utput result, the response achieved the steady state of tability more than 6 seconds with more than one Newton ıeter, torque.

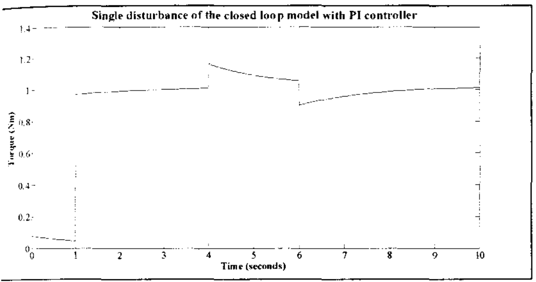

Figure 9. Output of the single disturbance for the closed loop model with PI controller

For comparison output results, the PD controller is eplaced by the PI controller in Fig.9. By using the PI ontroller, the response shows that the increases in the rise me from which is compared to the PD controller based on ıe tuning of the gains P and I from a high to a low value. 'rom the output result with the PI controller also, the esponse achieved the steady state earlier than the output esult with the PD controller. The output result of a single isturbance with the PI controller only takes a less time 'om which is compared to the PD controller with the one Iewton meter of the torque to stop the simulation.

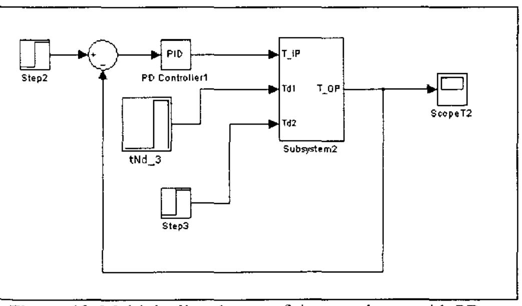

Figure 10. Multiple disturbance of timer and step with PD controller

The improvement from the open loop model to the closed loop model with a single disturbance shows that the PD and the PI controllers affected to the output results. Both models are compared also with multiple disturbances of timer and step as shown in Fig.10.

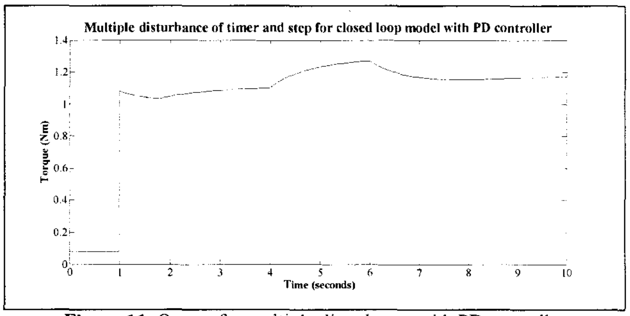

Figure 11. Output for multiple disturbance with PD controller

By using the PD controller to the closed loop model for the multiple disturbances, the output result in Fig.11 shows the response with the disturbances of timer and step. The disturbance of timer began in between 4 and 6 seconds but the disturbance of step began after a second. The multiple disturbances with the PD controller are increased the torque of the response after 6 seconds to achieve the steady state. This steady state shows that a slow response of the time from which is compared to the two previous models.

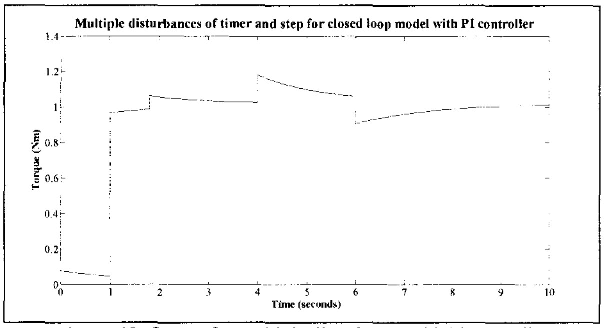

Figure 12. Output for multiple disturbance with PI controller

By the way, the output result for the multiple disturbances with the PI controller in Fig.12 gives a quite similar response of the output result for the single disturbance in Fig.9. For the output result for the multiple disturbances with the PI controller in Fig.12, the response of the two disturbances is figured clearly. The response in Fig.12 with the PI controller is also compared with the response in Fig.11 with the PD controller. In Fig.11, the disturbances are not clearly figured for the response but in Fig.12 the disturbances are clearly figured. These output results for the response in Fig.11 and Fig.12 are caused by the low and the high rise time. Each output results are summarized in Table 1 as following:

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TABLE 1. Comparison stability for PD and PI controller

stability Analysis	Controllers	
	Proportional and Derivative (PD)	*Proportional and Integral (PI)*
egree of system tability	Improves the system stability	As this PI controller increases the compensated by 1, the system show less stable
teady state erformance	To satisfy the steady state, the value of Kp must be suited. The constant of the steady state error based on the time, Td=0 and derivative portion provides no input.	Improves the steady state as its infinite gain at zero frequency
ansient response	Improves the transient response with the reduction of the rise time. From the transient response, the maximum overshoot is reduced better than PI controller.	The suitable values of Kp and Ti are selected to improve the transient response. From the transient response, the rise time is increasing and the maximum overshoot is reduced but shows clearer signal.

V. CONCLUSIONS

The multiple disturbances for the proposed model show ıat a better response of the simulation results output with ıe cascaded loop. These responses reduce disturbance with ıe gains tuning of the PD or PI controller for the inverted endulum proposed model, but low performance for the pen loop model from which is compared to the closed loop ıodel. By the way, this simulation results output is not enied at all as the single disturbance gives less problems to ıe response from which is compared to the multiple isturbances for the stability study. Therefore, the xperimental results are followed up for further study.

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APPENDIXES

Parameters are defined as following:

J_m = DC motor moment inertia
J_d = disc moment inertia
J_p = pendulum moment inertia
B_m = viscous friction
F_c = Coulomb friction
F_s = static friction
K_1 = torque constant / back e.m.f
K_2 = spring constant
$d1$ = disturbance 1
$d2$ = disturbance 2
u = PD or PI controller

$$x_1 = \theta_m \qquad a = K_1/J_m \qquad f = (m_p g l_p - K_2 - m_p l_p a_p)/J_p$$

$$x_2 = \theta_d \qquad b = B_m/J_m \qquad g = 1/J_m$$

$$x_3 = \theta_p \qquad c = K_1/J_d \qquad h = 1/J_d$$

$$x_4 = \dot{\theta}_m \qquad d = K_2/J_d \qquad i = 1/J_p$$

$$x_5 = \dot{\theta}_d \qquad e = (K_1 + m_d l_d a_d)/J_d$$

$$x_6 = \dot{\theta}_p$$

$$\dot{x}_1 = \dot{\theta}_m = x_4$$

$$\dot{x}_2 = \dot{\theta}_d = x_5$$

$$\dot{x}_3 = \dot{\theta}_p = x_6$$

$$\dot{x}_4 = \ddot{\theta}_m = -\frac{B_m}{J_m}x_4 - \frac{K_1}{J_m}x_1 + \frac{K_1}{J_m}x_2 + \frac{1}{J_m}\left(T_m \mp \left(F_{s_1}\right)\Big|_{\dot{\theta}_p=0} r_1\right)$$

$$\dot{x}_5 = \ddot{\theta}_d = \frac{K_1}{J_d}x_1 - \frac{1}{J_d}(K_1 + m_d l_d a_d)x_2 + \frac{1}{J_d}F_c r_1$$

$$\dot{x}_6 = \ddot{\theta}_p = \frac{K_2}{J_p}x_2 + \frac{1}{J_p}(m_p g l_p - K_2 - m_p l_p a_p)x_3 \pm \frac{1}{J_p}\left(F_{s_2}\right)\Big|_{\dot{\theta}_p=0} r_2$$

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