Faculty of Manufacturing Engineering

DESIGN OF LINEAR QUADRATIC REGULATOR CONTROLLER WITH ADJUSTABLE GAIN FUNCTION FOR ROTARY INVERTED PENDULUM SYSTEM

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DECLARATION

I declare that this thesis entitled "Design of linear quadratic regulator controller with adjustable gain function for rotary inverted pendulum system" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ………………………………

Name : Tang Teng Fong

Date : ………………………………

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Manufacturing Engineering.

Signature	:	……………………………………
Supervisor's Name	:	Associate Prof. Dr. Zamberi bin Jamaludin
Date	:	……………………………………

DEDICATION

To my beloved parents, Tang Chin Siang and Ong Geok Ee,

For taking good care and giving guidance in life and academic.

To my concerned sister, Tang Hoay Sean,

For giving moral support.

And also for those I love very much.

ABSTRACT

Design of controllers for non-linear systems has long drawn the attention of researchers especially in the fields of robotics, aerospace engineering and marine engineering. A classic example of a non-linear under-actuated control system is the balance control for a rotary inverted pendulum. Basically, the control approach for such system focusses on torque control of the servo-motor for the purpose of rotating the arm and stabilising the pendulum in its upright position at the shortest possible time. The aim of this research is to supplement and further enhance the control performance of a linear quadratic regulator (LQR) controller with focus on reduced response time and degree of oscillation of the pendulum with added robustness against input disturbance applied to the pendulum position and voltage to the motor. Initially, this thesis comprehensively analysed the LQR controller parameters based on minimal balance time of the pendulum. The LQR controller by itself produced high degree of oscillations, long balance time and poor robustness against input disturbance. As an enhancement over this approach, an adjustable gain was added to the existing LQR control structure. The results showed that for a 30° balancing control, the LQR controller with adjustable gain managed to reduce as much as 70% in the balance time and 98% in the degree of oscillation, while improved its robustness by producing faster balance time and lower oscillation upon excitation by input disturbance forces. In conclusion, the LQR controller with adjustable gain has significantly improved the control performance of the rotary inverted pendulum system.

ABSTRAK

Reka bentuk pengawal untuk sistem tidak linear telah sekian lama menarik perhatian penyelidik-penyelidik terutamanya dalam bidang robotik, kejuruteraan aeroangkasa dan kejuruteraan marin. Satu contoh klasik sistem kawalan tak lelurus yang kurang pacu gerak, ialah kawalan imbangan untuk bandul terbalik berputar. Pada asasnya, kaedah kawalan sistem sebegini berfokus kepada kawalan tork pada motor servo bagi tujuan memutar lengan dan menstabilkan bandul dalam kedudukan tegak pada masa paling singkat yang mungkin. Tujuan penyelidikan ini ialah untuk menambah dan meningkatkan lagi prestasi pengawal pengatur linear kuadratik (LQR) dengan fokus kepada pengurangan masa tindak balas bandul dan darjah ayunan dengan peningkatan keteguhan terhadap gangguan input pada kedudukan bandul dan voltan kepada motor. Tesis ini pada awalnya menganalisa secara komprehensif parameter pengawal LQR berdasarkan masa minima imbang bandul. Pengawal LQR dengan sendirinya menghasilkan darjah ayunan yang tinggi, masa imbang yang panjang dan tahap keteguhan yang rendah terhadap gangguan input. Sebagai satu peningkatan atas pengawal ini, satu pekali boleh laras ditambah kepada struktur pengawal LQR sedia ada. Keputusan yang diperolehi untuk kawalan keseimbangan 30° menunjukkan yang pengawal LQR dengan pekali boleh laras telah menurunkan sehingga 70% masa imbang dan 98% darjah ayunan serta memperbaiki tahap keteguhan sistem dengan menghasilkan masa imabang yang lebih cepat dan darjah ayunan yang lebih rendah apabila terdedah kepada gangguan input. Secara kesimpulannya, pengawal LQR yang ditambah baik dengan pekali boleh laras telah memperbaiki secara jelas prestasi kawalan sistem bandul terbalik berputar ini.

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LIST OF ABBREVIATIONS

AC/DC	-	Analogue converter/ digital converter
ACO	-	Ant colony optimization
ADC	-	Analogue-to-digital converter
D	-	Derivative
DAC	-	Digital-to-analogue converter
DAQ	-	Data acquisition system
DC	-	Direct current
DOF	-	Degree of freedom
EMECS	-	Electro-Mechanical Engineering Control System
FL	-	Fuzzy logic
FRF	-	Frequency response function
FSF	-	Full state feedback
GA	-	Genetic algorithm
GPIO	-	General-purpose input/output
I	-	Integral
LIP	-	Linear inverted pendulum
LQR	-	Linear quadratic regulator
P	-	Proportional
PC	-	Personal computer
PID	-	Proportional integrated derivative
PWM	-	Pulse-width modulation

RIP	-	Rotary inverted pendulum
SMC	-	Sliding mode controller
TORA	-	Translational oscillations with a rotational actuator
VSC	-	Variable structure control
VTOL	-	Vertical take-off and landing

LIST OF SYMBOLS

Mathematical Symbol:

$\approx$	-	Approximately equivalent
∞	-	Infinity
Ω	-	Ohm
$\frac{\partial f}{\partial x}$	-	Partial derivative
π	-	Pi
$\pm$	-	Plus or minus
-	-	Minus
%	-	Percentage
/	-	Divide
+	-	Plus
=	-	Equal
A	-	Ampere
cos	-	Cosine
dB	-	Decibel
f(x)	-	Function notation
G	-	Gravitational acceleration
Hz	-	Hertz
kg	-	Kilogram
m	-	Meter
N	-	Newton

^{o}	-	Degree
rad	-	Radian
s	-	Second
sin	-	Sine
t	-	Time
^{T}	-	Transpose
V	-	Volt

System Model Symbol:

$\ddot{\theta}_1$	-	Angular acceleration of arm
$\ddot{\theta}_2$	-	Angular acceleration of pendulum
$\ddot{\alpha}_2$	-	Angular acceleration of pendulum (downward position)
θ_1	-	Angular position of arm
θ_2	-	Angular position of pendulum
α_2	-	Angular position of pendulum (downward position)
$\dot{\theta}_1$	-	Angular velocity of arm
$\dot{\theta}_2$	-	Angular velocity of pendulum
$\dot{\alpha}_2$	-	Angular velocity of pendulum (downward position)
R_m	-	Armature resistance
τ_1	-	Control torque
e	-	Control voltage
c_1	-	Distance to centre of arm mass
c_2	-	Distance to centre of pendulum mass
J_1	-	Inertia of arm

J_2	-	Inertia of pendulum
l_1	-	Length of arm
l_2	-	Length of pendulum
m_1	-	Mass of arm
m_2	-	Mass of pendulum
K_b	-	Motor back-emf constant
K_u	-	Motor driver amplifier gain
K_t	-	Motor torque constant
C_1	-	Viscous friction co-efficient of arm
C_2	-	Viscous friction co-efficient of pendulum

Control System Symbol:

$\dot{x}$	-	Future state
A	-	System matrix
B	-	Control matrix or input matrix
C	-	Output matrix
D	-	Feed forward matrix
$G(s)$	-	Transfer function of system model
$G_{PID}(s)$	-	Transfer function of PID controller
J	-	Quadratic performance index
K_d	-	Derivative gain (PID controller)
K_d	-	State-feedback gains (LQR controller)
K_i	-	Integral gain
m	-	Adjustable gain
Q	-	Diagonal weight matrix

R	-	Weight factor
S	-	Riccati solution
Ts	-	Sampling time
u	-	Input
$u(t)$	-	Input signal
x	-	Current state
$y(t)$	-	Output

LIST OF PUBLICATIONS

Journal:

1. Fong, T.T., Jamaludin, Z. and Abdullah, L., 2014. System Identification and Modelling of Rotary Inverted Pendulum. *International Journal of Advances in Engineering & Technology (IJAET)*, 6(6), pp. 2342–2353.

Conference:

1. Fong, T.T., Jamaludin, Z., Hashim, A.Y.B. and Rahman, M.A.A., 2014. Design and Analysis of Linear Quadratic Regulator for a Non-linear Positioning System. *3rd International Conference on Design and Concurrent Engineering (iDECON)*, Melaka, 22nd – 23rd September 2014. [Accepted]

CHAPTER 1

INTRODUCTION

This chapter introduces the research work on control system design and development for rotary inverted pendulum (RIP). Sections included in this chapter are the background of the RIP system, problem statement, outlines of research, objectives, scopes, and the content of this thesis. In addition, a segment on contribution to knowledge based on the research work done is also included.

1.1 Background

Since the last few decades, control system design for non-linear and under-actuated systems has generated great interest among researchers. These interests cover a wide spectrum of applications that include control of a space booster rocket, satellite, an automatic aircraft landing system, and stabilisation of a robot. Rotary inverted pendulum (RIP) system is an example of a classical under-actuated system. The RIP consists of a rigid rod called pendulum, which is rotating freely in the vertical plane. The vertical pendulum is naturally unstable with the oscillation as it hangs downward at the equilibrium point. A swing-up action using rotary actuation of a pivot arm in the horizontal plane by a servo-motor would then result in the vertical pendulum achieving upright equilibrium point. A robust and stable controller must be applied in order to control the torque of the servo-motor for the purpose of rotating the arm and stabilising the pendulum in upright position.

The balancing control of a pendulum in the upright position is studied in this research. Firstly, the system model was derived mathematically using Lagrange's