BOUNDED BACK-STEPPING CONTROLLER FOR NONLINEAR SYSTEMS

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BOUNDED BACK-STEPPING CONTROLLER FOR NONLINEAR SYSTEMS

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Electrical Engineering)

Faculty of Electrical Engineering
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JULY 2015

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For my wife Sahazati..
For my children Huwaida and Muhammad Hadif..
For my parents Zainon Hashim and Kamarudin Abd. Razak..

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ABSTRACT

Back-stepping controller is a recursive design approach that offers flexible design steps to stabilize nonlinear systems. However, the well known back-stepping control technique is a full state feedback that is highly dependent to system parameters and system dynamics. As such, back-stepping approach normally produces large magnitude control signal which at times implausible that may lead to actuator saturation. In order to overcome this drawback, this thesis proposes a new bounded back-stepping controller technique. The design is based on a classical Lyapunov with LaSalle invariance set principle. LaSalle invariance set principle relaxes the negative definiteness of the derivative of a Lyapunov function while deducing the asymptotic stability of closed loop system. Hence, system trajectories are confined inside the stability region. In the controller design, the universal Sontag's formula is improved and merged with the back-stepping technique. To handle with the uncertainties and exogenous disturbances, a pseudo function is utilized during the Lyapunov redesign phase. In order to observe the efficacy of the proposed method, a strict feedback numerical nonlinear system with time varying exogenous disturbance is stabilized. The effectiveness of the proposed method is shown through control signal which can be bounded without impact to the closed loop stability and robustness. That is, the proposed control method guarantees global asymptotic stability upon perturbation in initial states with invariant set of solution. The proposed control method also guarantees the asymptotic disturbance rejection and it also robust towards uncertainties. In addition, the control law is smooth and continuous. The proposed method is used to develop a fixed pitch variable speed control for a numerical representation of a two-mass wind turbine system that focuses on the nacelle. Simulation results show that the proposed approach requires less control energy to guarantee the asymptotic tracking of turbine rotor speed for optimum tip-speed-ratio. Thus, it produces a maximum power output from the wind turbine while preserving robustness towards wind intermittent.

ABSTRAK

Pengawal langkah-belakang adalah satu pendekatan reka bentuk rekursi yang menawarkan langkah-langkah reka bentuk yang fleksibel bagi menstabilkan sistem tak lelurus. Walau bagaimanapun, teknik kawalan langkah-belakang adalah kaedah suapbalik penuh yang amat bergantung kepada pemalar dan dinamik sesuatu sistem. Justeru itu, teknik kawalan langkah-belakang kebiasaannya menghasilkan magnitud isyarat kawalan yang besar yang kadang kala tidak munasabah dan boleh membawa kepada ketepuan penggerak. Untuk mengatasi kelemahan ini, tesis ini mencadangkan teknik baru pengawal langkah-belakang yang disempadani. Reka bentuknya adalah berdasarkan kaedah klasik Lyapunov dengan prinsip set tak berubah LaSalle. Prinsip set tak berubah LaSalle melegakan kepastian negatif daripada terbitan fungsi Lyapunov ketika menyimpulkan kestabilan asimptot sistem gelung tertutup. Oleh itu, trajektori sistem adalah terhad di dalam rantau kestabilan. Dalam reka bentuk pengawal, formula Sontag sejagat dipertingkatkan dan digabungkan dengan teknik kawalan langkah-belakang. Untuk mengatasi ketidakpastian dan gangguan luaran, fungsi palsu digunakan semasa fasa reka bentuk semula Lyapunov. Untuk melihat keberkesanan kaedah yang dicadangkan, sistem kawalan ini digunakan untuk menstabilkan sistem berangka tak linear dengan gangguan luar yang berubah terhadap masa. Kaedah yang dicadangkan ini menunjukkan keberkesanannya apabila isyarat kawalan boleh disempadani tanpa memberi kesan buruk kepada keteguhan dan kestabilan gelung tertutup. Iaitu, kaedah kawalan yang dicadangkan menjamin kestabilan asimptot global apabila terdapat gangguan di keadaan awal dengan penyelesaian set tak berubah. Kaedah kawalan yang dicadangkan juga menjanjikan asimptot penolakan gangguan yang mantap terhadap ketidakpastian. Di samping itu, undang-undang kawalannya adalah lancar dan berterusan. Kaedah kawalan yang dicadangkan digunakan untuk membangunkan kawalan pit-tetap kelajuan boleh ubah untuk sistem numerik turbin angin dua jisim yang memberi tumpuan kepada nasel sahaja. Keputusan simulasi menunjukkan bahawa kaedah kawalan yang dicadangkan memerlukan tenaga yang rendah untuk menjamin pengesanan kelajuan pemutar yang asimptot bagi tip-nisbah kelajuan yang optimum. Oleh itu, kuasa keluaran maksimum dapat dihasilkan disamping ianya teguh terhadap ketidakpastian angin.

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LIST OF ABBREVIATIONS

ANN - Artificial Neural Network
ASR - Asymptotic Stability Region
FL - Fuzzy Logic
GA - Genetic Algorithm
GSA - Gravitational Search Algorithm
LMI - Linear Matrix Inequality
LQR - Linear Quadratic Regulator
LQG - Linear Quadratic Gaussian
LUT - Look-up-table
MPC - Model Predictive Control
MRAC - Model Reference Adaptive Control
NMRAC - Nonlinear Model Reference Adaptive Control
N-PID - Nonlinear Proportional-Integral-Derivative
PD - Proportional-Derivative
PI - Proportional-Integral
PID - Proportional-Integral-Derivative
PSO - Particle Swarm Optimization
QFT - Quantitative Feedback Theory
SMC - Sliding Mode Control
SSE - Sum of Squared Error
SSRSE - Sum of Squared Rotor Speed Error
VSC - Variable Structure Control

LIST OF SYMBOLS

R - Rotor blade radius
υ - Wind speed
ρ - Air density
$C_p(\lambda, \beta)$ - Power coefficient
λ - Tip speed ratio
β - Pitch angle
γ - Gearing ratio
ω_r - Rotor speed
ω_g - Generator speed
J_r - Rotor inertia
J_g - Generator inertia
K_r - Rotor external damping
K_g - Generator external damping
B_r - Rotor stiffness
B_g - Generator stiffness
T_m - Aerodynamic torque
T_g - Generator torque / Electromagnetic torque
T_{hs} - High-speed shaft torque
T_{ls} - Low-speed shaft torque
θ_g - Generator-side angular deviation
θ_r - Rotor-side angular deviation
P_{capt} - Captured power
T_{capt} - Captured torque
$\Re^n$ - Real number vector with n-size
$C_1,\ C_2$ - Control parameters for back-stepping controller
$C_{1_{upper}},\ C_{2_{upper}}$ - Maximum control parameter for back-stepping controller
$C_{1_{lower}},\ C_{2_{lower}}$ - Minimum control parameter for back-stepping controller
α_0 - Decaying parameter for bounded controller
$\alpha_1,\ \alpha_2$ - Decaying parameters for pseudo function
ξ - Sum of uncertainties and disturbances

ε	-	Parameter for pseudo function
$\mathcal{A}$	-	Antecedent linguistic term for fuzzy logic controller
$\mathcal{B}$	-	Consequent linguistic term for fuzzy logic controller
P	-	Dimension of the input space for fuzzy logic controller
N_i	-	The number of linguistic terms of the i^{th} antecedent variable

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The field of control engineering is greatly advanced in order to fulfils great industrial demand. Industrial sectors such as manufacturing, aerospace [7, 8], robotics [9], transportation [10, 11, 12], traffic flow [13], maritime [14], wind turbine system [15], flight control design [16, 17] and many more are expanding rapidly. Most of the systems are nonlinear and to gain the asymptotic stability and robustness of these systems requires advanced control techniques. Nonlinear systems do not fulfil superposition principle as linear systems do. Nonlinear systems absorb nonlinear phenomena such as chaos, saturation, limit cycle, finite escape time, having multiple isolated equilibrium points, and unpredictable. Thus, solving nonlinear systems requires advance control techniques. The presence of uncertainties and exogenous disturbances in the nonlinear systems dynamic is sometimes inevitable, and give catastrophic effect to the stability and robustness of closed loop systems. As such, developing a robust control for nonlinear systems with uncertainties and exogenous disturbances offer challenge to control research community.

The need to limit the magnitude and energy of a control signal is due to multifarious causes. One of the causes is to avoid actuator saturation, and to respect the actuator's allowable input ranges. As quoted in [18] 'saturation is probably the most encountered nonlinearity in control engineering'. In practice, the requirement to bound the control signal is a must because of the electrical constraints and the mechanical constraints of the system under controlled. In fact, stabilizing unstable nonlinear system with fast settling time often requires large control energy. Moreover, the control signal might reach unreasonable magnitude when the system is perturbed with large initial condition. This problem becomes unambiguous when a full state feedback control technique such as back-stepping is used. In a family of nonlinear control techniques, back-stepping is a new approach that has been developed in 1990 by Petar V. Kokotovic [19]. Back-stepping is known as a full state feedback approach based on systematic Lyapunov control technique [20]. As such, back-stepping depends highly on system parameters and dynamics. For that reason, back-stepping controller normally produces high magnitude control signal with large power depending on the system order and the location of the perturbed initial state.

1.2 Problem Statement

In this thesis, two main problem statements are outlined. These statements explain the shortcomings of the ideal back-stepping controller in term of the control signal magnitudes, control signal energy and the asymptotic stability condition based on ideal Lyapunov function.

Statement 1

Quite often, control law primarily aims at the asymptotic stability and asymptotic disturbance rejection of the closed loop system. However, very little research focuses on the control signal magnitude and the power produced by the controller while achieving the asymptotic stability and the asymptotic disturbance rejection [21, 22]. Hence, bounded control problem has become an incessant research in control engineering field. For illustration, it is easy to stabilize unstable system by forcing their poles to the left-hand-side of the S-plane so that the closed-loop system stable. Theoretically, placing the closed-loop poles near to $-\infty$ may result in fast regulation rate but require high energy as a trade-off. For example, consider a linear system

$$\dot{x} = Ax + Bu \tag{1.1}$$

where the state $x \in \Re^n$, system matrix $A \in \Re^{n \times n}$, input matrix $B \in \Re^{n \times m}$ and control input $u \in \Re^m$. By pole placement approach, states x can be regulated to equilibrium $x = 0$ by a simple state feedback control law $u = -Kx$ [23]. Without considering the magnitude and the amount of energy in the control signal u, the designer will ponder how to find K such that the closed loop system $\dot{x} = (A - BK)\,x$ stable. For this motive, designer may simply choose any K so that the eigenvalues of a new system matrix $(A - BK)$ positioned at the left-hand-side of the S-plane. For instance, let consider numerical values for system (1.1) as

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{1.2}$$

Then, placing the closed loop poles at -5 and -6 yields $K = [16 \;\; 32]^T$. Figure 1.1 and Figure 1.2 depict the regulated x_1 and the control signal respectively. Placing closed loop poles more toward $-\infty$ will result in faster regulation rate (Figure 1.1), but will increase the feedback gain K and hence, increase the magnitude of the initial control signal (Figure 1.2). This observation shows that avoiding excessive control signal is crucial in the control design yet really important in practice.

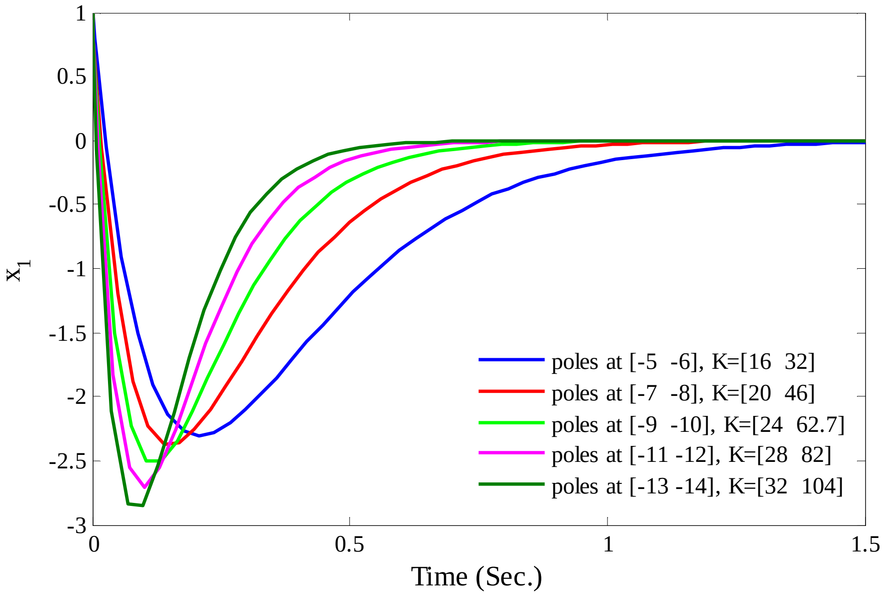

Figure 1.1: Regulated x_1

This thesis treats back-stepping controller as its core nonlinear control technique. Nevertheless, normal back-stepping is a full-state feedback which depends highly on system parameters and dynamics [21, 22]. Therefore, back-stepping controller normally produces high magnitude control signal with large signal power depending on the system dimension and the location of the perturbed initial state. Large control magnitude disrespects the actuator constraint and implausible in practice. This phenomena may result in singularity error to the closed-loop system and gives computational burden to the processor. Hence, this thesis proposes bounded back-stepping controller that limits the magnitude of the control signal and reduces the control signal power, namely a *bounded back-stepping controller*. Figure 1.3 shows