

## **Conditional Residual Time Modelling Using Oil Analysis: A Mixed Condition Information Using Accumulated Metal Concentration and Lubricant Measurements**

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This paper reports on modelling decision problems associated with condition monitoring by providing some scientific support to maintenance decision making. In this study, a Spectrometric Oil Analysis Programme (SOAP) is used as monitoring measures for diesel engines used in ships. Using the residual time at a monitoring check as the condition of the diesel engines, we seek to establish the relationship between the residual time and the total wear concentrations which are available from SOAP data. These metal concentrations are treated as concomitant variables which are influenced by the residual time. We also used lubricant measurements data acting as covariates that may increase/decrease the residual time. The formula for finding a residual time distribution is presented using a filtering technique and a method for estimating the model parameters is discussed. Once the distribution of the residual time is known, a decision model can be established to recommend the optimal maintenance actions in terms of cost, availability or any criterion of interest.

Key words: residual time, oil analysis, modelling decision problems, filtering technique

### **1. INTRODUCTION**

The residual time of a machine tends to be a highly uncertain and may depend upon several variable factors. In the case of oil analysis, the moment lubricant oil enters an operating machine; it begins to deteriorate as it becomes contaminated with wear metal debris, oxidation by-products and external substances such as water, dirt and/or process material. A high level of oil contamination is usually an indication of the presence of a mechanical or lubricant degradation, which if left unattended could lead to extensive damage or failure of the machine. Using a SOAP data, it provides us with two types of condition monitoring data, i.e. metal concentration data and lubricant condition data. In theory, metal concentration is a good indicator that indicates the wear processes that are occurring within the engines. By defining the residual time as a representation of the wear process the relationship between the residual time and metal concentration may be established. Since wear is the roots cause of the observed metal concentrations the relationship between wear (residual time) and metal concentration is one way namely the amount of wears determines the total metal concentrations in a stochastic way, but not vice versus. We call for now these metal concentration data as concomitant variables. The lubricant condition monitoring on the other hand determines if the lubricant itself is fit for continue used based on some performance measures. Changes in the operating state of a machine i.e. stress and load can have a substantial effect on the results. Even more importantly, the lubrication schedule within the plant and the types of additions and changes made to the working lubricant within the machine will have a profound effect on the lubricant analysis. Therefore, this information by itself has great value in determining the condition of both the lubricant and the machine to be used in maintenance decision making.

In a parallel work, (Hussin and Wang, 2006) we had shown how the residual time of an engines used in ships can be modelled using total metal concentrations based on SOAP analysis. However, SOAP analysis can detect metals element up to 8 microns in size, which implied that the larger metals are not detected. Hence, we might not capture more information that could tell us about the exact accumulated metal concentration in engines. However, SOAP also provides lubricant performance measures as an indicator of lubricant performance, which could influence the engine wear. Hence, we are interested to develop a residual time prediction model that combined the information given by metal concentration and the lubricant data.

## 2. MODELLING DEVELOPMENT

This development of a new model is similar with a model which had developed in other works, (Wang and Christer, 2000; Wang, 2002; Wang, 2004), (Zhang, 2004) and (Wang and Zhang, 2005) but with a few new extensions as we embed lubricant data as a factor that could increase or decrease the residual time. For the purpose of model building, we list notation as follows:

### 2.1 Notations

$x_i$  is a residual time at time  $t_i$ .

$y_i$  is a type of concomittent monitoring information which reflects the condition of the engine state.  $y_i$  is random and the relationship between  $x_i$  and  $y_i$  is described as  $p(y_i | x_i)$ .

$z_i$  is a type of covariate monitoring information which may influence  $x_i$  but is assumed not having influence in  $y_i$ . This is because if both of the measurements are correlated, it is sufficient to use only one measurement. Thus, at this early stage, a simple analysis has been carried out from data that support our assumption, that there are no correlation between  $y_i$  and  $z_i$ .

$x_i^-$  is the residual time before information  $z_i$  is taken at time  $t_i$ . In our case, both information  $y_i$  and  $z_i$  are available at the same time.

$x_i^+$  is the residual life after the information  $z_i$  is taken at time  $t_i$ .

$Y_i$  and  $Z_i$  are the monitoring history for  $y_i$  and  $z_i$  which takes the values of  $\{y_1, y_2, \dots, y_i\}, \{z_1, z_2, \dots, z_i\}$ .

### 2.2 Formulations

Our objective is to find the residual time distribution given the information of metal concentration and lubricant data, i.e.  $p(x_i^+ | Y_i, Z_i)$  at time  $t_i$ . To start with, we defined the residual time adopted from (Wang and Christer, 2000) as

$$x_i^- = \begin{cases} x_{i-1}^+ - (t_i - t_{i-1}) & \text{if } x_{i-1}^+ > t_i - t_{i-1}, \quad \text{no maintenance intervention,} \\ \text{not defined} & \text{otherwise} \end{cases} \quad (1)$$

The relationship between  $x_i^-$  and  $x_i^+$  can be shown as below

$$x_i^- = x_i^+ e^{B\Delta z_i} \quad (2)$$

where  $B$  is a parameter that defines the influence of  $\Delta z_i$  and  $\Delta z_i$  is  $z_i - z_{i-1}$ . Noted that if  $\Delta z_i$  is a vector than this relationship can be presented as

$$x_i^- = x_i^+ e^{\sum_k^i \sum_s^M B_s \Delta z_k} \quad (3)$$

where  $M$  is the size of the vector. For the sake of simplicity, we will use a scalar to explain the influence of  $\Delta z_i$  in the model. We will deal with this vector while fitting the model with data. Hence, equation (2) and (3) can be explained as follows,

1. if  $\Delta z_i = 0$ , then  $x_i^+ = x_i^-$  which means the residual time remains the same.
2. if  $\Delta z_i > 0$ ,  $x_i^+ < x_i^-$  which means the residual time becomes shorter, the system is deteriorating.
3. if  $\Delta z_i < 0$ ,  $x_i^+ > x_i^-$  which means the residual life becomes more longer, the system is improving.

We seek to establish the  $p(x_i^+ | Y_i, Z_i)$  and adopted from (Wang and Christer, 2000), it can be written as

$$\begin{aligned} p(x_i^+ | Y_i, Z_i) &= p(x_i^+ | y_i, Y_{i-1}, Z_i) \\ &= \frac{p(x_i^+, y_i | Y_{i-1}, Z_i)}{p(y_i | Y_{i-1}, Z_{i-1})} \end{aligned} \quad (4)$$

and after some manipulation, equation (4) can be written as

$$p(x_i^+ | Y_i, Z_i) = \frac{p(y_i | x_i^+) p(x_{i-1}^+ = x_i^- + t_i - t_{i-1} | Y_{i-1}, Z_{i-1})}{\int_0^\infty p(y_i | x_i^+) p(x_{i-1}^+ = x_i^- + t_i - t_{i-1} | Y_{i-1}, Z_{i-1}) dx_i^+} \quad (5)$$

Equations (4) and (5) are important that explained how the condition monitoring data is taking into account in the model with two key issues that need further explanation.  $x_i^+$  measures the residual time from new, taking consideration the concomittent variables and covariates. Since  $Y_0$  and  $Z_0$ , are not available at  $t_0$  in most cases, we therefore set  $p(x_0^+ | Y_0, Z_0) = p(x_0)$ , which is the *pdf* of the machine life.

The next stage is to establish the relationship between the observed information,  $y_i$  and  $z_i$ , with the residual time,  $x_i^+$ , which can be established by a probability distribution (Wang and Christer, 2000). Generally, we expect that a short residual time,  $x_i^-$ , will generate a high reading in  $y_i$ . However, taking consideration of covariates,  $z_i$  should increase or decrease the residual time,  $x_i^+$ . This relationship is modelled in equation (1). The value of  $p(x_0)$  and  $p(y_i | x_i^+)$  can be freely chosen from any distribution and it can be shown that (5) can be determined recursively if  $p(x_0 | Y_0)$  and  $p(y_i | x_i^+)$  are known. As an example, Weibull distribution is chosen to represent both  $p(x_0)$  and  $p(y_i | x_i^+)$ . Generalizing to the *ith* terms,  $p(x_i^+ | Y_i, Z_i)$  can be written as

$$\begin{aligned} p(x_i^+ | Y_i, Z_i) &= \\ &= \frac{\left( e^{b(x_i^+ + t_i)} \right)^\eta e^{-\left( a e^{b(x_i^+ + t_i)} y_i \right)^\eta} e^{b\eta \sum_{j=2}^i \psi_j(x_i^+)} e^{-a^\eta \sum_{j=2}^i \left( e^{b\psi_j(x_i^+)} y_{j-1} \right)^\eta} \left( \psi_1(x_i^+) \right)^{\beta-1} e^{-\left( a \psi_1(x_i^+) \right)^\beta}}{\int_0^\infty \left( e^{b(x_i^+ + t_i)} \right)^\eta e^{-\left( a e^{b(x_i^+ + t_i)} y_i \right)^\eta} e^{b\eta \sum_{j=2}^i \psi_j(x_i^+)} e^{-a^\eta \sum_{j=2}^i \left( e^{b\psi_j(x_i^+)} y_{j-1} \right)^\eta} \left( \psi_1(x_i^+) \right)^{\beta-1} e^{-\left( a \psi_1(x_i^+) \right)^\beta} dx_i^+} \end{aligned} \quad (6)$$

where

$$\psi_j(x_i^+) = x_i^+ e^{\sum_{k=j}^i \sum_s^M B_s \Delta z_k} + \sum_{k=j+1}^i (t_k - t_{k-1}) I_{k,i} e^{\sum_{m=j}^{k-1} \sum_s^M B_s \Delta z_m} + t_j \quad j = 1, \dots, i$$

$$\text{and } \begin{cases} I_{k,i} = 0 & \text{if } k > i \\ I_{k,i} = 1 & \text{otherwise} \end{cases}$$

### 3. PARAMETER ESTIMATION

To be able to use equation (6) we need to know the parameters within equation (6). The model parameters are estimated in two steps, with the first step is to estimate the parameters in  $p(x_0)$ , namely  $\alpha$ , the scale, and  $\beta$ , the shape parameters of the Weibull distribution as a failure distribution. This can be done because from our assumption that  $y_i$  is dependent on  $x_i$ , but  $x_i$  is independent of  $y_i$ . The second step is to estimate  $a$ ,  $b$ ,  $\eta$ ,  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$  which is in  $p(y_i | x_i)$  from the observed data.

At every time checking  $t_i$ , two pieces of information are valuable to us. The first is the observed monitoring information,  $y_i$  with their history,  $Y_{i-1}$  and the second is if the engine has survived over  $t_i$ , their residual time is  $x_{i-1} > t_i - t_{i-1}$ . Furthermore, if the last observation is the failure at time,  $t_f$  (assuming we have this information, which in practice is hard to obtain),  $t_f > t_n$ , where  $t_n$  is the time of the last monitoring check, its contribution to the likelihood function is  $p(x_n = t_f - t_n | Y_n)$ . The likelihood function for a single item using this approach becomes,

$$L = \left( \prod_{i=1}^n p(y_i | Y_{i-1}, Z_{i-1}) \int_{t_i - t_{i-1}}^{\infty} p(x_{i-1}^+ | Y_{i-1}, Z_{i-1}) dx_{i-1}^+ \right) p(x_n = t_f - t_n | Y_n) \quad (7)$$

where  $p(y_i | Y_{i-1})$  can be written as,

$$\begin{aligned} p(y_i | Y_{i-1}, Z_{i-1}) &= \int_0^{\infty} p(y_i | x_i^+) p(x_i^+ e^{B \Delta z_i} | Y_{i-1}, Z_{i-1}) dx_i^+ \\ &= \int_0^{\infty} p(y_i | x_i^+) \frac{p(x_i^+ e^{B \Delta z_i} + t_i - t_{i-1} | Y_{i-1}, Z_{i-1})}{\int_{t_i - t_{i-1}}^{\infty} p(x_{i-1}^+ | Y_{i-1}, Z_{i-1}) dx_{i-1}^+} dx_i^+ \end{aligned} \quad (8)$$

After some manipulation, the log likelihood function of all observation points can be written as

$$\begin{aligned} \log L &= \eta \log(ae^{b(t_f)}) + \log \eta + (\eta - 1) \log y_n - (ae^{b(t_f)} y_n)^\eta + \beta \log \alpha + \log \beta + \\ &(\beta - 1) \log(\psi_1(x_n^+)) + \sum_{i=2}^n \log \eta + (\eta - 1) \log y_{i-1} + \eta \log(ae^{b(\psi_i(x_n^+))}) - (ae^{b(\psi_i(x_n^+))} y_{i-1})^\eta \end{aligned}$$

where

$$\psi_j(x_i^+) = x_i^+ e^{\sum_{k=j}^i \sum_s^M B_s \Delta z_k} + \sum_{k=j+1}^i (t_k - t_{k-1}) I_{k,i} e^{\sum_{m=j}^{k-1} \sum_s^M B_s \Delta z_m} + t_j \quad j = 1, \dots, i$$

$$\text{and } \begin{cases} I_{k,i} = 0 & \text{if } k > i \\ I_{k,i} = 1 & \text{otherwise} \end{cases}$$

### 4. FITTING THE MODEL TO THE DATA

In order to fit data to the model, we need to prepare data that is suitable for the model's requirements. To prepare the required data from metal element information, we used the analysis that yields total wear metal concentrations, (Hussin and Wang, 2006). For data preparation from oil performance measurements, instead of using the original oil data values, we applied a concept called independent component analysis (ICA). This is carried out for two main reasons; the first is to normalize the scale of each individual oil performance measurement and the second is to identify the possibility of reducing the dimensions of the monitored variables for oil performance. Normalization of the scale of the parameters is important, as each parameter has its own scale and it was noted from the literature, (Lukas and Anderson, 1996) and discussion with persons familiar with similar types of data that these variables are very much correlated among themselves. Therefore, ICA can be used to calculate the contribution of each parameter.

#### 4.1 Independent Component Analysis (ICA)

ICA of a random vector  $X$  consists of finding a linear transformation  $\hat{s} = Wx$  so that the components  $\hat{s}_i$  are as independent as possible, in the sense of maximizing some function  $F(\hat{s}_1, \dots, \hat{s}_m)$  that measures independence. To begin the discussion of ICA, we can express the entire system of  $n$  measured signals and  $m$  observations as

$$x = As : \quad \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix} \quad (9)$$

We refer to each original independent signal as  $s$ . Each measured signal  $x$  can be expressed as a linear combination of the original independent signals.  $A$  is an  $m \times n$  mixing matrix that generates  $x$  from  $s$ . Hence, the goal of ICA, given the observation  $x$ , is to calculate matrix  $A$  and the set of independent  $s$  values. In order to perform this task, our intention is to find a matrix  $W$  (the inverse of  $A$ ) such that, if it is applied to  $x$ , it yields a dataset  $\hat{s}$  that is as independent as possible and thus approaches the unknown signals  $s$ .

$$\hat{s} = Wx : \quad \begin{bmatrix} \hat{s}_1 \\ \vdots \\ \hat{s}_n \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & & \vdots \\ w_{n1} & \cdots & w_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \quad (10)$$

In our case we have 4 variables from lubricant measurement, i.e. INS, KV, TBN and WC. INS measures the build-up of insoluble combustion-related debris and oxidation products. High INS values will cause lacquer formation on hot surfaces, sticking of piston rings and wear of the cylinder liner and bearing surfaces, which indicates a contamination of the system. Kinetic viscosity (KV) is the most important property of a lubricant. Increased oil viscosity will cause wear of bearings and running surfaces, thus high KV indicates that the system has been contaminated. WC is a quantification of water contamination, where water in a lubricant will promote corrosion and oxidation. An increase again shows that the system has been contaminated. The total base number (TBN) of a used lubricant is a measurement of its ability to neutralize acids. A low TBN will cause more corrosion and thus indicates oil contamination. (Lukas and Anderson, 1996). It should be noted that each of these variables influences the others, so that applying ICA as a technique to separate linearly mixed sources seems promising. Several approaches to making ICA estimations are available. In this research, the FastICA package developed for Matlab application and designed by Aapo Hyvärinen at Helsinki University of Technology has been applied, (Hyvärinen and Oja, 1997). Thus, for every set of data, the transformation is made from the original value of lubricant variables to a set of independent variables. The transformed data will be used for

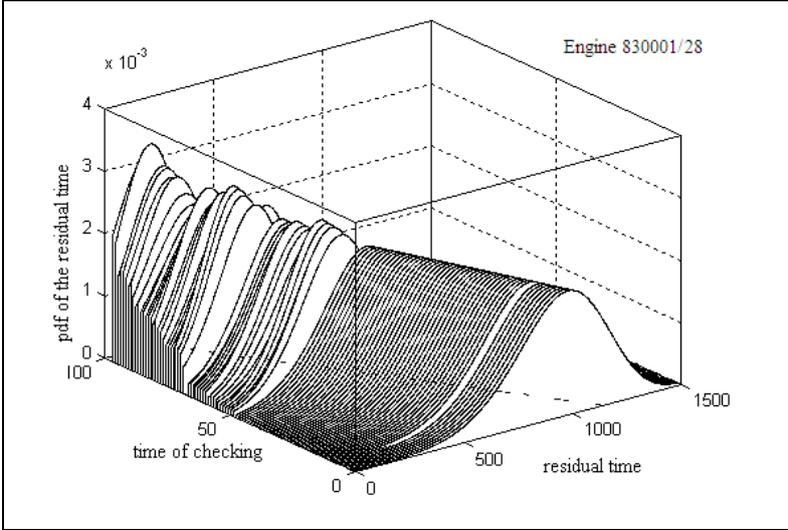
further analysis. As noted above, ICA has the possibility of reducing the dimensions of the variables, although this is not its main purpose. This analysis was conducted on our data set, and while conducting the whitening process, the eigenvalues of the first principal component of uncorrelated variables showed a range of values from 55 to 95%. This strongly suggests that the dimensions could not be reduced, hence we decided to use the entire set of variables in the model. Having all the data required in hand, we first estimated the parameter values for our model. We randomly selected 22 sets of data for this purpose and the results are shown below.

Table 1: Estimated parameters from observed data

$a$	$b$	$\eta$	$B_1$	$B_2$	$B_3$	$B_4$
0.00106	0.00021	1.19712	0.03321	0.01672	0.03516	0.06309

The variances of the estimated parameters are relatively small for all parameters, which tell us something about variability around the mean. Thus, we can say that the parameter estimates were good. The covariance of each parameter is literally small, indicating that there are no relationships between variables. Using the estimated parameters, we calculated our residual model,  $p(x_i^+ | Y_i, Z_i)$ , using the remaining dataset. Figure 1 below shows one of the results for residual time distribution. The dotted straight line in the plot indicates the actual residual life. It is shown that the variance of the pdf of the residual time becomes smaller as we have more information.

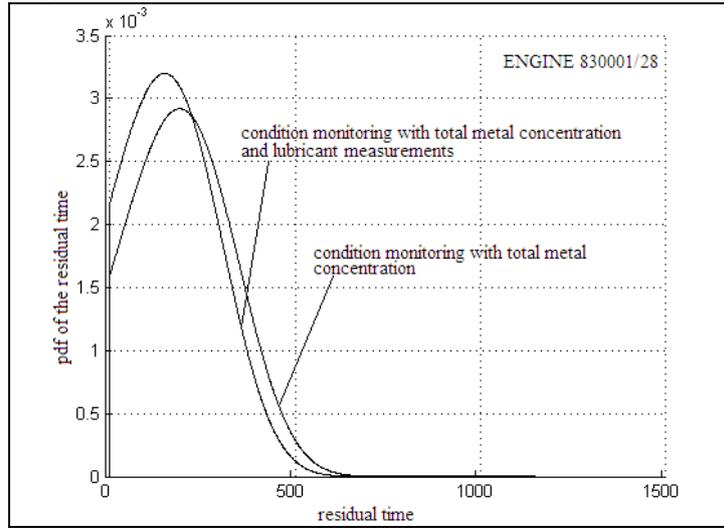
Figure 1: pdf and actual residual time for engine 830001/28



**5. MODEL COMPARISON**

To show the comparison of this new model with the model developed in other work, (Hussin and Wang, 2006), Figure 2 below shows the plots of distribution of residual life at the last monitoring point for a testing dataset. From Figure 2 it can be seen that the new model gives a new prediction that is significantly better than the previous one. Thus, it is concluded that the more condition monitoring data is used, the better the model estimation becomes.

Figure 2: pdf residual time of engine 830001/28 at the last observation point using total metal concentration.



### 5.1 The Decision Model

The essential decision to make at different monitoring point is whether we should replace the engine or not given all information available. If the answer is yes then what is the best time for such replacement, and should we wait until a suitable production window such as a scheduled shutdown arrives? Suppose, we need to develop a decision model, therefore a decision variables itself might depend on several criteria's such as cost, reliability, safety or any criterion of interest. To show an example, we adopt a decision model developed by (Wang and Christer, 2000) as

$$C(t) = \frac{C_f P_i(x_i < T - t | Y_i) + C_p (1 - P_i(x_i < T - t | Y_i))}{t + (T - t)(1 - P_i(x_i < T - t | Y_i)) + \int_0^{T-t} z p_i(z | Y_i) dz} \quad (11)$$

where

$C(t)$  is the total expected cost per unit time

$t$  is the current monitoring point measured from new

$T$  is the planned replacement time

$P_i(x_i < T - t | Y_i)$  is the probability of the residual time at time  $t$

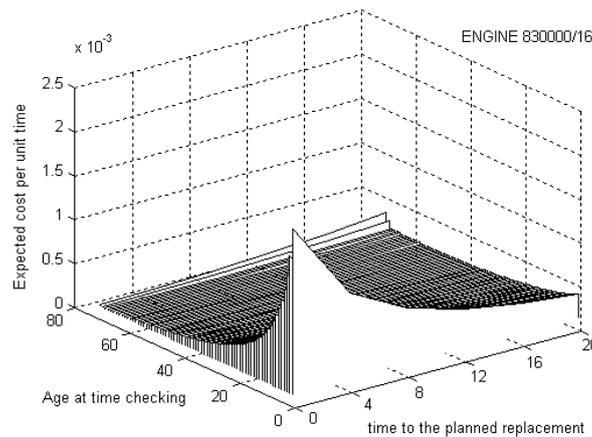
$C_f$  is a failure cost

$C_p$  is a preventive cost

The decision model above assumes that the cost monitoring cost is negligible. Suppose that the mean cost of a failure replacement is 10 times higher than the mean cost for a preventive maintenance, using the estimated parameter and equation (11) above we compute the decision model on a life dataset. At every at every  $t$  time, we could identify the best time to carry out a preventive replacement,  $T$ . We show one example. The diesel engine is monitor for 72 data points from new and the interval between monitoring is 9.2 days and failed at 658.2 days of operation. We notice that from

Figure 3 that the expected cost per day is decreasing at early stage until it starts to increase at 69th monitoring point. This suggested that if preventive replacement is carried out, the cost should be minimized. This shows that our model works well to recommend the preventive replacement before the failure occurs.

Figure 3: Expected cost per day in terms of planned replacement at time  $t$  given that the current monitoring check is time  $t$



## 6. CONCLUSION

This paper has outlined a new development of conditional residual time where the previous approach is enhanced with some data that we assume will increase or decrease the residual time. This was done by first assuming that the residual time is a function of observed concomittent monitoring data,  $y$ . Then, we set up a relationship indicating how the residual time will change with the covariates,  $z$ . Issues such as the possibility of the covariates being correlated with each other were solved by using ICA. By providing the data required by the model, we then estimated the model parameters using the maximum likelihood approach. The results from parameter estimation were sufficiently good to justify proceeding with fitting the model to the data. The results of this approach were satisfactory, showing better prediction than the previous model with the same dataset. Thus it is concluded that more information leads to better a prediction.

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