Faculty of Mechanical Engineering

MODELING OF VIBRATION TRANSMISSION AND
PREDICTION OF STRUCTURE-BORNE NOISE IN BUILDINGS

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Master of Science in Mechanical Engineering

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MODELING OF VIBRATION TRANSMISSION AND PREDICTION OF STRUCTURE-BORNE NOISE IN BUILDINGS

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**A thesis submitted
in fulfillment of the requirements for the degree of Master of Science
in Mechanical Engineering**

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2016

DECLARATION

I declare that this thesis titled, 'Modeling of vibration transmission and prediction of structure-borne noise in buildings' is the result of my own research except as cited in the references. The Thesis has not been accepted for any degree and is not currently submitted in candidate of any other degree.

Signature : ....

Name : ....

Date : ....

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Mechanical Engineering

Signature   : ....

Supervisor Name : ....

Date   : ....

DEDICATION

"Be kind whenever possible. It is always possible."

Dalai Lama

ABSTRACT

Vibration originating from mechanical services are often the source of structure-borne noise in buildings. The transmitted vibration waves from the service equipment can propagate through the building structures and can eventually radiate sound which causes an audible low frequency noise causing disturbance inside the building. Models and discussions on of the vibration transmitted through the building structures are still lacking, particularly on how it is radiated as sound. This project proposes the development of firstly, a generic analytical model of a 2D portal frame structure consisting of column and beam elements. The results of the velocity of the structures are validated with those from Finite Element (FE) model. Bending waves in the building structure are presented through the operation deflection shapes diagram. Secondly, the 2D FE model is extended to a 3D FE model to include plate elements to represent walls, floors and roofs in a building. Using the hybrid FE/SEA analysis in VA One software, sound pressure level (SPL) in the building environment can be predicted using the injected power data of the motor obtained from the reception plate experiment. With the induction motor speed of 50 Hz, the overall SPL are 44 dB and 42 dB for the upper and lower cavities of the building respectively. These levels depend on the input power injected by the motor which is also directly affected by the mechanical faults in the motor i.e. mass unbalance and structural looseness. It is demonstrated that the SPL varies due to the faults in the motor. The result shows that the model can be applied as a preliminary predictive guide on the building design to minimize noise and vibration generated by service equipment.

ABSTRAK

Getaran yang terhasil daripada perkhidmatan mekanikal merupakan sumber utama pence-maran bunyi dalam bangunan. Gelombang getaran yang berpunca dari peralatan perkhid-matan boleh mengaktifkan getaran pada struktur bangunan dan akhirnya memancarkan bunyi yang menyebabkan gangguan bunyi frekuensi rendah di dalam bangunan. Model dan perbincangan mengenai bagaimana getaran yang dihantar melalui struktur bangunan masih kurang , terutamanya bagaimana ia terpancar sebagai bunyi. Pertama, projek ini mencadangkan pembangunan model generik analitikal struktur kerangka portal 2D yang terdiri daripada unsur lajur dan tiang. Model analitikal disahkan dengan model unsur ter-hingga (FE). Lenturan gelombang dalam struktur bangunan dibentangkan melalui gambar rajah bentuk operasi pesongan. Kedua, model 2D FE diteruskan kepada model 3D FE un-tuk merangkumi unsur-unsur plat bagi mewakili dinding, lantai dan bumbung di bangunan. Melalui penggunaan analisis hibrid FE/SEA dalam perisian VA One, tahap tekanan bunyi (SPL) dalam persekitaran bangunan itu boleh diramal dengan menggunakan data kuasa motor yang diperoleh daripada eksperimen plat penerima. Kelajuan motor yang digunakan adalah sebanyak 50 Hz, SPL keseluruhan adalah 44 dB dan 42 dB untuk kaviti atas dan bawah bangunan tersebut. Aras tekanan bunyi ini bergantung kepada kuasa input motor yang juga melibatkan kerosakan mekanikal dalam motor iaitu jisim yang tidak seimbang dan kelonggaran struktur. Ia menunjukkan bahawa SPL berbeza disebabkan oleh kerosakan dalam motor. Hasil kajian menunjukkan bahawa model ini adalah mencukupi untuk mem-beri panduan awal dalam reka bentuk bangunan untuk mengurangkan pencemaran bunyi dan getaran yang dihasilkan oleh peralatan perkhidmatan bangunan.

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LIST OF ABBREVIATIONS

ANSI   **A**merican **N**ational **S**tandard **I**nstitutes

BE   **B**oundary **E**lement

FE   **F**inite **E**lement

HVAC   **H**eating, **V**entilation and **A**ir **C**onditioning

MATLAB   **Mat**rix **Lab**oratory

NC   **N**oise **C**riteria

ODS   **O**perating **D**eflection **S**hape

RC   **R**einforced **C**oncrete

SEA   **S**tatistical **E**nergy **A**nalysis

SDOF   **S**ingle **D**egree **O**f **F**reedom

LIST OF PHYSICAL CONSTANT

Speed of sound	c	$=$	$343\ \text{ms}^{-1}$
Velocity amplitude reference level	ν_{ref}	$=$	$1\text{x}10^{-9}\ \text{ms}^{-1}$
Sound power reference level	W_{ref}	$=$	$1\text{x}10^{-12}\ \text{W}$

LIST OF SYMBOLS

A	Cross-sectional area
a_1, a_2	Wave
B	Plate bending stiffness
E	Young's modulus
E_1, E_2, E_t	Energy
E_{m1}, E_{m2}	Modal energy
ϵ	Strain
F	Force
I	Second moment of area of cross section
$j = \sqrt{-1}$	Imaginary unit
K	Dynamic stiffness
k	Wave number
k_{sp}	Hysteric characteristic of the soil
L	Length
L_ν	Vibration velocity level
$\ddot{m}_{\mathrm{R}}$	Mass per unit area of the panel
M_{R}	Total mass of the plate
M	Moment
η	Damping loss factor
η_{12}, η_{21}	Coupling loss factor
n_1, n_2	Modal density
ϕ	Mode shape
θ	Angular displacement
ρ	Density
P	Structure-borne sound power
P_{in}	Source input power
S_{R}	Surface area of the plate

t	Plate thickness
$\bar{W}$	Sound power
W_{diss}	Dissipated power
W_{in}	Input power
W'	Transmitted power
W_{12}, W_{21}	Power flow between subsystems 1 and 2
ω	Angular frequency
$\Delta\omega$	Frequency bandwidth
$\bar{Y}_{\text{S}}$	Mobility of the source
$\bar{Y}_{\text{R}}$	Mobility of the receiver
$\bar{Y}_{\text{p}}$	Point Mobility
$\bar{Y}_{\text{t}}$	Transfer mobility
ν	Poisson's ratio
$\bar{\nu}_p$	Normal velocity amplitude
$\bar{\nu}_{S_f}$	Root mean square free velocity
X	Ratio
Z	Impedance

LIST OF PUBLICATIONS

Journal Articles

Putra, A., **Cheah, Y.M.**, Muhammad, N., Rivai, A. and Wai, C.M., (2014). The effect of perforation on the dynamics of a flexible panel. *Advances in Acoustics and Vibration*, 2014.

Cheah, Y.M., Putra, A., Lim, K.H., Ramlan, R., and Muhammad, N., (2016). Identifying structure-borne noise transmission in building. *International Review of Mechanical Engineering*, 2016.

Proceedings

Cheah, Y.M., Putra, A., Muhammad, N. and Ramlan, R., (2015). Fundamental model of structure-borne vibration transmission in buildings using the portal frame approach. *Proceedings of Mechanical Engineering Research Day 2015*, MERD'15, 2015, pp.93-94.

Cheah, Y.M., Putra, A., Ramlan, R. and Muhammad, N., (2016). Identification of noise in room due to HVAC system. *Proceedings of Mechanical Engineering Research Day 2016*, MERD'16, 2016, pp. 208-209.

CHAPTER 1

INTRODUCTION

1.1 Background

As a number of building service equipments, such as pumps, ventilators and air conditioners, emergency generators, cooling towers and mechanical parking towers are installed in buildings, inevitably, the operation induced structure-borne vibration. This can be a common noise and vibration problem in the building. Long-term exposure to the ambient noise, can lead to noise-induced stress related health effects such as depression, sorrow, social isolation and aggression (Passchier-Vermeer and Passchier, 2000). For these reasons, noise control is essential.

Occupants of the interior space are subjected to noise due to direct transmission through the structure junction and the radiated sound from the surfaces of the room. The background noise criteria are based on two factors, the first is the perceived loudness of the noise relative to that of normal activities and second is the sound quality of the background noise (Schaffer, 1991). The recent ANSI S12.2 (2008) room noise criteria standard contains both a survey and an engineering method to specify room noise criteria (Schomer, 2009). The standard employs A-weighted sound level and extended Noise Criteria (NC), respectively. It is based on human hearing (subjective responses) and it is sensitive to the standard deviation of random noise or low frequency dominating in the 16 to 125 Hz octave band such as the sound that can be produced by Heating, Ventilation and Air Conditioning (HVAC) systems or other equipment. According to ANSI S12.2 (2008), the background

noise level of an unoccupied private office and conference rooms should not exceed a sound level of 35 dB(A). If the ambient noise level exceeds 35 dB(A), it will decrease intelligibility of the speech, for example, complaints of annoyance and stress can result.

The majority of noise sources come from vibrating equipment installed in the building. The transmitted vibration from service equipment activates the building structures to vibrate and eventually radiate sound which causes audible low frequency noise disturbance inside the building.The vibration waves follow different paths until they reach a room inside the building as illustrated in Figure 1.1. The illustration can be seen to mostly concern the airborne path rather than the structure-borne path (Schaffer, 1991). However, in practice, the structure-borne path can also come from the connection of the duct to the wall and from the roof.

Path A: Structureborne path through floor
Path B: Airborne path through supply air system
Path C: Duct breakout from supply air duct
Path D: Airborne path through return air system
Path E: Airborne path through mechanical equipment room wall
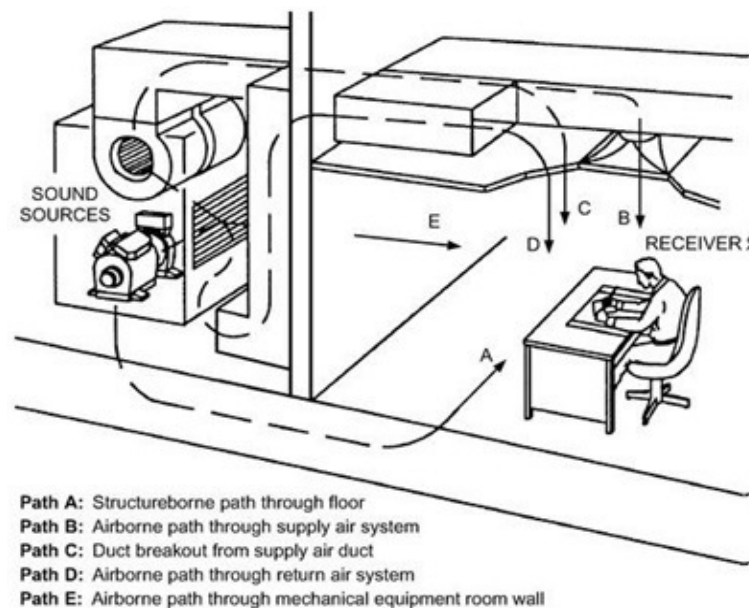
Figure 1.1 Typical paths of noise and vibration propagation from air-conditioning systems (Schaffer, 1991)

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1.2 Problem statement

Over a period of time, the vibrating equipment from the machinery are capable of injecting high level vibration input power that is hazardous to the building. The structural excitation on the building does not only lead to noise pollution, but may also causes structural damage. Discussions on the structure-borne noise behavior in buildings are still progressing. Recent work regarding the structure-borne vibration mostly concentrated on determination of the 'vibration strength' of the vibrating source (Späh and Gibbs, 2009; Ohlrich, 2011). There appears to be lack of studies regarding propagation of vibration and the noise radiation due to the vibration transmitted into the building.

Therefore, analytical and numerical analysis methods of a portal frame approach are of interest and the main discussion in this study. This study is focused on the structural response of a multi-storey portal frame building in order to understand the behavior of the structure with propagating waves. Then, in order to develop more complete models, the model is extended to include plate elements using numerical approaches. The established numerical approach employing Hybrid Finite Element and Statistical Energy Analysis is used to identify sound pressure level in the receiving room.

1.3 Objective

This study embarks on the following objective:

- To develop an analytical model for the structure-borne vibration in a building using portal frame structure approach.

- To validate the 2D analytical model using the proposed numerical model.

- To predict the sound pressure level in the receiving room using experimental data from reception plate experiment.

1.4 Methodology

This research started by developing the analytical model using a portal frame structure approach. The result from the analytical model is then validated using Finite Element model (FE). The FE model is then expanded into three dimensional model to be used for identifying sound pressure levels in the receiving room. To complete the analysis, the model is also extended to include plate elements and the power input from the source is identified by using the reception plate technique. Later, the power is supplied into the Hybrid FE/SEA model to calculate the sound pressure level due to the transmitted vibration in the building environment.

1.5 Scope of study

A simple 2D analytical portal frame model consists of beam and column elements is used to represent the building. The results of the analytical model are validated with those from 2D Finite Element (FE) model. It is tedious and time consuming to carry out the exact integration of a 3D analytical model. For this reason, the 3D FE portal model is constructed using Finite Element Method. In order to complete the analysis, the 3D FE model is extended to hybrid FE/SEA model in VA One Software to include the plate elements and cavities. The model is then used to predict the sound pressure level in the receiving room due to the input power inject by the motor. Only two machine faults studied in this project i.e. mass unbalance and structural looseness.

1.6 Thesis outline

In this chapter, some of the background and motivation for this study has been discussed. The problem is identified in this chapter. The discussion of the background also answers the aims and the scopes of the study on structure-borne vibration and noise transmission in building.

Chapter 2 discusses the literature reviews where established works from past researches are taken into account for this study. The basic background theory of longitudinal waves in columns and bending waves in beams are presented in this chapter. The detail formulation and assumptions of dynamic stiffness matrix for the building elements is also discussed in this chapter. The basic mathematical modeling of vibration input power for multiple contact points are also discussed in this chapter.

Chapter 3 presents the methodology of the two dimensional analytical model employing the assembly procedure for both stiffness matrix of each elements. The derivation to estimate the vibration velocity level for the case of a small portal frame is also included in this chapter. This chapter also includes portal frame experiment and reception plate experiment. Reception plate method is used to identify the input power of the induction motor which acts as a source to the portal frame model.

Chapter 4 reviews the analytical model from Chapter 3. The simple model is then extended consisting of five-floors to simulate a real building. The two-dimensional (2D) analytical model is validated with results from a numerical model. It is useful to predict the vibration response of the structure.

However, a two dimensional analytical model is not sufficient to study the noise generated in the room. In Chapter 5, a three dimensional (3D) numerical model is extended from

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the two-dimensional numerical model in Chapter 4 for a better understanding of the structural behaviour. The sound pressure level in the receiving room is predicted using a Hybrid FE/SEA approach.

Chapter 6 summarizes the findings of this study and proposes future work and recommendation for further study.

CHAPTER 2

LITERATURE STUDY

2.1 Introduction

In this chapter, established research work done for structural-borne vibration and noise transmission in building is reviewed. A brief principle to the theory of dynamic stiffness matrix approach for columns and beams is also included in this chapter. Euler-Bernoulli theory is applied to determine the bending waves in a beam element, while assuming only longitudinal waves occur in column element.

2.2 Past findings on noise generation and transmission

Building service equipment are often the major sources of interior noise. Operation of the equipment can induce mechanical vibration that propagates into the receiving room through building structure at the contact points between the equipment to the structure. The vibration can also create a secondary radiation of noise from the walls and floors in the room. The effects of transmitted vibration include feel-able movement of the building floors, shaking of items on shelves, and rumbling sounds. The level of the vibration generated from the machine depends on a number of parameters related to the operation of the machine. Machine deterioration due to poor maintenance and faults can also contribute to the vibration level. The significant factors of the system are the characteristics of the machine, the vibration propagation in the building, the building foundation response and the room acoustic response.

Heavy monolithic construction was first investigated to predict sound transmission through complete building system (Quirt, 2009). The principle of statistical energy analysis (SEA) is well suited for heavy monolithic construction, with structural elements such as concrete floors and masonry walls. The building elements such as floors or walls are treated as homogeneous and isotropic, where these elements are characterized at reverberant levels and most energy losses are due to adjoining elements (Craik, 1996). However, if the proposed system is anisotropic and highly damped, in result, the vibration levels vary across the surface of the structural assembly, which limits the applicability of simple SEA models.

On the other hand, for lightweight framed construction, study of sound transmission in Canada has focused on typical North American wood-framed building. Nightingale et al. (2002) proposed that the power flow via each flanking path is defined by five transmission factors whose combined effect is characterized by a path transfer function specific to the type of the excitation and the construction detail. Figure 2.1 shows the factors controlling the transmission of the structure-borne sound from the source to the room beside. Unlike heavy monolithic construction, the structure-borne source in lightweight construction is much more localized. Hence, in Nightingale et al. (2002) work, he suggests that direction of transmission relative to the framing members becomes an additional parameter needed for an accurate prediction.

For this project, the most common type of building structures in Malaysia will be discussed. Reinforced concrete (RC) frame structures are the predominant structural system in Malaysia. RC is designed to provide resistance to gravity and lateral loads through bending in beams and columns.
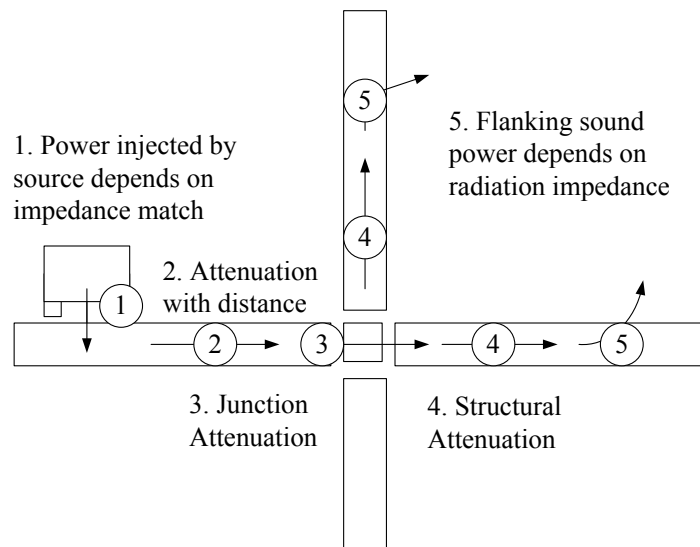
Figure 2.1 Five factors that control flanking transmission via floor or wall junction
from the source (Adapted from Nightingale et al. (2002)).

2.2.1   Vibration transmission through building structures

There are several methods which can be used to predict structure-borne noise gen-
erated by vibration through building structures. The introduction of the European standard
on estimating structure-borne noise in building, EN (2009) provides methods for estimating
structure-borne noise, however, due to the uncertainty in structure-borne sources, the calcu-
lations are rather complex and difficult to apply (Smith, 2011).

The estimation of sound levels due to service equipment in buildings is a complex
task and structure borne noise sources and transmission are not completely understood.

2.2.1.1   Analytical methods

The excitation of structure-borne sound is due to machinery that is installed on foun-
dations, in buildings, vehicle, ships etc. The current construction trend is that the buildings
are build continuously using light weight structures such as steel frames and pre-stressed

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concrete. This results in structures with inherently low damping in comparison to older buildings which are built with bricks. This phenomena also enables larger span floors with lower first resonances (Grootenhuis, 1990). Hence, these modern construction methods tend to result in buildings which are more susceptible to vibration within the frequency range of concern, typically between 5 and 200 Hz (Talbot and Hunt, 2000).

In recent years, many studies of transmission of structure-borne sound in buildings focus on buildings above railway tunnels. Ljunggren (1991) conducted the research on the transmission of structure-borne sound in buildings in the special case where the sound originates from underground railway traffic. In his study, it was demonstrated that a major part of the sound is transmitted upwards in the building in the form of quasi-longitudinal waves. The model is presented based on Ketten-Leiter theory, where the load bearing walls/columns and the floor are modelled as an infinite cascade of longitudinal rods alternating with impedance elements. Ljunggren's method was applied in Hassan (2001)'s recent work, a finite ground impedance is taken into account as well as wave reflection from the rood. As this study focused on low frequencies, from comparison between result from the approximate and the complete models, it is found that the agreement between the two models is in general fairly good for buildings with columns, but not for bearing walls.

Cryer (1994) used the dynamic stiffness method to model a two dimensional building to study vibration transmission in building. The model demonstrates the dynamic behaviour of the portal frame using analytic solutions of an elastic bar and Euler beam. The model does include the longitudinal and transverse behaviour of the elements such as floor and columns. In Hunt (1997)'s study, dynamic stiffness matrix is used for predicting the vibration response of a larger structure. The dynamic modeling is obtained by assembling columns and beams

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into repeating units. The method is extended to structures that are infinitely long, thereby reducing computational times. The numerical value of damping is found not to be a critical factor on account of radiation to infinity through the structure and into the piled foundation.

2.2.1.2   Numerical methods

Lightweight constructions are progressing faster than the conventional heavy structure buildings. Reliable prediction tools for noise transmission in lightweight buildings are in needs, hence analytical solutions have been established for a number of simple structures (Cremer et al., 2005). Galbrun (2010) conducted a study on modeling of vibration transmission through plate/beam structures typical of lightweight buildings. SEA was used as the framework of analysis for prediction, but the theories examined were independent from SEA. The result obtained indicated that simple point models are only applicable to single plate and beam system and to the parallel opposite plates connected along their center to a beam. Such application is limited to frequencies below 2 kHz. In Craik et al. (1991)'s studies, it is shown that SEA is unreliable for low frequencies investigation as there are only few resonant modes. It was found that it is the modes in the receiving subsystem that affect the power flow for transmission between plates. Finite element models do not suffer from the same limitation as SEA models.

Studies on transmission of structure-borne sound in buildings due to ground-borne noise have been conducted by Andersen and Jones (2006) and Fiala et al. (2007). A coupled finite element (FE) and boundary element (BE) scheme were applied in their studies. Fiala et al. (2007) proposed a numerical model of the structural and acoustic response of a building due to incoming wave field generated by high-speed surface railway traffic. The research

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concentrated on the structure and acoustic response of a multi-storey portal frame office building up to a frequency of 150 Hz to the passage of a Thalys high-speed train at constant velocity. The method was based on the Green's function of a layered half-space to calculate the floor radiation inside the building. Here, it is noted that the classical problem with numerical models is always the computational time. The study presented the modes of the structure and an example of the structure response is shown in Figure 2.2. According to this study, a FE/BE model, such as a tunnel, takes about five seconds per frequency in the two-dimensional case, whereas the computation time for the three-dimensional models is about two hours per frequency on a typical personal computer. Therefore, a full three-dimensional analysis takes approximately 1000-2000 times longer than the two dimensional analysis. In addition, this difference is increased when larger numerical models are considered.

The basic structure consists of a reinforced concrete portal frame structure containing columns of dimensions 0.3×0.2 m. This frame structure supports three 0.3 m thick horizontal slabs. The structure has a reinforced concrete central core which surrounds the static cast. The thickness of the core walls is 0.15 m. The structural model is extended with the in-fill walls of three rooms besides the core. Room 1 has dimensions $5 \times 6 \times 3$ m, and is located in the first floor, behind the core wall; room 2, which has the same dimensions, is located on the second floor; a smaller room 3 with dimensions $5 \times 4 \times 3$ m is located on the first floor, besides the core. The masonry in-fill walls are 0.06 m thick.

The finite element size is chosen as 0.25 m, which is fine enough for computations up to 200 Hz. The total model has 85518 degrees of freedom. A constant hysteretic structural damping of $\beta_\delta = 0.025$ is assumed.

6.3.2 The modes of the structure

mode 7, 3.73 Hz	mode 8, 3.83 Hz	mode 100, 207.61 Hz
mode 2, 6.66 Hz	mode 4, 11.96 Hz	mode 300, 131.83 Hz

Figure 6.5: (a) Quasi-static transmission of flexible foundation modes on the superstructure and (b) Flexible modes of the superstructure with clamped foundation

Figure 2.2 (a) Quasi-static transmission of flexible foundation modes on the superstructure and (b) flexible modes of the superstructure with clamped foundation (Adapted from Fiala et al. (2007)).

According to the Rubin criterion [Rub75], all the modes up to $1.5 f_{\max}$ have to be taken into account in the modal superposition in order to have a kinematic base that is sufficient up to a frequency of $f_{\max}$. In the present study, all the foundation and superstructure modes up to 300 Hz have been accounted for. A few modes are displayed in figure 6.5. The lowest mode of the superstructure with a clamped base is at 2.60 Hz, and only 12 modes of the superstructure have been found under 20 Hz. These low frequency modes are the global torsional and bending modes of the whole building. Above 50 Hz, however, the modal density tends to be very high and the high frequency modal shapes show local bending modes

2.2.2 Characterization of structure-borne source

The transmitted vibration waves from machinery do not only lead to noise pollution, but may also be hazardous to the building structure. Structural collapses can come without any early warning sign and are difficult to predict. Recent international cases includes the Sampoong Super Store collapse in Seoul, Republic of Korea (Gardner et al., 2002) and Rana Plaza collapse in the Greater Dhaka Area, the capital of Bangladesh (Yardley, 2013). The most important causes of Sampoong Super Store collapse were the reduced slab depth and the excessive loads applied to the building due to the change of use of the space. Similiarly, the main reason of the collapse of Rana Plaza is that the building was poorly constructed. The building was initially built for commercial use and later additional three floors were added to the building to house garment factories. Large power generators placed on these upper floors produced vibration, which aggravated the building structures. Finally, even though symptoms of structural distress were evident in several locations before the collapse of the building, the people in positions of knowledge and authority took no action resulting in the death of hundreds of people.

In order to prevent sudden structural damage, as well as to predict noise transmission, information of the structure-borne source strength is important. Recent work regarding the structure-borne noise is mostly concentrated on the characterization of the structure-borne sound source. The structure-borne sound source is introduced as a 'black box' and its effect on the connected structure is represented by its properties at the contact points (Cremer et al., 2005). The properties can be described in terms of source activity and a mobility matrix of the connection points (Moorhouse, 2007). The activity can be in the form of the velocity of the free (uncoupled) source (Cremer et al., 2005; Fulford and Gibbs, 1999) and the blocked

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Figure 2.3 Aerial view of the Rana Plaza following the disaster

Figure 2.4 Rescue crews at the site of the Sampoong Super Store collapse

force vector (Gardonio and Brennan, 2004). For a single contact and single component of excitation (Mondot and Petersson, 1987), the structure-borne sound power P from a source is,

$$P = \mathrm{Re}\big[\bar{W}\big] = \frac{\big|\bar{\nu}_{S_f}\big|^2}{\big|\bar{Y}_S + \bar{Y}_R\big|}\mathrm{Re}\big[\bar{Y}_R\big] \tag{2.1}$$

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where $\bar{\nu}_{Sf}$ is the root mean square of free velocity of the source, and $\bar{Y}_S$ and $\bar{Y}_R$ are the complex source and receiver mobility, respectively. Späh and Gibbs (2009) proposed a method for characterization of structure-borne sources based on the concept of the reception plate where the total structure-borne sound power from the machine under test is assumed equal to the power dissipated by a plate attached to the machine. The reception plate method is proposed as a laboratory test. Alber et al. (2011) also employed the reception plate method for prediction of structure-borne sound due to vibrations in taps and valves.

Bonhoff and Petersson (2009); Mathiowetz and Bonhoff (2013) focused on developing mathematical models to determine the power injected by the structure-borne source to the receiving structure. However, these works only focus on determination of the 'vibration strength' of the mechanical sources. There appears to be a lack of studies regarding propagation of vibration and the noise radiation in the space due to the vibration transmitted into the building. It is essential to understand how vibration waves travel in the building structure. In the interest of understanding the behavior of the building, analytical analysis will be the first step to investigate the phenomena.

2.3 Governing equation

2.3.1 Fundamental of structural waves

Concrete frame structures are one of the most common types of modern buildings. This type of building consists of a frame or skeleton of concrete. Horizontal members of the frame are known as beams and vertical members are known as columns. The column is the primary load-carrying element of the building and humans walk on flat panels of concrete referred to as slabs (floors). Figure 2.5 shows the elements of a frame structure. The structure

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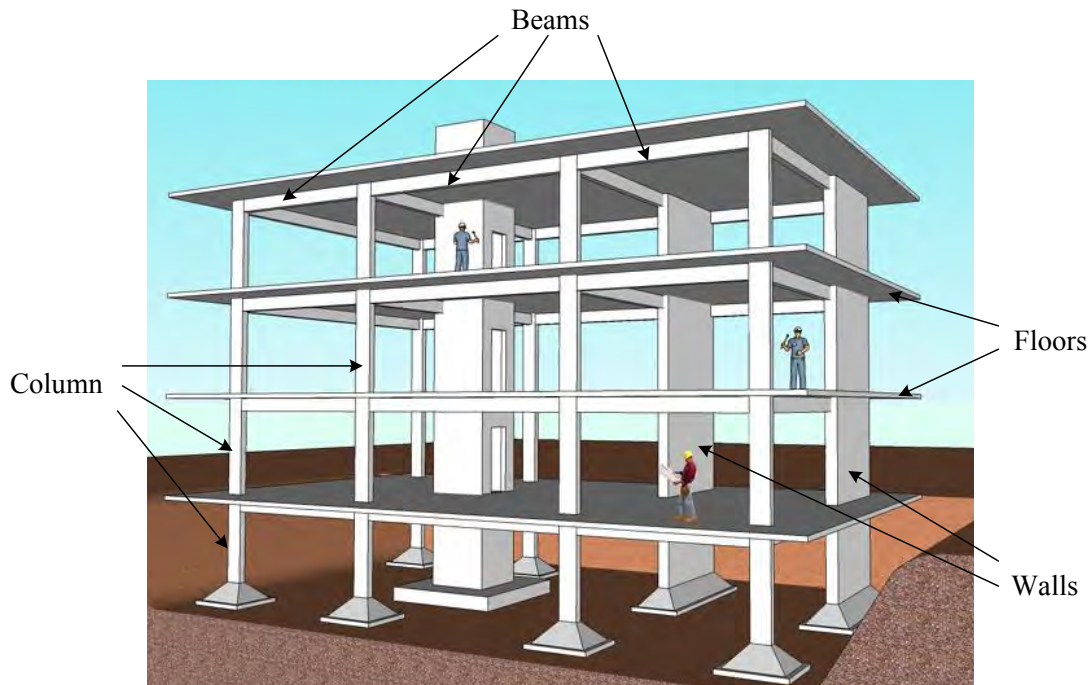
Figure 2.5 Elements in a concrete frame structure
(Adapted from www.understandconstruction.com)

is a connected frame of members, each of which are firmly connected to each other (moment connection) in order to resist the various loads that act on a building (Allen and Iano, 2013). The concrete frame rests on foundations, which transfer the forces from the building to the ground.

2.3.2   Longitudinal wave motion in columns

One of the two most important elements in a framework is column. Longitudinal waves are waves in which the direction of the particle displacement coincides with the direction of wave propagation (Cremer et al., 2005). Pure longitudinal waves can occur only in solids whose dimensions in all directions are much greater than wavelength. However, for most cases of practical structures, at least one of the dimensions is small compared with a wavelength. Hence, the waves that travel along the column cannot be a pure longitudinal

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one. Rather, the waves that travel in the columns are defined as quasi-longitudinal waves.

According to Thomson (1993), the equation of motion of axial vibration of a column in the y-direction is:

$$EA\frac{\partial^2 u(y,t)}{\partial y^2} = \rho A\frac{\partial^2 u(y,t)}{\partial t^2} \tag{2.2}$$

where $u(y,t)$ is the longitudinal displacement through the column with a uniform cross-sectional area A, Young's modulus E and density ρ. In order to find the general solution, Eq. (2.2) is assumed for a time harmonic solution of the form:

$$u(y,t) = U(y)e^{j\omega t} \tag{2.3}$$

This allows the solution to be written as

$$U(y) = a_1 e^{-jky} + a_2 e^{jky} \tag{2.4}$$

where the wave-number is defined as:

$$k = \omega\sqrt{\frac{\rho}{E}} \tag{2.5}$$

In Eq. (2.3), $a_1 e^{j(\omega t - ky)}$ is a wave varying harmonically in space and time, which propagates in the positive y-direction. Also, $a_2 e^{j(\omega t + ky)}$ is a wave varying harmonically in space and time, which propagates in the negative y-direction. The amplitudes of the waves are represented by a_1 and a_2.

Figure 2.6 illustrates the direction of the displacement and forces. The displacements at the boundaries of the element are expressed as:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{-jkL_y} & e^{jkL_y} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{2.6}$$

where L_y is the length of the column in the direction of y. Also, if the element is subjected to axial loads F_1 and F_2, the applied forces are expressed in terms of displacement by:

$$F(y) = EA\frac{\partial u}{\partial y} \tag{2.7}$$

then the vector of forces is expressed as:

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = EA \begin{bmatrix} jk & -jk \\ -jke^{-jkL_y} & jke^{jkL_y} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{2.8}$$

and the dynamic stiffness is defined by:

$$Ku = F \tag{2.9}$$

From Eq. (2.6) and (2.8), the dynamic stiffness matrix is equal to:

$$K = EA \begin{bmatrix} jk & -jk \\ -jke^{-jkL_y} & jke^{jkL_y} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ e^{-jkL_y} & e^{jkL_y} \end{bmatrix}^{-1} \tag{2.10}$$

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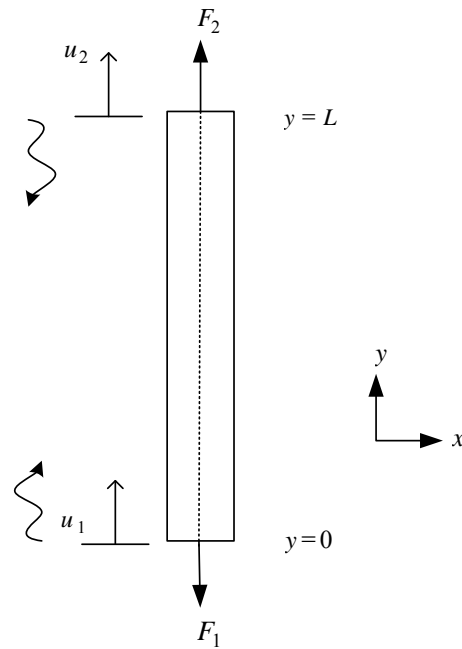
Figure 2.6 Longitudinal waves travel in column element

2.3.3 Bending waves in beams

For bending in a beam, bending waves are assumed to propagate through the beam. The equation of motion for the flexural vibration of a beam is fourth order and there are four types of free wave solution. The relative amplitudes depend on the excitation, assumed to be concentrated at the two ends, which comprises forces and moments. For wavelengths which are greater than six times the thickness of the beam (Petyt, 1990), Euler-Bernoulli beam theory can be applied. The equation of motion is:

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + \rho A\frac{\partial^2 u(x,t)}{\partial t^2} = 0 \qquad (2.11)$$

The solution in time harmonic form can be written as:

$$U(x) = a_1 e^{-jkx} + a_2 e^{jkx} + a_{n1} e^{-kx} + a_{n2} e^{kx} \qquad (2.12)$$

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where the wave-number is defined as:

$$k = \sqrt{\omega} \left(\frac{\rho A}{EI} \right)^{\frac{1}{4}}$$

(2.13)

As can be seen in Eq. (2.12), a_1 and a_2 are the amplitudes of waves propagating in the positive and negative x-direction, where a_{n1} and a_{n2} are the wave amplitudes of the near-field in the region $x>0$. The direction of the displacements, forces and waves can be seen in Figure 2.7.
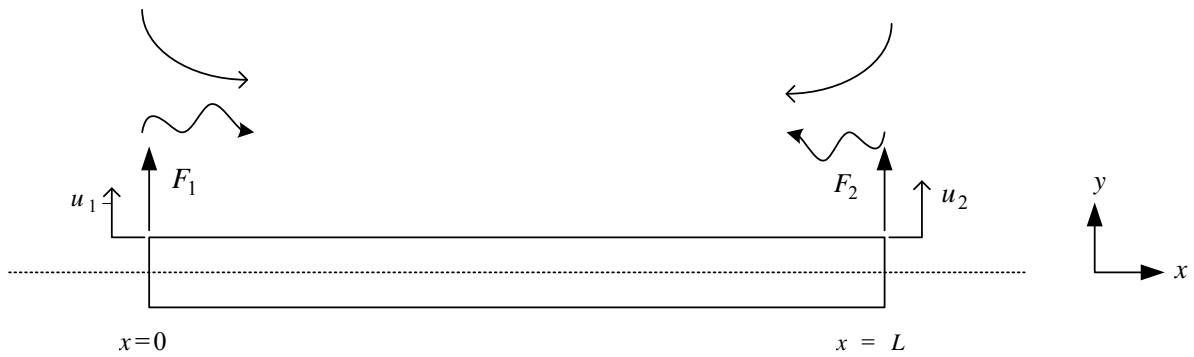

Figure 2.7 Bending waves propagating in beam element

A beam element requires two degrees of freedom at each end, translational displacement and rotational displacement. The latter is defined as:

$$\theta(x,t) = \frac{\partial u(x,t)}{\partial x}$$

(2.14)

Then, the vector of the degrees of freedom are is given by:

$$
\begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkL_x} & e^{jkL_x} & e^{-kL_x} & e^{kL_x} \\ -jke^{-jkL_x} & jke^{jkL_x} & -ke^{-kL_x} & ke^{kL_x} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_{n1} \\ a_{n2} \end{bmatrix}
\tag{2.15}
$$

The shear force and bending moment acting on the beam are given by:

$$
F(x) = -EI\frac{\partial^3 u}{\partial x^3} \quad ; \quad M(x) = EI\frac{\partial^2 u}{\partial x^2}
\tag{2.16}
$$

from which the general matrix of forces is equal to:

$$
\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = EI \begin{bmatrix} jk^3 & -jk^3 & -k^3 & k^3 \\ k^2 & k^2 & -k^2 & -k^2 \\ -jk^3 e^{-jkL_x} & jk^3 e^{-jkL_x} & k^3 e^{-kL_x} & -k^3 e^{kL_x} \\ -k^2 e^{-jkL_x} & -k^2 e^{jkL_x} & k^2 e^{-kL_x} & k^2 e^{kL_x} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_{n1} \\ a_{n2} \end{bmatrix}
\tag{2.17}
$$

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Hence, the dynamic stiffness matrix of a beam is the solution of:

$$
K = EI
\begin{bmatrix}
jk^3 & -jk^3 & -k^3 & k^3 \\
k^2 & k^2 & -k^2 & -k^2 \\
-jk^3 e^{-jkL_x} & jk^3 e^{-jkL_x} & k^3 e^{-kL_x} & -k^3 e^{kL_x} \\
-k^2 e^{-jkL_x} & -k^2 e^{jkL_x} & k^2 e^{-kL_x} & k^2 e^{kL_x}
\end{bmatrix}
\times
$$

$$
\begin{bmatrix}
1 & 1 & 1 & 1 \\
-jk & jk & -k & k \\
e^{-jkL_x} & e^{jkL_x} & e^{-kL_x} & e^{kL_x} \\
-jke^{-jkL_x} & jke^{jkL_x} & -ke^{-kL_x} & ke^{kL_x}
\end{bmatrix}^{-1}
\tag{2.18}
$$

where I is the second moment of cross sectional area and L_x is the length of the beam in the direction of x. Many publication results include the calculation of dynamic stiffness matrix for the Euler-Bernoulli beam element. Such calculations are provided by Bishop and Johnson (2011), Gorzynski and Thornton (1974) and Warburton (1976) who surveyed the exact FE method. Unlike Timoshenko beam theory, Euler-Bernoulli beam theory does not taken into account the effects of transverse shear strain. It is suitable where beam thickness is less than $1/6$ of the wavelength. For these reasons, Euler-Bernoulli beam theory is applied in this study.

2.3.4 Input power of multiple contact points

Prediction and measurement on the structure-borne sound source has been studied from past researches especially lightweight building elements (Gibbs, 2013) where a plate is used as the receiver. It is a straight-forward prediction which concentrates on the calculation

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for receiver structure. The structure-borne sound source can be modeled into a single and multiple contact point force. Mobility is defined as the ratio of the resulting velocity ν to the driving force F at a given point (Gardonio and Brennan, 2004)

$$Y = \frac{\nu}{F} \tag{2.19}$$

and the impedance is the reciprocal of mobility i.e. the ratio of the force F acting on the system to the resulting velocity ν given as

$$Z = \frac{F}{\nu} \tag{2.20}$$

Considering a vibrating source has impedance z_{S} which is free suspended and vibrates with velocity ν_f. The velocity of the source in this case is known as 'free velocity'. Assuming the source is attached on a rigid surface as seen in Figure 2.8, the source is restrained to move at the contact point and it is now injecting force onto the surface. The force in this case is now called as 'blocked force' which is also defined as

$$f_{\mathrm{B}} = z_{\mathrm{S}}.\nu_f \tag{2.21}$$

Considering the source is rigidly connected to a receiver structure as shown in Figure 2.9, where both the source and structure are assumed moving at the same velocity ν. The blocked force at the contact point is now the sum of the forces from the source f_{S} and the force which is applied on the receiver f_{R} (Brennan and Ferguson, 2004). The blocked force can thus be expressed as
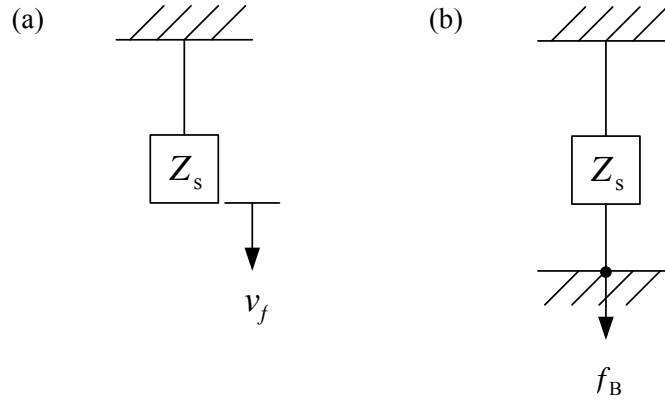
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Figure 2.8 A vibrating source with (a) free velocity and (b) blocked force

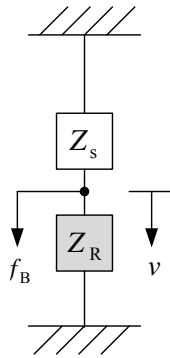$$f_B = f_S + f_R = (z_S + z_R)\nu \tag{2.22}$$

Figure 2.9 A source connected to a receiver

Assuming the source is connected to a receiver structure through N contact points, the formulation can be expressed in terms of matrices and vectors. The input power can be expressed as

$$P_{\text{in}} = \frac{1}{2}\text{Re}\left\{\widetilde{\mathbf{F}}_R^H\widetilde{\mathbf{v}}\right\} = \frac{1}{2}\text{Re}\left\{\widetilde{\mathbf{v}}^H\mathbf{Z}_R^H\widetilde{\mathbf{v}}\right\} \tag{2.23}$$

where $\widetilde{\mathbf{F}}_R = \{f_1 f_2 f_3...f_N\}^{-1}$ and $\widetilde{\mathbf{v}} = \{\nu_1 \nu_2 \nu_3...\nu_N^{-1}\}$ are column vectors of size $N \times 1$ and $\mathbf{Z}$ is the impedance of $N \times N$ matrix and the superscript H denotes the conjugate transpose

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while the curly sign indicates the vector.

Using Eq. (2.22) and substituting it into Eq. (2.23), this gives

$$P_{\text{in}} = \frac{1}{2}\text{Re}\Big\{\widetilde{\mathbf{v}}_f^H \, |\mathbf{Y}_{\text{S}} + \mathbf{Y}_{\text{R}}|^{-H} \, \mathbf{Y}_{\text{R}}|\mathbf{Y}_{\text{S}} + \mathbf{Y}_{\text{R}}|^{-1}\widetilde{\mathbf{v}}_f\Big\} \tag{2.24}$$

The input power is defined in decibels as sound power level. The scale and the reference value used in this work are listed below:

$$L_w = 10\log_{10}\left(\frac{W}{W_{\text{ref}}}\right), W_{\text{ref}} = 1 \times 10^{-12}\text{W} \tag{2.25}$$

2.3.5   Statistical Energy Analysis

Statistical Energy Analysis (SEA) has been widely used to calculate the energy flow between the connected resonant systems. It is used to predict the average values over a frequency band of interest. The local models of subsystems are described statistically and the average response of the subsystems is predicted. It has an advantage where it is suitable for modeling vibro-acoustic systems when detailed information about the system properties are not available.

The primary variable in SEA is the modal energy. The loss factor is used to characterize the energy loss in the subsystem and the coupling loss factors are used to characterize the power flow between the subsystems (Lyon, 2014). A simple power flow between two subsystems using SEA is illustrated in Figure 2.10. The total energy in each system is represented by E_1 and E_2, where n_1 and n_2 are the modal density for each subsystem respectively. The input power is W^{in} and transmitted power is W', and the dissipated power of each system is marked as W^{diss}.