A Markovian Approach to Determine Optimal Means for SME Production Process

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Abstract

The determination of optimum process mean has become one of the focused research area in order to improve product quality. Depending on the value of quality characteristic, an item can be reworked, scrapped or accepted by the system which is successfully transform to the finishing product by using the Markovian model. By assuming the quality characteristic is normally distributed, the probability of rework, scrap and accept is obtained by the Markov model and next the optimum of process mean is determine which maximizes the expected profit per item. In this paper, we present the preliminary analysis of selecting the process mean by referring to SME production process. By varying the rework and scrap cost, the analysis shows the sensitivity of the Markov approach to determine process mean.

Keywords: Markovian model; optimum process mean; product quality

1. INTRODUCTION

The optimum process mean setting is used for selecting the manufacturing target. The selection of the appropriate process parameters is of major interest and importance in satisfying such a desirable goal. The selection of optimum process mean will directly affect the process defective rate, production cost, rework cost, scrap cost and the cost of use. The objective of this study is to find the optimum values of process parameters or machine settings that will achieve certain economical objectives which are usually refer as maximum profit. A number of models have been proposed in the literature for determining an optimum target (mean and/ or variance). According to Taguchi (1986), if the process mean approaches the target value and the process standard deviation approaches zero, then the process is under optimum control.

The targeting problem has many applications, the most common being the canning or filling problem in which the quality characteristic is the amount of fill, or ingredient, put in a can. Traditionally, the product characteristic within the specifications is the conforming item. Since the probability of producing a good item, p, can be controlled by setting the production process mean, it is possible to find the value of process mean that will maximize the net profit per fill item [1].

For the filling or canning industry, the product needs to be produced within the specific limits. In the case of production process where products are produced continuously usually the specification limits are implemented based on quality evaluation system that focuses primarily on the cost of non-conformance. The manufacturing cost per unit considers the fixed and variable production costs and the constant inspection cost. The variable production cost is proportional to the value of the quality characteristic. Consider a certain quality characteristic, where a product is rejected if its products performance either falls above an upper specification limit or falls below a lower specification limit. If the product measurement level is higher than upper limit it can be reworked or scrapped if it is falls below than lower limit. If the process target is set too low, then the proportion of non-conforming products becomes high. In consequence the manufacturer may experience high rejection costs associated with non-conforming products.

2. LITERATURE REVIEW

There are considerable attentions paid to the study of economic selection of process mean. The initial process targeting problem addressed is the can filling problem. The first real attempt to deal with this problem was in [2]. He considered the problem of finding the optimal process mean for a canning process when both upper and lower control limits are specified.

The author assuming that a quality characteristic follows normal and gamma distribution. A simple method is suggested in determining the optimum target mean that minimizes the total cost. According to Springer, the financial loss during the production where the item is above the upper specification limit is not necessarily equal to the loss for the lower specification limit. The same problem as in [2] except that only the lower limit was specified, and used a trial and error procedure to tabulate a set of values for the specified lower limit [3]. Furthermore, the assumption is on undersized and oversized items are reprocessed at a fixed cost. Other method is the model to determine the optimum process targets with the assumption that the cans meet the minimum content requirement are sold in regular market at a fixed price [4]. In contrast the under filled cans are sold at a reduced price in a secondary market. Thus, the customer is compensated for poor quality but does not pay for the redundant quality product.

Taguchi [5] presented the quadratic quality loss function for redefining the product quality. According to him, product quality is the society's loss when the product is sold to the customer. Other researchers considered performance as the variable to maximize the expected profit per item in obtaining the optimum process mean [6]. The profit is depending on the normal characteristic value and each item is classified to three grades; items that sold to primary market, items that sold to secondary market and rework items. In order to evaluate the quality cost of a product for two different markets, the quadratic quality loss function is adopted [7]. The aim is to obtain the optimum process mean based on maximizing the expected profit per item.
A great majority of process target models in the literature were derived assuming a single-stage production system, except for [8] and [9]. Al-Sultan [8] developed an algorithm to find the optimal machine setting when two machines are connected in series. According to him, the rework items are sent back to the first stage. Markovian approach is introduced by [9] for formulating the model in multi-stage serial production system by assuming that the quality characteristic is normally distributed with both-sided specification limits is known. The purpose of the research is to study production system where the output of a stage could be accepted, rework or scraped. The approach is to generate the absorption probabilities into scrap, rework, and accept states for each production stage. In this paper we will implement the Markovian approach as the preliminary analysis for determining the optimum process mean which fulfil the maximum expected profit.

3. MODEL FORMULATION

A transition probability matrix P to describe transitions among three states is proposed for the model formulation [9]. State 1 indicates processing state at first stage production. State 2 indicates that the item has been processed successfully and transform into finishing product. While Stage 3 represents that the item has been scrapped. Fig. 1 show a single-stage production system which is used as the preliminary study for this research.

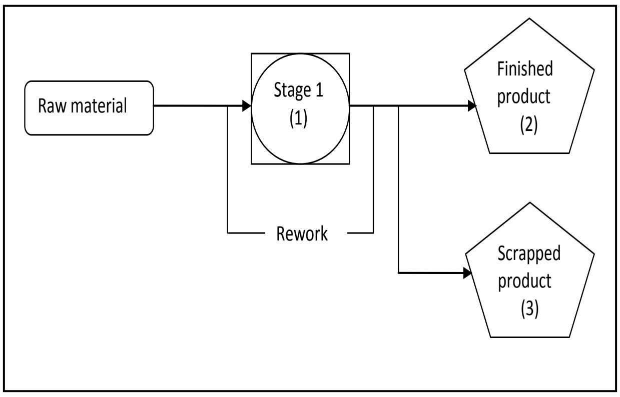

Fig. 1 Single-stage production system process flow

The single-step transition probability matrix can be expressed in equation (1).

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Where p_{11} is the probability of an item being rework, p_{12} is the probability of an item being accepted, and p_{13} is the probability of an item being scrapped. By assuming that the quality characteristic is normally distributed, these probabilities can be expressed as in Fig. 2.

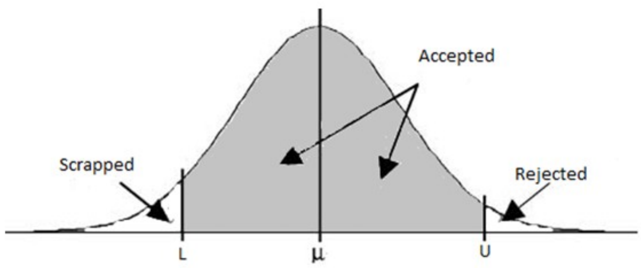

Fig. 2. Normal Distribution graph for accepted, reworked and scrapped region

The details formulation of P_{11}, P_{12} and P_{13} is as follows.

$$p_{11} = \int_U^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \qquad (2)$$

$$p_{12} = \int_L^{U} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \qquad (3)$$

$$p_{13} = \int_{-\infty}^{L} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \qquad (4)$$

Therefore, the expected profit per item can be expressed as follows:

$$EPR = SP.f_{12} - PC - SC.f_{13} - RC.(m_{11} - 1) \qquad (5)$$

Where SP is represent the selling price per item, PC is processing cost per item and SC and RC is scrapped cost and rework cost per item, respectively. f_{12} represent the long-term probability of an item has been processed successfully at Stage 1 and being accepted. While f_{13} represent the long-term probability of the item is being scrapped as the item is rejected from Stage 1. The element of m_{11} is represent the expected number of times in the long run that the transient state 1 is occupied before accepted or scrapped. The m_{11} element is obtained by following fundamental matrix ***M***:

$$\boldsymbol{M} = (\boldsymbol{I} - p_{11})^{-1} = m_{11} = \frac{1}{1 - p_{11}} \qquad (6)$$

Where ***I*** is the identity matrix. The long-run probability matrix, ***F***, can be expressed as follows:

$$\boldsymbol{F} = \boldsymbol{M} \times \boldsymbol{R} = \begin{array}{c c} & \begin{array}{cc} 2 & 3 \end{array} \\ 1 & \begin{bmatrix} \frac{P_{12}}{1 - P_{11}} & \frac{P_{13}}{1 - P_{11}} \end{bmatrix} \end{array} \qquad (7)$$

Substituting for f_{13} and m_{11}, the expected profit equation (5) can be written as follows:

$$EPR = SP\left(1 - \frac{p_{13}}{1 - p_{11}}\right) - PC - SC\left(\frac{p_{13}}{1 - p_{11}}\right) - RC\left(\frac{p_{11}}{1 - p_{11}}\right) \qquad (8)$$

From this equation, one would like to find the value of optimum mean that maximizes the expected profit.

4. PRELIMINARY ANALYSIS

To perform the preliminary analysis, one of the small and medium enterprise (SME) companies is selected as a case study. The selected parameter is as follows. The selling price per item, SP is 1.20, processing cost per item, PC=0.50, rework cost, RC=0.30, scrapped cost, SC=0.55, standard deviation is set to 1 while the lower and upper limit is set to 28 and 31, respectively as shown in Table I..

Table I: Selected parameters for model formulation

μ	EPR
28	-0.1766
29	0.4151
30	0.6125
31	0.5474

Graph in Fig. 3 show the expected profit was optimum at mean 30.09 with maximum EPR of 0.6134.

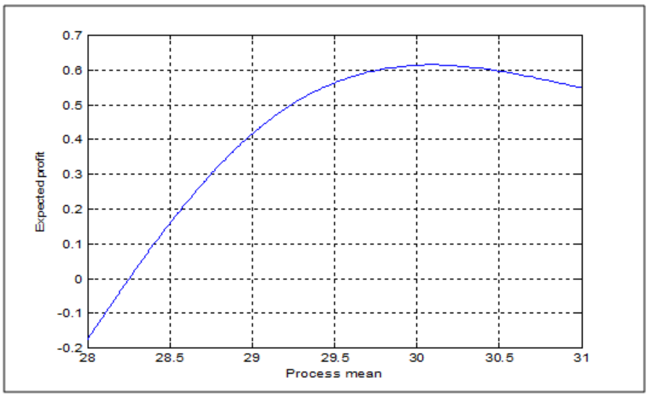

Fig. 3 Expected profit versus process mean

5. SENSITIVITY ANALYSIS

The sensitivity analysis is performed to illustrate the possible impact of parameters on the optimal process mean and also the optimal expected profit. The rework and scrap cost is varied for the single-stage production system and their effects are shown in Fig. 4 and Fig. 5.

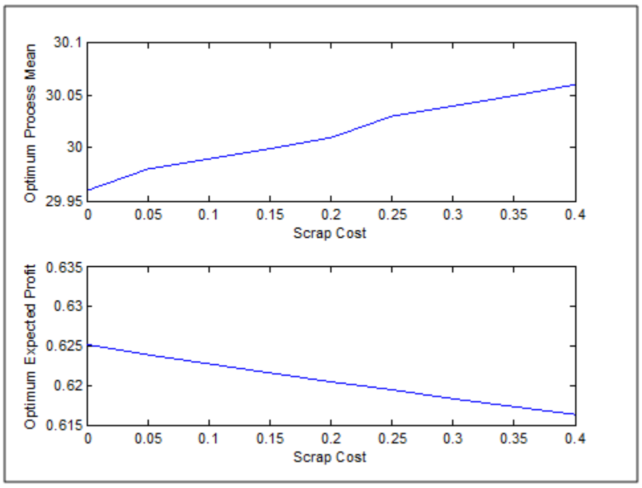

Fig.4 Effect of rework cost on optimal value of process mean and expected profit

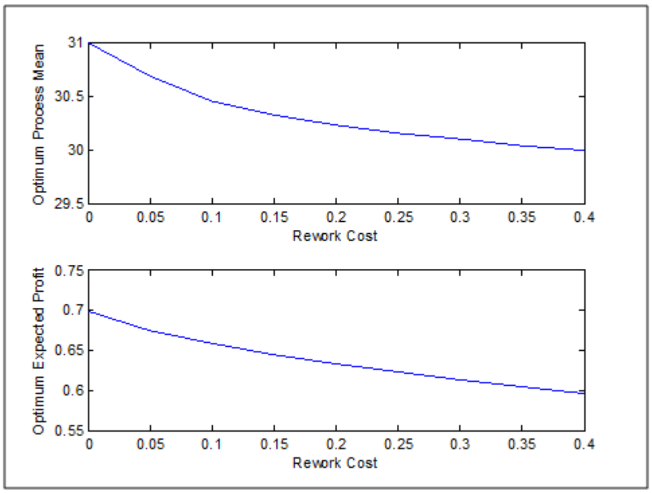

Fig. 5 Effect of scrap cost on optimal value of process mean and expected profit

From the Fig.4 and Fig.5 show the behaviour of the optimum process mean and optimum expected profit with the variation of scrap cost and rework cost. It can be shown that the optimum process mean and optimum expected profit are sensitive to the changes of scrap and rework cost values.

The analysis is follow the behaviours of the optimum process mean and the optimum expected profit with the variation of the scrap and rework costs for the production system. The result is shown in Table II. For cases 1-4, as scrap cost increases, the optimum process mean is increase and the optimum expected profit is decrease. While for cases 5-8, the expected profit is decrease when the rework cost is increase while the optimum process mean is decrease. It is observed that the optimum expected profit is decreases as the scrap cost and rework cost is increase for the single stage production system.

Table II: Optimum process mean and the optimum expected profit

Cost parameter	Case#	Parameter value	Optimum process mean	Optimum expected profit
Scrap Cost	1	0.45	30.05	0.6153
	2	0.50	30.06	0.6143
	3	0.55	30.09	0.6134
	4	0.60	30.10	0.6125
Rework Cost	5	0.20	30.22	0.6333
	6	0.25	30.15	0.6299
	7	0.30	30.09	0.6134
	8	0.35	30.04	0.6047

6. DISCUSSION AND CONCLUSION

In this paper, we present the preliminary analysis by using the Markovian approach to find the optimum process mean that is maximizes the expected profit. From the analysis, it is observed that when either the scrap cost or rework cost is increase, the expected profit will be decrease. Due to this condition, the manufacturer will try to reduce the rework or scrap items. By determining the optimum process mean, the manufacturer also needs to produce items that are approach to target value so that they will gain maximum profit without ignoring the specification limit. For future work, we are considering to hybrid Artificial Intelligence (AI) techniques in order to find the optimal target value.

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