PERISTALTIC TRANSPORT OF A VISCOUS FLUID IN A POROUS CHANNEL WITH SUCTION AND INJECTION

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Abstract:
Peristaltic pumping by a sinusoidal traveling wave in the porous walls of a two-dimensional channel filled with a viscous incompressible fluid is studied under long wavelength and low Reynolds number assumptions. The fluid is injected into the channel perpendicular to the low porous layer with constant velocity v_0 and is sucked out into the upper permeable layer with the same velocity v_0. The physical quantities of interest like pumping, the velocity, the stream function and the frictional force are discussed for various parameters of interest governing the flow like permeability, amplitude ratio etc. It is observed that the behaviour of frictional force is opposite to that of pressure rise.

1 Introduction

The study of peristaltic pumping has received considerable attention for the past few decades because of its importance in both biological and mechanical situations. Peristalsis consists of narrowing and transverse shortening of a portion of the tube which then relaxes, while the lower portion becomes shortened and narrowed. Some Bio-medical instruments are manufactured based on the principles of peristaltic pumping. A detailed review on peristalsis was presented by Jaffrin and Shapiro (1971). The analysis given for a single fluid was extended by Brasseur *et al.* (1987) for a two fluid model in a channel. All these investigations are made with the assumptions that the wall of the duct is impermeable.

Viscous fluid flow through and past porous media has wider applications in many branches like Bio-medical Engineering, Chemical engineering and such other important fields. Tang and Fung (1975) and Gopalan (1981) reported that the lung can be described as a channel bounded by two thin layers of porous media. Shivakumar *et al.* (1986) discussed the flow in a channel of varying gap with permeable walls. A thesis on the peristaltic pumping in a channel with flexible porous wall has been presented by Reese (1985). Sreenadh and Arunachalam (1986) studied the Couette flow between two permeable beds with suction and injection. In view of the several physiological applications it is necessary to study the peristaltic transport of a viscous fluid in a channel with suction and injection.

In this paper peristaltic flow of a viscous fluid in a channel with blowing and suction is investigated under long wavelength and low Reynolds number assumptions. The fluid is injected into the channel perpendicular to the lower porous bed with constant velocity v_0 and is sucked out to the upper permeable bed with the same velocity v_0. The velocity, the stream function, the pressure rise and friction force are obtained. The results are deduced and discussed.

2 Mathematical formulation and solution

Consider the peristaltic pumping of a viscous fluid in a porous channel of half width a (Figure 1). A longitudinal train of progressive sinusoidal waves takes place on the upper and lower permeable walls of the channel. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity v_0 and is sucked out of the upper permeable wall with the same velocity v_0. For simplicity, we restrict our discussion to the half width of the channel as shown in figure1. The wall deformation is given by

$$H(X, t) = a + b \, \mathrm{Sin} \, \frac{2\pi}{\lambda} (X - ct) \tag{1}$$

where b is the amplitude, λ is the wave length and c is the wave speed.

2.1 Equations of motion

Under the assumptions that the tube length is an integral multiple of the wavelength λ and the pressure difference across the ends of the tube is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y). The transformation between these two frames is given by

$$\begin{aligned} x &= X - ct \;, \; y = Y \\ u(x, y) &= U(X - ct, Y) - c \\ v(x, y) &= V(X - ct, Y) \end{aligned} \tag{2}$$

where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame. Further, we assume that the wavelength is infinite. So the flow is Poiseuille type at each local cross – section.

We use the following non – dimensional quantities,

$$\bar{x} = \frac{x}{\lambda}; \quad \bar{y} = \frac{y}{a}; \quad \bar{t} = \frac{ct}{\lambda} \quad \bar{u} = \frac{u}{c}; \quad \bar{\psi} = \frac{\psi}{ac}$$

$$\bar{\phi} = \frac{b}{a}; \quad \bar{h} = \frac{H}{a}; \quad \alpha = \left(\frac{am}{\sqrt{k}}\right)^{-1}; \quad R_e = \left(\frac{ac}{\vartheta}\right);$$

$$\sigma = \frac{a}{\sqrt{k}}; \quad \bar{p} = \frac{a^2}{\lambda \mu c} p; \quad \overline{Q_1} = \frac{Q_1}{c} \quad , \; \overline{v_0} = \frac{v_0}{c}$$

where R is Reynolds number, ϕ is the amplitude ratio, α is the permeability (including slip) parameter.

The equation governing the motion (ignoring the bars) is

$$\frac{\partial^2 u}{\partial y^2} - k\frac{\partial u}{\partial y} = P \quad (3)$$

where $k = R_e v_0$, and $P = -\frac{\partial p}{\partial x}$

$$Q_l = \frac{P}{\sigma^2} \text{ (Darcy's law)} \quad (3a)$$

The non-dimensional boundary conditions are :

$$\psi = 0 \text{ at } y = 0 \quad (4)$$

$$u = \frac{\partial \psi}{\partial y} = -1 - \alpha\frac{\partial u}{\partial y} \text{ at } y = h \quad (5a)$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \quad (5b)$$

where ψ is the stream function.

2.2 Solution

Solving the equation (3) with the boundary conditions(4) and (5), we obtain the velocity as

$$u = -1 + \frac{P}{k^2}\left[\left(e^{ky} - e^{kh}\right) + \alpha k\left(1 - e^{kh}\right) - k(y - h)\right] \quad (6)$$

Integrating the equation (6) and using the boundary condition $\psi = 0$ at $y = 0$, we get

$$\psi = -y + \frac{P}{k^2}\left[\left(\frac{e^{ky}}{k} - \frac{1}{k} - ye^{kh}\right) + \alpha k\left(1 - e^{kh}\right)y - k\left(\frac{y^2}{2} - hy\right)\right] \quad (7)$$

The volume flux q through each cross-section in the wave frame is given by

$$q = \int_0^h u\,dy$$

$$= -h + \frac{P}{k^3}\left[e^{kh}(1 - kh) - 1 + \frac{h^2k^2}{2} + h\alpha k^2(1 - e^{kh}\right] \quad (8)$$

The instantaneous volume flow rate Q (X, t) in the laboratory frame between the central line and the wall is

$$Q(X, t) = \int_0^H U(X, Y, t)\,dY$$

$$= \frac{p}{K^3}\left[e^{kh}(1 - kh) + \frac{h^2k^2}{2} + \alpha hk^2(1 - e^{kh}) - 1\right] \quad (9)$$

From equation (3.8) we have

$$\frac{dp}{dx} = \frac{-2k^3(q + h)}{2e^{hk}(1 - hk) + 2\alpha hk^2(1 - e^{hk}) + h^2k^2 - 2} \quad (10)$$

Averaging equation (9) over one period yields the time mean flow (time-averaged volume flow rate) $\bar{Q}$ as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\,dt$$

$$= q + 1 \quad (11)$$

2.3 The Pumping Characteristics

Integrating the equation (10) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \frac{-2k^3(q + h)}{2e^{hk}(1 - hk) + 2\alpha hk^2(1 - e^{hk}) + h^2k^2 - 2}\,dx \quad (12)$$

The time-averaged flux at zero pressure rise is denoted by $\bar{Q}_0$ and the pressure rise required to produce zero flow rate is denoted by Δp_0.

The dimensionless friction force F at the wall across one wavelength is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right)dx$$

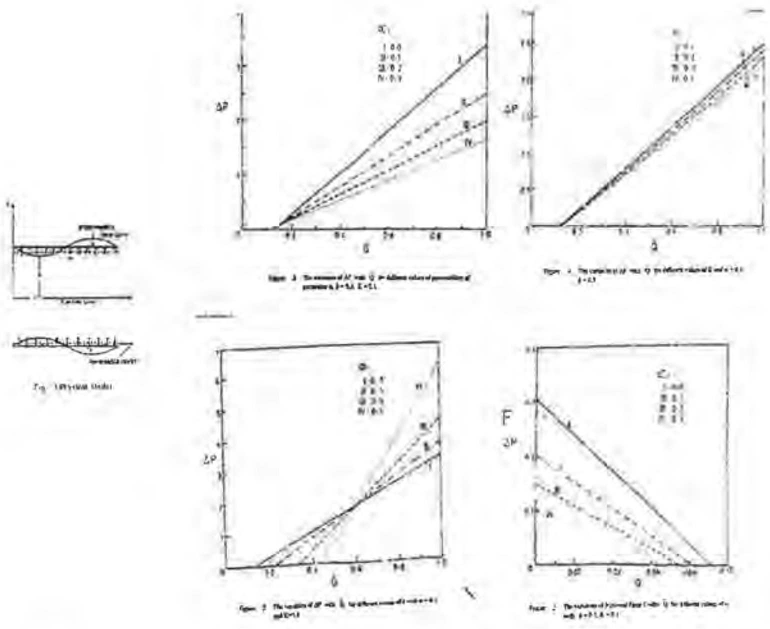
$$= \int_0^1 \frac{2hk^3(q+h)}{2e^{hk}(1-hk)+2\alpha hk^2(1-e^{hk})+h^2k^2-2}dx \qquad (13)$$

3 Discussion of the results

From equation (12) we have calculated the pressure difference as a function of $\bar{Q}$ for different values of α for fixed ϕ, k and is depicted in figure 2. It is observed that for a given α, Δp increases with increase in $\bar{Q}$. For a given Δp, the flux $\bar{Q}$ depends on α and it increases with increasing in slip parameter α. For a given $\bar{Q}$, Δp decreases with increasing α.

The variation of pressure rise with time averaged flow rate is calculated from equation (12) for different amplitude ratio's ϕ and is shown in figure 3 for fixed k = 0.1 and α = 0.1. We observe that for a given ϕ, Δp increases with increasing $\bar{Q}$.

From equation (12) we have calculated the pressure rise with time–averaged flow rate for a different values of k and is shown in figure 4 for fixed $\alpha = 0.1$ and $\phi = 0.3$. It is observed that for a given k, Δp increases with increasing $\bar{Q}$. For a given Δp, $\bar{Q}$ increases with increasing suction / injection parameter k. For a given flux $\bar{Q}$, Δp decreases with increasing k. From figure 5 we observe that the frictional force shows opposite behaviour compared to the pressure rise.



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