UNIVERSITI TEKNIKAL MALAYSIA MELAKA

Faculty of Mechanical Engineering

EFFECT OF STIFFNESS NONLINEARITY ON THE TRANSDUCTION COEFFICIENT OF A VIBRATION BASED ELECTROMAGNETIC ENERGY HARVESTING DEVICE

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Master of Science in Mechanical Engineering

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A thesis submitted
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DECLARATION

I declare that this thesis entitle "EFFECT OF STIFFNESS NONLINEARITY ON THE TRANSDUCTION COEFFICIENT OF A VIBRATION BASED ELECTROMAGNETIC ENERGY HARVESTING DEVICE" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science.

Signature : . . . .

Supervisor Name : . . . .

Date : . . . .

DEDICATION

To my beloved

family

ABSTRACT

The concept of harvesting energy from ambient to power applications has been gaining attention from researchers in recent times due to its self-sustaining power source solution. Traditionally, the linear system has been used to harvest the energy. However, the limitation due to the difficulty in matching between the natural frequency of the device and the ambient frequency makes it less favourable. This is even worst when the ambient frequency varies with time, which leads to the narrow bandwidth for the device to perform. Presently, the nonlinear system has been introduced to overcome the limitation of the linear system. This thesis analyses the improvement of bandwidth performance at the maximum response in the nonlinear system. Despite of the bandwidth performance, the closed form performance characteristic for the nonlinear system in terms of transduction coefficient and optimum power has yet to be established. A theoretical study is conducted using the harmonic balance method to analyse the dynamic characteristics of the system. The harmonic balance method is further employed to determine the jump frequencies and to deduce the new optimum power expression for the nonlinear system. Apart from that, the initial conditions selection that creates the limitation to the nonlinear system is investigated using the basin of attraction. In this thesis, a proof of concept device with adjustable magnet gap is fabricated. The nonlinearity of the device is varied by adjusting the magnet gap. The dynamic characteristics of the proposed device are investigated experimentally. The quasi-static measurement is adapted to estimate the amount of nonlinearity using the force - deflection relationship. In the dynamic measurement, the dynamic characteristics are studied in terms of displacement and induced voltage. Further studies involve the effect of nonlinearity on the transduction coefficient and setting the limit of using the linear transduction coefficient on the nonlinear system. The limit is characterized from the analysis of the harmonic ratio. Ultimately, the optimum power at the maximum response of the device for the nonlinear system is measured to study the new optimum power expression deduced theoretically. From the theoretical results, it shows that the dynamic characteristics of the nonlinear system are affected by the nonlinearity and damping of the system. Meanwhile, the new optimum power expression is found to be proportional to the transduction coefficient and thus opens up question on the lower bound limit of the transduction coefficient. As for the limitation of the nonlinear system, the results show that a strong nonlinear system that may have bandwidth widen to a much frequency is restricted by the initial conditions selection especially near the maximum response region. In the experimental analysis, the quasi-static results reveal that the nonlinear system gradually converges to linear system as the magnet gap increases. The bandwidth region in the response curves obtained from the dynamic measurements also shows decrement as the magnet gap increases. In the study involving the transduction coefficient, the limit is characterized based on the harmonic ratio analysis. The results show that the linear transduction coefficient is applicable to the nonlinear system when the harmonic ratio is less than five percent at the multi-stable solutions region. For the highest power obtained experimentally, it is achieved when the transduction coefficient is the highest in the strongest nonlinear system with the transduction coefficient being considered at the maximum displacement.

ABSTRAK

Konsep penuaian tenaga dari persekitaran untuk tujuan penjanaan kuasa semakin menarik perhatian para penyelidik sejak kebelakangan ini atas kebolehannya untuk menjana kuasa sendiri. Secara tradisinya, sistem linear sering digunakan untuk menuai tenaga. Walau bagaimanapun, prestasinya yang dihadkan oleh kesukaran untuk memadankan antara frekuensi tabii peranti dan ambien menjadikannya kurang menggalakkan. Prestasi sistem linear menjadi lebih teruk apabila frekuensi ambien berubah dengan masa dan menyebabkan pengurangan julat frekuensi lebar jalur peranti tersebut. Pada masa ini, sistem tak linear telah diperkenalkan untuk mengatasi kekurangan sistem linear. Tesis ini menganalisis keupayaan sistem tak linear dalam meningkatkan julat lebar jalur. Selain prestasi lebar jalur, prestasi bagi sistem tak linear dari segi pekali transduksi dan kuasa optimum masih belum dibuktikan sepenuhnya. Satu kajian teori telah dijalankan dengan menggunakan kaedah penyeimbangan harmonik untuk menganalisis ciri-ciri dinamik sistem tersebut. Kaedah penyeimbangan harmonik kemudiannya digunakan untuk menentukan frekuensi lompatan dan untuk menerbitkan ungkapan kuasa optimum bagi sistem tak linear. Selain itu, pemilihan keadaan awal yang menyumbang kepada kekurangan prestasi sistem tak linear dikaji menggunakan lembangan tarikan. Dalam tesis ini, alat peranti yang merealisasikan konsep dengan sela magnet boleh laras telah dibangunkan. Ketaklelurusan peranti tersebut diubah dengan melaraskan sela magnet. Ciri-ciri dinamik peranti yang dibangunkan itu dikaji dengan kaedah ujikaji. Pengukuran kuasi-statik digunapakai untuk menganggar tahap ketaklelurusan dengan menggunakan hubungan daya - pesongan. Dalam pengukuran dinamik, ciri-ciri dinamik peranti tersebut dikaji dari segi sesaran dan voltan teraruh. Kajian yang selanjutnya melibatkan kesan ketaklelurusan pada pekali transduksi dan penetapan had menggunakan pekali transduksi linear pada sistem tak linear. Had ini ditentukan melalui analisis nisbah harmonik. Akhirnya, kuasa optimum yang dijana oleh peranti untuk sistem tak linear diukur untuk mengkaji ungkapan kuasa optimum yang diterbitkan melalui kajian teori. Keputusan kajian teori menunjukkan bahawa ciri-ciri dinamik sistem tak linear dipengaruhi oleh ketaklelurusan dan redaman dalam sistem tersebut. Sementara itu, ungkapan kuasa optimum didapati berkadaran dengan pekali transduksi dan ini menimbulkan persoalan kepada had batas bawah pekali transduksi itu. Bagi had sistem tak linear pula, keputusan kajian menunjukkan bahawa sistem tak linear dengan tahap ketaklelurusan yang tinggi berserta julat frekuensi yang besar pada lebar jalur dihadkan oleh pemilihan keadaan awal terutama berhampiran kawasan respon maksimum. Dalam analisis eksperimen, keputusan dari pengukuran kuasi-statik mendedahkan bahawa sistem tak linear secara beransur-ansur menumpu kepada sistem linear dengan peningkatan sela magnet. Kawasan lebar jalur dalam tindak balas lengkungan yang diperolehi daripada pengukuran dinamik juga menunjukkan penyusutan apabila sela magnet meningkat. Dalam kajian terhadap pekali transduksi, had ini dicirikan berdasarkan analisis nisbah harmonik. Hasil kajian menunjukkan bahawa pekali transduksi linear boleh digunakan untuk sistem tak linear apabila nisbah harmonik kurang daripada lima peratus pada kawasan penyelesaian stabil berbilang. Untuk kuasa tertinggi yang dijana oleh peranti secara ujikaji, ia dicapai apabila pekali transduksi adalah yang tertinggi dalam sistem tak linear dengan ketaklelurusan paling kuat dan pekali transduksi diambil pada frekuensi dimana sesaran maksimum terhasil.

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LIST OF ABBREVIATIONS

DC Direct Current

FFT Fast Fourier Transform

FKM Fakulti Kejuruteraan Mekanikal

HBM Harmonic Balance Method

IoT Internet-of-things

LVDT Linear Variable Differential Transformer

MEMS Micro-electromechanical system

MMS Multiple Scales Method

N/A Not Applicable

SDOF Single-Degree-of-Freedom

UTeM Universiti Teknikal Malaysia Melaka

LIST OF SYMBOLS

a	Acceleration
A	Cross sectional area
A_b	Amplitude
b	Harmonic term
B	Flux density through the coil
c	Damping, Damping coefficient $c_m + c_e$
c_e	Electrical damping coefficient
c_m	Mechanical damping coefficient
C_1	Fundamental harmonic
C_3	Second harmonic
d	Gap between magnets
d_p	Piezoelectric strain coefficient
D	Electrical displacement
E	Young’s Modulus
E_e	Electric field
f	Frequency

$f(x)$ Nonlinear spring function

F Mechanical force

F_e Excitation force per unit mass

F_m Electromotive force

F_0 Gravitational constant

F_1 Function of the excitation frequency

h Height

i Electric current

I Moment of inertia

k Spring stiffness

K Transduction coefficient, Electromechanical coupling coefficient, Electromagnetic coupling coefficient

k_1 Linear spring stiffness

k_3 Nonlinear spring stiffness

l Length

L Coil inductance

m Seismic mass, Effective mass, Proof mass

M Number of harmonics

N Number of coil turns

p Slow varying function of time

$P_{avg\ max}$ Average maximum power

P_{inst} Instantaneous power

P_{opt} Optimum power

P Power output

q Slow varying function of time

R Resistance

R_{int} Internal resistance of the coil

R_{load} Load resistance

t Thickness, Time

T_n Different time scales

$u, \dot{u}, \ddot{u}$ Non-dimensional displacement, velocity, acceleration

U_1 Half-power point response

U_m Maximum response

U_{max} Maximum response

V Voltage

w Width

x Seismic mass deflection/motion

$\dot{x}$ Velocity of the seismic mass

y Harmonic base excitation $Y\cos(\omega t)$

$\dot{y}$ Velocity of the housing

Y Amplitude of the input displacement

$y(t)$ Sinusoidal excitation

$\ddot{y}(t)$ Input acceleration, $\omega^2 Y\cos(\omega t)$

$z, \dot{z}, \ddot{z}$ Relative displacement, velocity, acceleration

α Nonlinearity of the system

ζ Damping factor, Damping ratio

ζ_e Electrical damping factor

ζ_m Mechanical damping factor

δ Mechanical strain

σ Mechanical stress

ω Angular frequency, Angular speed $2\pi f$

ω_e Excitation frequency

ω_n Undamped natural frequency $\sqrt{k_1/m}$

Ω Non-dimensional frequency

Ω_1 Half-power point frequency

Ω_m Maximum response frequency

Ω_{nr} Steady state response for non-resonant branch

Ω_r Steady state response for resonant branch

Ω_{max} Non-dimensional jump-down frequency

Ω_{up} Non-dimensional jump-up frequency

$\Delta\Omega_l$ Bandwidth region for linear system

$\Delta\Omega_s$ Bandwidth region for nonlinear softening system

ε Bookkeeping parameter

ε_p Dielectric constant for piezoelectric material

β Phase angle

τ_i Independent variable ωt

τ Non-dimensional time $\omega_n t$

ϕ Coil diameter

ρ Resistivity of the material

∞ Infinity

$\varepsilon = \dfrac{3\alpha}{32\zeta^2}$

$\dot{}$ $d/d\tau$

$\ddot{}$ $d^2/d\tau^2$