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Improved heading direction interpretation via optical flow using selected region of interest

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Abstract

In this paper, a technique of motion processing and analysis based on optical flow to gather information of heading direction from a vision system is presented. Instead of using the complete frame to determine the heading direction, in this paper, a region of interest (ROI) is used to calculate the heading direction. The selection of this ROI is based on the vector's magnitude dispersion criteria. This value is used as a visual feedback to a control system. The performance of proposed technique is compared to the true heading and the error is evaluated using root mean square error (RMSE). From the results, it shows that by appropriate selection of the region where the information from the optical flow is used, the visual feedback information results in less overshoot and undershoot in terms of its response.

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Keywords: heading direction; angle estimation; optical flow; region of interest; vector magnitude; skewness.

Nomenclature

k	temporal coordinate
n	iteration counter
t	time
\tilde{u}, \tilde{v}	estimation value for ν
\bar{u}, \bar{v}	neighborhood average of ν
I_t	temporal gradient
E_{lk}	error in the optical flow constraint over a particular block of pixels
E_{of}	optical flow error
E_s	smoothness constraint
E_{total}	energy function
ν	optical flow vector
x	image-plane coordinate
\mathbf{R}	set of real numbers

Greek symbols

δ	derivative
α	weighting factor

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Δ	Laplace operator
Σ	summation
∇I	spatial gradient

1. Introduction

It is well known that in a control system, in order to control a variable in process or plant, information of the variable must be clearly clarified, in which such information can be found by measuring the variable. Usually, these measurements are made with purpose for monitoring or controlling processes operation, or to carry out analysis. The measurement can be made by using suitable instruments, which consists few elements such as sensors, signal conditioning unit and display or recording device. In any control system, these measurements play an important role because it provides information in implementing feedback mechanism to control the plant or the system.

There are various types of sensors available, regard on the purpose of the measurement itself. In certain criteria, sensor is selected based on its cost, size, weight, reliability and accuracy. Among existing sensors, vision systems are possibly the most complicated sensors available, due to their significance and complexity. Commonly, vision systems are used for operations that require information from environment [1].

When dealing with vision system, it is usually involved image processing and analysis. Image processing and analysis is considered as a cross-disciplinary field, hence, it can be treated from quite different points of view, ranging from image sensors, machine vision perception, robotic and computer vision. It can be seen from related work, for example work done by Ghazali et. al. in agriculture [2], and Kanawathi et.al. in motion detection [3]. In the other hand, it is well acknowledged that in computer vision, selection of computational algorithms is highly application dependent and incorporate factors such as robustness of algorithms, amount of valuable information that can be mined from image sequences, as well as efficiency and computational accuracy [4]. Therefore, many algorithms have been developed in recent year in various parts in computer vision to suite with various applications, such as pattern recognition, motion analysis and others.

In this paper, emphasis has been given to motion processing and analysis, which focuses on optical flow technique and capabilities to use it as a measurement apparatus. This is due to the fact that vision system has a strong potential of either replacing or supporting many conventional sensors. For example, in mobile robots application, an optical navigation sensor, consists of complementary metal-oxide-semiconductor (CMOS) image sensor, usually found in computer mice, through some algorithm, configuration and customization, can be employed, which provides consistent reading output regardless on changes in the distance between the surface and the sensor. It is considerably good replacement for traditional encoders because the traditional encoders cannot operate properly on slippery planes, where errors due to the poor surface conditions are difficult to detect. In addition, it is also inexpensive, reliable, accurate and very fast enough to be used in an ordinary computer [5]. Other work regarding on this matter can also be observed in unmanned air vehicle (UAV) [6, 7], automotive [8, 9] and humanoid robotics [10, 11].

Besides, optical flow computes the motion field through the changes on the scene that can be detected. Indirectly, it gives information about motion caused by observer and objects. In other words, optical flow takes an idea of relative motion. For example, consider single point in 3-D world which projected to the image plane, to calculate relative velocity, ones must know information such as linear and rotational velocity, as well as camera's focal length. However, for velocity of multiple points in image plane, usually it is represented as optical flow field, where it is generated from motion and texture intensity over plane. Generated optical flow vector can consists of erroneous vectors, so it is necessary to find out the vector that best represents the motion [4]. To derive optical flow, methods that usually used are block-based processing and differential-based method.

Thus, this field has attracted interest among researchers, and subsequently led to propose suggestion on the algorithms, idea, strategy and method to encounter the issues arise on this field. With advancement of the knowledge related to this field, the applications related to this technique can be observed. For example, to find reference yaw angle for unmanned air vehicle (UAV), variation in the optical flow is used, although with restriction, where it is limited to optical flow generated from detection of junctions and obstacles avoidance. Furthermore, optical flow also be used to detect and track and moving object. Through analyzing motion field on images, detection and segmentation on moving objects can be achieved [12].

2. Motion estimation and optical flow

In motion estimation, in order to perform or carried out analysis, it is needed to capture the sequence of images at any particular or dedicated time. For example, the displacement of an image plane coordinates $\mathbf{x} \in \mathbf{R}^2$ from time t to t' can be represented by two sequential images. Optical flow is defined as a sequential rate of change of the image-plane coordination, $\dot{\mathbf{x}}$ at a particular point $(\mathbf{x}, t) \in \mathbf{R}^3$ as denoted by the spatial-temporal deviation of the image intensity $f(\mathbf{x}, t)$.

To derive an optical flow, it is assumed that the intensity remains constant along motion trajectory. Let image intensity distribution is defined as $f(\mathbf{x},t)$. Using intensity constancy assumption, which denotes that, small translation does not have an effect on the intensity values of a point, give

$$\dot{f}(\mathbf{x},t) = 0 \tag{1}$$

where \mathbf{x} varies by t according to motion trajectory. From (1), it is also can be expanded as the following equation:

$$\frac{\delta f(\mathbf{x},t)}{\delta x} \frac{\delta x}{\delta t} + \frac{\delta f(\mathbf{x},t)}{\delta y} \frac{\delta y}{\delta t} + \frac{\delta f(\mathbf{x},t)}{\delta t} = 0 \tag{2}$$

$$\nabla I \bullet \mathbf{v} + I_t = 0 \tag{3}$$

where $(x,y) \in \mathbf{x}$, $\nabla I = [\delta f(\mathbf{x},t)/\delta x \quad \delta f(\mathbf{x},t)/\delta y]$, $\mathbf{v} = [\delta x/\delta t \quad \delta y/\delta t]^T = [u \quad v]^T$ and $I_t = \delta f(\mathbf{x},t)/\delta t$. Here $\delta f(\mathbf{x},t)/\delta x, \delta f(\mathbf{x},t)/\delta y, \delta f(\mathbf{x},t)/\delta t$ denotes spatial and temporal gradient and $\delta x/\delta t, \delta y/\delta t$ denotes optical flow vector in horizontal and vertical direction, respectively. Equation (3) is known as an optical flow equation, or optical flow constraint. However, optical flow vector is cannot always be estimated with sufficient accuracy due to constraints such as aperture problem, brightness violation etc. Therefore, it makes a sense that (3) does not always yield reliable flow estimates.

Computation of optical flow is widely done using Lucas-Kanade or Horn-Schunck methods. In Lucas-Kanade method, although it cannot handle rotation motion, it is possible to estimate a purely translational motion vector, through assumption that the motion vector remains unchanged over a particular block of pixels [13]. While in Horn-Schunck method, motion field is estimated that satisfies (3) with minimum pixel-to-pixel variation among flow vectors [14]. Optical flow is an important task in computer vision with many interesting application. Applications involves includes motion and speed estimation, where Horn-Schunck method is utilized [3, 17], as well as motion estimation for dynamic legged locomotion, where Lucas-Kanade method is used [11].

2.1. Horn-Schunck method

In this method, motion field that satisfies optical flow constraint with minimum pixel-to pixel variation (PTPV) among the flow vectors is derived. For this purpose, the error in the optical flow constraint is defined as the following equation:

$$E_{of}(\mathbf{v}) = \nabla I \bullet \mathbf{v} + I_t \tag{4}$$

In order to satisfies(3), $E_{of}(\mathbf{v})$ need to be minimized. Furthermore, PTPV of the velocity vectors can be computed by using the following defined error:

$$E_s^2(\mathbf{v}) = \sum \|\nabla \mathbf{v}(\mathbf{x},t)\|^2 \tag{5}$$

$$E_s^2(\mathbf{v}) = \left(\frac{\delta u}{\delta x}\right)^2 + \left(\frac{\delta u}{\delta y}\right)^2 + \left(\frac{\delta v}{\delta x}\right)^2 + \left(\frac{\delta v}{\delta y}\right)^2 \tag{6}$$

Here we assume that the spatial and temporal coordinates are continuous variable. This error denotes the difference of the optical vector compared to its neighbours, where also be called as a smoothness constraint [14].

To estimate spatial and temporal gradient, the following computational method has been proposed through utilization of finite differences method.

$$\frac{\delta f(\mathbf{x},t)}{\delta x} \approx \frac{1}{4} \left\{ \begin{array}{l} f(n_1+1, n_2, k) - f(n_1, n_2, k) + f(n_1+1, n_2+1, k) \\ -f(n_1, n_2+1, k) + f(n_1+1, n_2, k+1) - f(n_1, n_2, k+1) \\ +f(n_1+1, n_2+1, k+1) - f(n_1, n_2+1, k+1) \end{array} \right\} \tag{7}$$

$$\frac{\delta f(\mathbf{x}, t)}{\delta y} \approx \frac{1}{4} \left\{ \begin{array}{l} f(n_1, n_2 + 1, k) - f(n_1, n_2, k) + f(n_1 + 1, n_2 + 1, k) \\ -f(n_1 + 1, n_2, k) + f(n_1, n_2 + 1, k + 1) - f(n_1, n_2, k + 1) \\ +f(n_1 + 1, n_2 + 1, k + 1) - f(n_1 + 1, n_2, k + 1) \end{array} \right\} \quad (8)$$

$$\frac{\delta f(\mathbf{x}, t)}{\delta t} \approx \frac{1}{4} \left\{ \begin{array}{l} f(n_1, n_2, k + 1) - f(n_1, n_2, k) + f(n_1 + 1, n_2, k + 1) \\ -f(n_1 + 1, n_2, k) + f(n_1, n_2 + 1, k + 1) - f(n_1, n_2 + 1, k) \\ +f(n_1 + 1, n_2 + 1, k + 1) - f(n_1 + 1, n_2 + 1, k) \end{array} \right\} \quad (9)$$

Here $(n_1, n_2) \in \mathbf{x}$ and k is temporal coordinate.

To minimize weighted sum of error in optical flow constraint and a measure of the PTPV of the velocity field,

$$E_{total} = \int (E_{of}^2(\mathbf{v}) + \alpha^2 E_s^2(\mathbf{v})) d\mathbf{x} \quad (10)$$

where E_{total} denotes energy function and α^2 is a weighting factor. The derivatives of (10) can be determined as follows:

$$\frac{\partial E_{total}}{\partial u} = 2(\nabla I \bullet \mathbf{v} + I_t) \frac{\delta f(\mathbf{x}, t)}{\delta x} + 2\alpha^2 (\Delta u(\mathbf{x}, t)) = 0 \quad (11)$$

$$\frac{\partial E_{total}}{\partial v} = 2(\nabla I \bullet \mathbf{v} + I_t) \frac{\delta f(\mathbf{x}, t)}{\delta y} + 2\alpha^2 (\Delta v(\mathbf{x}, t)) = 0 \quad (12)$$

In approximating the value of u and v , based on (11) and (12), the following iterative scheme is used:

$$u^{n+1} = \bar{u}^n - \frac{\delta f(\mathbf{x}, t)}{\delta x} \cdot \frac{\frac{\delta f(\mathbf{x}, t)}{\delta x} \bar{u}^n + \frac{\delta f(\mathbf{x}, t)}{\delta y} \bar{v}^n + \frac{\delta f(\mathbf{x}, t)}{\delta t}}{\alpha^2 + \left(\frac{\delta f(\mathbf{x}, t)}{\delta x}\right)^2 + \left(\frac{\delta f(\mathbf{x}, t)}{\delta y}\right)^2} \quad (13)$$

$$v^{n+1} = \bar{v}^n - \frac{\delta f(\mathbf{x}, t)}{\delta y} \cdot \frac{\frac{\delta f(\mathbf{x}, t)}{\delta x} \bar{u}^n + \frac{\delta f(\mathbf{x}, t)}{\delta y} \bar{v}^n + \frac{\delta f(\mathbf{x}, t)}{\delta t}}{\alpha^2 + \left(\frac{\delta f(\mathbf{x}, t)}{\delta x}\right)^2 + \left(\frac{\delta f(\mathbf{x}, t)}{\delta y}\right)^2} \quad (14)$$

where n is defined as iteration counter, and \bar{u}, \bar{v} denotes neighborhood average of the flow vector u and v , respectively.

2.2. Lucas-Kanade method

Another method used to generate optical flow is the Lucas-Kanade method. In Lucas-Kanade method, it is assumed that the motion vector remains unchanged over a particular block of pixels. This is due to overcome an aperture problem. In this method, the error in the optical flow constraint over a particular block of pixels is defined as

$$E_{lk} = \sum (\nabla I \bullet \mathbf{v} + I_t)^2 \quad (15)$$

Minimize (15) over u and v yield

$$\frac{\partial E_{lk}}{\partial u} = \sum 2(\nabla I \bullet \mathbf{v} + I_t) \frac{\delta f(\mathbf{x}, t)}{\delta x} = 0 \quad (16)$$

$$\frac{\partial E_{lk}}{\partial v} = \sum 2(\nabla I \bullet \mathbf{v} + I_t) \frac{\delta f(\mathbf{x}, t)}{\delta y} = 0 \quad (17)$$

Solve (16) and (17) simultaneously yield

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} \sum \frac{\delta f(\mathbf{x}, t)}{\delta x} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta x} & \sum \frac{\delta f(\mathbf{x}, t)}{\delta x} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta y} \\ \sum \frac{\delta f(\mathbf{x}, t)}{\delta x} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta y} & \sum \frac{\delta f(\mathbf{x}, t)}{\delta y} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta y} \end{bmatrix}^{-1} \begin{bmatrix} -\sum \frac{\delta f(\mathbf{x}, t)}{\delta x} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta t} \\ -\sum \frac{\delta f(\mathbf{x}, t)}{\delta y} \cdot \frac{\delta f(\mathbf{x}, t)}{\delta t} \end{bmatrix} \quad (18)$$

where \tilde{u}, \tilde{v} denotes an estimation value for u, v , respectively.

In this paper, these two methods will be used in generating optical flow. Image pre-processing, which includes image resizing and image sequences averaging are used to improve the images, which reduce image shaking and computational time needed for optical flow calculation. After optical flow computation, generated optical flow vector field will be segmented into parts known as ROI, and selection of ROI will be done based on vector's magnitude dispersion measurement. Besides, optical flow vector for selected ROI is computed, and calculation of horizontal and vertical components for each vector is implemented. Resultant vector is calculated using these vectors' horizontal and vertical components, and information regarding on magnitude and angle is extracted. Obtained angle is an estimated angle representing heading direction. The estimated angle then further filtered using moving averaging filter and exponential moving averaging filter to obtain smooth results. Estimation results are then evaluated using RMSE, where method with small RMSE is considered the best results.

3. Methodology

Methodology used in this work is represented in Fig. 1. In this work, condition where static environment with moving camera is considered, as in Fig. 2, where camera moves from left to right. Optical flow is generated using Horn-Schunck and Lucas-Kanade methods. Optical flow vector field is then segmented into nine regions, which defines the regions of interest (ROI). The reason behind on ROI selection is due to the fact that condition where static environment is used. Therefore, it is assumed that there have plenty of vectors with significant magnitude in optical flow field that represents the movement of a camera, which is considered as a true motion. As mentioned in previous section, generated optical flow usually consists of erroneous vector that does not represents true motion. Some region may contain optical flow vectors which are extremely small or nearly zero magnitude, due to the reason that there exists a region where no variation on brightness texture is observed. Thus, selection of ROI is necessary. Selection of ROI is based on measurement of optical flow vector's magnitude dispersion. In general, random variables are expected to be scattered following normal distribution, due its convenient properties [15]. However, in real world, the distribution of the data not exactly follows this type of distribution. Therefore, it is convenience that other criterion also should be identified. In this work, distribution's skewness is included as selection criterion. If the vectors magnitude dispersion has zero skewness, as well as value of median and mean are same, the distribution is assumed to be symmetry, which is close to normal distribution's pattern. However, if there exist skewness on distribution, the tendency of distribution skewness is checked, whether the value or coefficient of skewness have either positive or negative sign. In this work, Pearson's median skewness coefficient has been used [16]. For ROI selection, negative skew is selected, due to the fact that distribution has relatively few values that less than mean value. In other words, to determine region with dense of vectors, vectors with large magnitude is considered. If the number of vectors with magnitude larger than mean value is observed more than the number of vectors with magnitude smaller than mean value, it means that significant number of vectors exists at that region. Selection of a region with large mean value is implemented if skewness criterion has been fulfilled.

In addition, for performance evaluation, percentage of root mean square error (RMSE) criterion is used. This criterion is selected due to the reason that it is frequently used to measure differences between value predicted by the model or estimator with the value actually observed, as well as it is a good measure of accuracy.

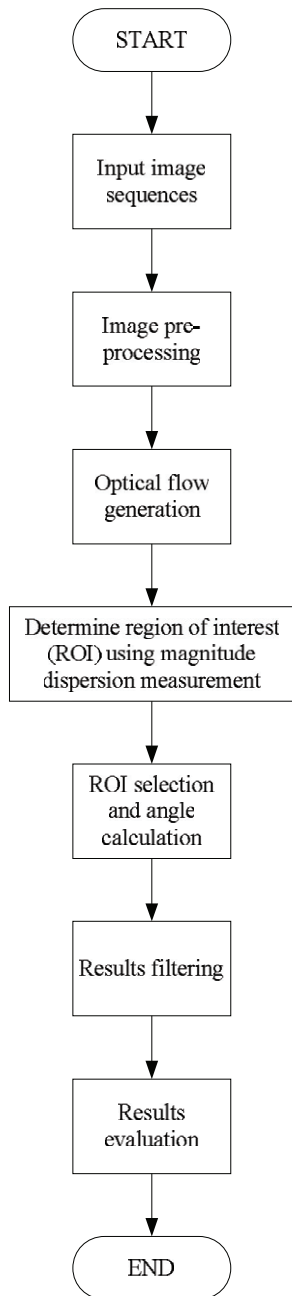


Fig. 1. Methodology workflow

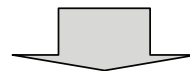


Fig. 2. Moving camera with static environment (camera moves from left to right)

4. Results and discussion

Capabilities as measurement apparatus is shown through proposed method. Using Horn-Schunck and Lucas-Kanade method, optical flow field is generated, where sample of generated optical flow is shown in Fig. 3.

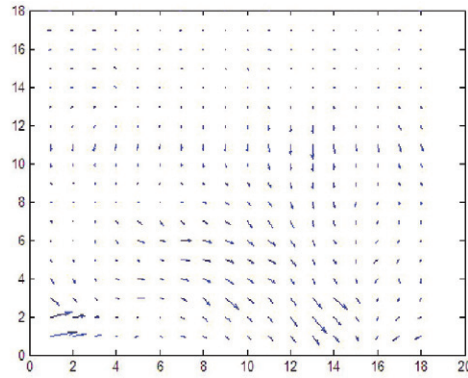


Fig. 3. Generated optical flow from image sequences (sample)

Generated optical flow field is then divided into several regions, and selection of the region is implemented using vector's magnitude dispersion, as explained in previous section. From selected region, each flow vector is then been decoupled into horizontal and vertical component and vector addition and subtraction operation is implemented for each component. From total vectors for each component, information regards on angle of the resultant vector is extracted.

Experimental results are as shown in Fig. 4 until Fig. 7. Notice that in the graphs, RV means resultant vector, MA means moving average and EMA means exponential moving average.

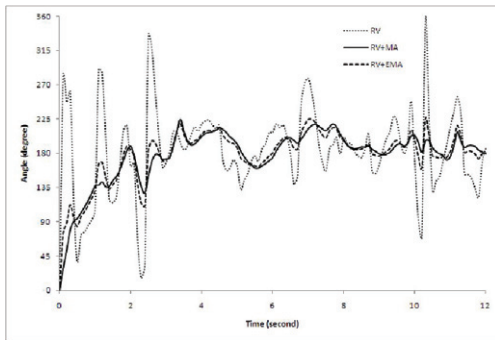


Fig. 4. Angle estimation from optical flow via Horn-Schunck method (without ROI)

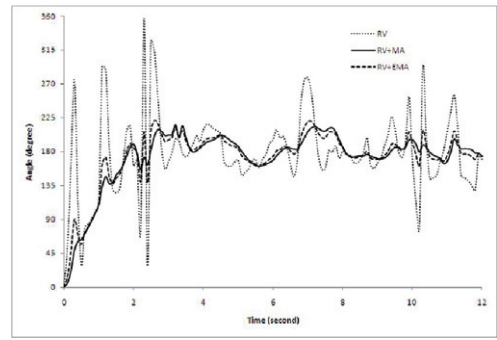


Fig. 5. Angle estimation from optical flow via Lucas-Kanade method (without ROI)

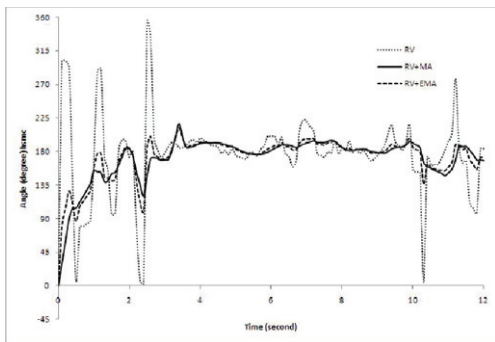


Fig. 6. Angle estimation from optical flow via Horn-Schunck method (with ROI)

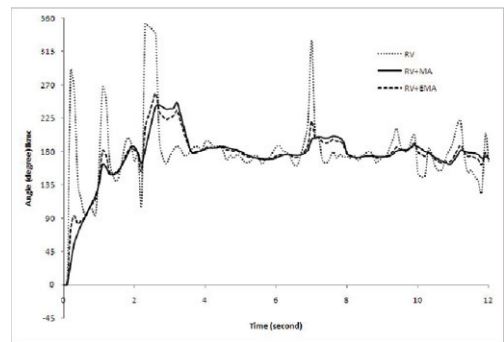


Fig. 7. Angle estimation from optical flow via Lucas-Kanade method (with ROI)

Using RMSE criterion, performances for both conditions are evaluated and concluded as in Table 1.

Table 1. Percentage of RMSE for angle estimation

Condition	Without ROI		With ROI	
	Horn-Schunck	Lucas-Kanade	Horn-Schunck	Lucas-Kanade
Resultant Vector	39.1	36.5	30.2	26.1
Resultant Vector + Moving Average	22.7	26.0	15.8	20.3
Resultant Vector + Exponential Moving Average	21.3	24.7	14.5	18.7

From the table, it shows that estimation error exist for generated optical flow field with ROI selection is relatively less than without it. From Fig. 6 and Fig. 7, it is clear that the resultant vector's graphs have less overshoot and undershoot compared with the results as in Fig. 4 and Fig. 5. Through ROI processing, significant results improvement has be done, through observation on resultant vector's graph in Fig. 4 and Fig. 5, where oscillation occurs frequently, while, it is significantly reduced when looking on Fig. 6 and Fig. 7. In addition, improvement of the results has been done through filtering. In this work, two types of filters are used: moving averaging filter and exponential moving averaging filter. From the graph, the results are enhanced when these type of filter are used.

5. Conclusion

In this paper, in interpreting heading direction, combination of optical flow and ROI processing using measurement of vector's magnitude dispersion has been used. Angle estimation is implemented using combination of vectors decoupling, vector addition and subtraction, and extraction of estimated angle, which can be manipulated as measured angle for heading direction. The result gathered from this work is then compared with the result where only optical flow method is applied in representing heading direction. From the results, optical flow with proposed ROI processing has shows improved performance. It is believed that through application of this method, information about current condition of the system is gathered, which is useful as an alternative measurement apparatus for the mechatronics system such as bipedal and mobile robot, especially when there exists faultiest in the system, where conventional sensors cannot give accurate or stable measurements. In addition, in case where current conventional sensor cannot give accurate measurement due to certain situation, for instance, excessive motion of single-leg boom arm due to impact during landing that cannot be detected by inertial measurements, combination of conventional sensor with optical flow is necessary.

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References

- [1] Niku, Saeed B., 2001. Introduction to robotics analysis, systems, applications, Prentice Hall Inc., USA.
- [2] Ghazali, K. H., Mustafa, M. M., Hussain, A., 2008. Machine Vision System for Automatic Weeding Strategy Using Image Processing Technique, *American-Eurasian J. Agric. & Environ. Sci.* 3:3, p. 451.
- [3] Kanawathi, J., Mokri, S. S., Ibrahim, N., Hussain, A., Mustafa, M. M., 2009. "Motion detection using Horn Schunck algorithm and implementation," *International Conference on Electrical Engineering and Informatics*, pp. 83-87.
- [4] Low, E. M. P., Manchester, I. R., Savkin, A. V., 2007. A Biologically Inspired Method for Vision-Based Docking of Wheeled Mobile Robots. *Robotics and Autonomous Systems* 55, p. 769.
- [5] Hyun, D., Yang, H. S., Park, H. R., Park, H. S., 2009. Differential Optical Navigation Sensor for Mobile Robots. *Sensors and Actuators A* 156, p. 296.
- [6] Srinivasan, M. V., Thurrowgood, S., Soccol, D., 2006. "An optical system for guidance of terrain following in UAVs," *Proceedings of the IEEE International Conference on Video and Signal Based Surveillance AVSS '06*, pp. 51–51.
- [7] Soccol, D., Thurrowgood, S., Srinivasan, M., 2007. "A vision system for optic-flow-based guidance of UAVs," *Proceedings of the 2007 Australasian Conference on Robotics & Automation*.
- [8] Vatani, N. N., Roberts, J., Srinivasan M. V., 2009. "Practical visual odometry for car-like vehicles," *Proceedings of the 2009 IEEE International Conference on Robotics and Automation (ICRA '09)*, pp. 3551–3557.
- [9] Kitt, B., Geiger, A., Latagahn, H., 2010. "Visual odometry based on stereo image sequences with RANSAC-based outlier rejection scheme," *IEEE Intelligent Vehicles Symposium (IV)*, pp. 486–492.
- [10] Pretto, A., Menegatti, E., Bennedict, M., Burgard, W., Pagello, E., 2009. "A visual odometry framework robust to motion blur," *IEEE International Conference on Robotics and Automation 2009 (ICRA '09)*, pp. 2250-2257.

- [11] S. P.N. Singh, P. J. Csonka, K. J. Waldron. "Optical flow aided motion estimation for legged locomotion," IEEE/RSJ International Conference on Intelligent Robots and Systems 2006; pp. 1738-1743.
- [12] Eresen, A., Imamoglu, N., Efe, M. O., 2012. Autonomous Quadrotor Flight with Vision-Based Obstacle Avoidance in Virtual Environment. Expert Systems with Applications 39, p. 894.
- [13] Lucas, B. D., Kanade, T., 1981. "An iterative image registration technique with an application to stereo vision," Proc. 7th International Joint Conference on Artificial Intelligence 1981, pp. 674-679.
- [14] Horn, B.K.P., Schunk, B. G., 1981. Determining Optical Flow. Artificial Intelligence 17, p. 185.
- [15] Weisstein, E. W. "Normal Distribution." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/NormalDistribution.html>.
- [16] Weisstein, E. W. "Pearson's Skewness Coefficients." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/PearsonsSkewnessCoefficients.html>.
- [17] Indu, S., Gupta, M., Bhattacharyya, A., 2011. Vehicle Tracking and Speed Estimation Using Optical Flow Method. International Journal of Engineering Science and Technology 3(1), p. 429.