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Research paper

Review on the Methods to Solve Combinatorial Optimization Problems Particularly: Quadratic Assignment Model

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Abstract

The quadratic assignment problem (QAP) is one of the fundamental combinatorial optimization problem (COPs) in the branch of optimization or operation research in mathematics, from the category of the Facilities Location Problems (FLPs). The quadratic assignment problem (QAP) be appropriate to the group of NP-hard issues and is measured as a challenging problem of the combinatorial optimization. QAP in Location Theory considers one of the problems of facilities tracing which the rate of locating a facility be determined by the spaces between facilities as well as the communication among the further facilities. QAP was presented in 1957 by Beckman and Koopmans as they were attempting to model a problem of facilities location. To survey the researcher's works for QAP and applied, the mapped research landscape outlines literature into a logical classification and discovers this field basic characteristics represented on the motivation to use the quadratic assignment problem applied in hospital layout and campus planning. This survey achieved a concentrated each QAP article search in three key databases: Web of Science, Science Direct, and IEEE Xplore. Those databases are regarded extensive adequate in covering QAP and the methods utilized in solving QAP.

Keywords- *Quadratic Assignment Problem, Metaheuristic Algorithms, Combinatorial Optimization Problems.*

1. Introduction

In theoretical computer science and applied mathematics contain problems of combinatorial optimization which are a subject that is made of finding an optimal object from a finite set of Objects [1] . In general, the problems in the real world are divided into two categories, called decision problems and optimization problem as shown in fig. 1 [2].

There are many specific issues of combinatorial optimization for example Facilities Location Problems FLP, Scheduling Problem SP, and Routing Problem RP. The quadratic assignment problem (QAP) is regarded the essential problems for combinatorial optimization among the other problems in the branch of optimization or operations research in mathematics, among the type of the amenities problems of setting [3]. The classic Quadratic Assignment Problem (QAP) is one of the greatest thought-provoking and inspiring combinatorial optimization issues in actuality [4]. Due to the importance of the QAP decision-making process, it is used in different applications such as data analysis, parallel computing, distributing computing and other research application. Among the various research applications, QAP is considered as one form of the NP-hardest problem category [5].

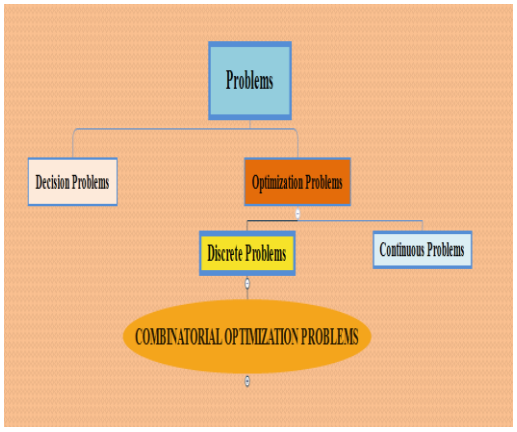

Figure 1: Problem Formulation

The quantity of relief problems modeled mathematically by QAPs has constantly increased and the multiplicity of fields they are a part of is astounding. Recalling a limited quantity of QAP applications allow us to state distributed computing, manufacturing, scheduling, placement problems, statistical data analysis and parallel. From a computational viewpoint, QAP as a problem is complicated. The solving of combinatorial problems as competently to be conceivable where competent typically means an appeal for the fast possibility. Therefore, a vital standard for the organization of the problems through time the well-identified algorithms required to discover an answer for the particular problem. The algorithms which used to solve the combinatorial optimization problems may be categorized by way of both Exact as well as Approximate algorithms.

The exact algorithms are guaranteed to obtain a solution (optimal) in the predetermined period through scientifically penetrating space solution. Usually, cases of size n>30 might not stand to be solved optimally through a thorough algorithm in a rational period [6]. In the meantime, various actual occurrences of scope happenings more than 30, this reason has made the problem is a challenge for researchers to use approximate methods. The objectives of this study are, to highlight the researchers` efforts on the QAP which is considered one of the most important and difficult problems due to, its involvement in many different fields of the real-world, and to find a new optimization method for solving the model of QAP. The rest of the paper is organized as follows. Section II Description of QAP. Section III Methods. Section IV includes the result. Finally, the conclusions are given in section V.

2. Description of QAP

QAP is considered as one of the site problems which reduce the momentum within the places of high mobility such as hospitals and campus and also the number of facilities to be allocated to these sites by calculating the matrix of distances between location and the calculating matrix of flows between facilities, where solving the QAP means examining the assignment that reduces the Transportation Cost in the among of the facilities. To have a QAP instance, there ought to be visibility and full distances list of accessible locations and material flow among facilities (Fij). Each N facility is interchangeable. Alternatively, there are N locations that can only provide for one facility. Therefore, QAP can be shown as: There is N facilities set and N locations set. For each locations' pair the specification of distance (Dij) and for each facilities' pair, a flow (Fij) is itemized. The difficulty in assigning entire facilities to alternative positions is aimed at minimizing the distances sum increased by

conforming flows [7]. Formally, let and be two N*N matrices and let P be the set of permutation of {1, 2, …, N}. Then, the QAP can be written as:

$$\text{Min}(\pi) = \text{Fij}\, D\pi(i), \pi(j)$$

Overall permutations P . For an example in Fig. 2 [8] the assignment π is shown:

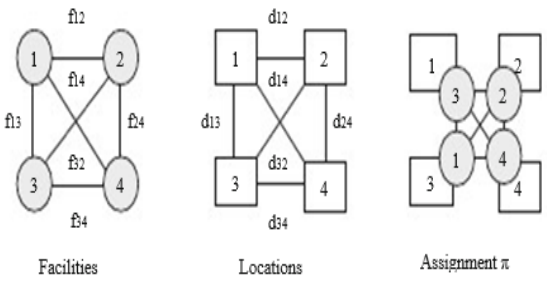

Figure 2: Assignment π = (3,2,4,1)

A) Assumptions of Model

- The objective purpose is Mini Sum.
- Every facility is assigned to one location
- The solution space is discrete and finite.
- The quantity of location and facilities is known.
- All choice variables of the model are binary (0–1) variables.

B) Inputs of Model QAP

- $F = [F_{ij}]$: flow matrix between facility i and j.
- $D = [D_{ij}]$: distances matrix between locations i and j.

C) Model Outputs (Decision Variables)

$$X_{ij} = \begin{cases} 1; \text{ if facility i assigned in location j} \\ 0; \text{ otherwise} \end{cases}$$

D) Complexity of QAP

Several scientists have been concerned with mathematicians, operations research, computer scientists, and economists worked on the QAP to modify an assortment of optimization problems. The QAP is measured as one of the most complicated forms of Combinatorial Optimization Problems which belongs to NP-Hard Problems [3], [9], [10], [11], and [12] and it is highly challengeable and attractive [13]. The QAP is a very interesting and challenging problem that can model several real-life problems [14]. Since applying exact methods to solve such problems requires high implementation time, especially when the problem size is greater than 30, we use approximate methods in solving this problem. From the viewpoint of computational complexity, the exact methods have been used to solve QAP with small instances of optimal results, however in middle or large cases optimal solutions cannot be found. Due to this reason, QAP has drawn the attention of researchers worldwide, and many articles have been published for the theory, applications, and solution techniques for solving QAP [15]. Despite the large research proposed, QAP remains one of the most complicated combinatorial optimization problems. In general, QAP situations with problem N > 30 cannot be resolved within passable computational times.

E) Applications of QAP

Since mathematical model of QAP is one of the nonlinear models in different sciences, dealing with the mathematical model of the case is very complicated, especially when the matter is transformed into an analysis of this model, whereas most of the operational research programs have a specific capacity for the number of variables and the number of constraints. As a mathematical model, there are large numbers in real life for QAPs applications in regards to ec onomic activities. In a backboard wiring [16] proposed QAP usage in minimizing the total connections between components, [17] presented a natural application using buildings location theory assignment in university grounds. [18] utilized it in economic problems. [19] related it to archeology. [20] presented facility layout design problem formulation to minimize work-in-process (WIP).

[21] analyzed a special QAP case known as index assignment with the purpose of decreasing vector-quantization channel errors. The utilization of vector-quantization is to map speech or images to digital signals. The discovery of a related mapping problem during microarrays layout configuration, which is a bioinformatics problem taken as a QAP by de [22]. A more recent utilization of the identical problem is in touchscreen devices on keyboards design [23]. An applied case of quadratic assignment problem in hospital department layout [24].

3. Methods

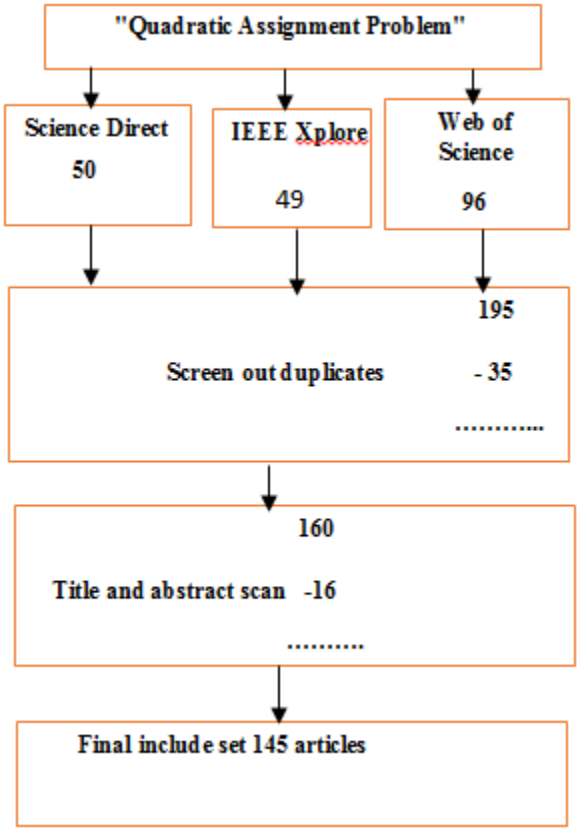

Figure 3: Searching Process

In this study, we presented the first classification in fig. 4 the classification involves on 145 papers obtained through the use of search engines such as IEEE Xplore, Since Direct and Web of Sciences, the study selection, progression means looking for literature sources, subsequently iterated by screening and filtering. The exclusion in the first iteration is the unrelated articles and duplicates done by going through the titles and abstracts thoroughly. Figure 6 presented the second classification, which divides the methods of approximate based on their technique to improving, hybrid, and parallel. This survey presented by Information Sources, Study Selection, and Search. In Information Sources were used three digital databases were chosen to perform target articles search: (1) Science Direct which offers access to science, journal articles; (2) IEEE Xplore library. (3) Web of Science. The selection's justification covers both QAP and methods utilized in solving QAP and providing an extensive analysis of researchers' hard work in a broad, but pertinent, array of disciplines. Then, the study selection progression means looking for literature sources, subsequently iterated by screening and filtering. The exclusion in the first iteration is the unrelated articles and duplicates done by going through the titles and abstracts thoroughly. The second iteration involves filtering of articles after a meticulous full-text reading of the articles screened from the initial step. Finally, the search was carried out at the beginning of August 2017 and limited to articles from 2002 to 2018 using Science Direct, IEEE Xplore, and Web of Science engines. Each search engine's options are utilized to rule out various categories of reports excluding conference articles and journals which are the two sources they are most likely to contain advanced and appropriate scientific researches applicable to the current survey on this evolving tendency of operation research.

4. Results

The first query search resulted in 196 articles: 50 from Science Direct, 49 from IEEE Xplore, and 96 articles from Web of Science, over the span from 2002 to 2018. Thirty-five (35) articles were duplicates among the three library databases, articles were excluded 16 papers, leaving 145 articles in the final included set. Those papers were read thoroughly for the chief purpose of finding out a general plan for the attended research on this emerging topic.

The search involves two classifications as follows:

A)The first classification

This section is a description of a survey of all methods that solve combinatorial optimization problems particularly Quadratic Assignment Problem, this classification includes 145 papers for the methods which used to solve QAP model was presented during the period from 2002 - 2018, we have found two main methods to solve QAP model in this classification the first method namely Exact Algorithms and the second method namely Approximate Algorithms as shown in fig. 4. The Approximate Algorithms include Metaheuristic algorithms which have three categories of algorithms:

- Evolutionary algorithms (EA)
- Local Search Algorithms(LSA)
- Swarm Intelligence (SI)
- Exact Algorithms 32 papers

- Approximate Algorithms (LS, EA, and SI) 91 papers
- Others 22 papers

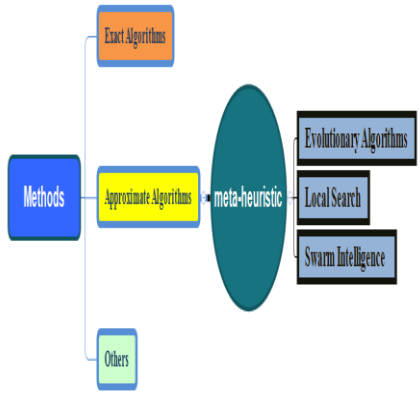

Figure 4 Methods to solve QAP

Fig. 5 shows the number of every category from algorithms as follows:

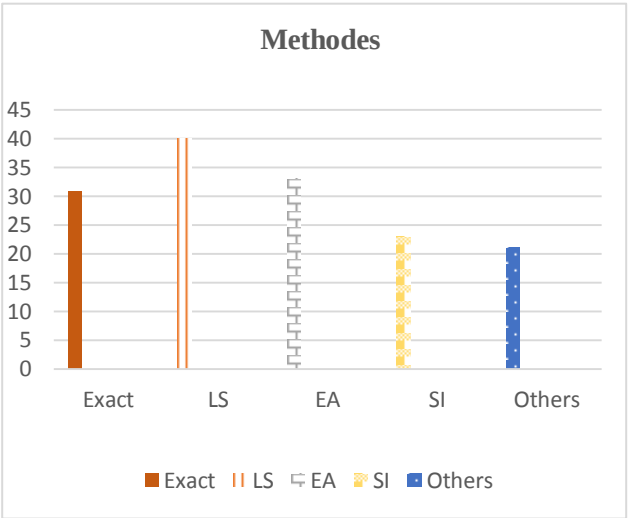

Figure 5: Number of papers every category of algorithms

The main algorithms which belong to Exact Algorithms such as dynamic programming [25], cutting plane techniques [26], and branch and bound procedures [27]. This review proved the method Exact Algorithms to be the best way for determining the optimal solution but if the dimension of the issue is greater than 25 need a long time for the computation, this reason has posed a major challenge and make researchers use the method of Approximate Algorithms. The approximate Algorithms was used to find a good solution and near to the optimal solution but with a reasonable computation time, local search demonstrates the maximum fruitful wide-ranging method for discovering high-quality solutions to tough combinatorial optimization problems within the practical stage. The iterative examination of localities of clarifications is essential to developing the existing solution through confined modifications. With regarding various problems, this approach exposed to be an encouraging and responsible for enhanced solutions comparing to initiating the local search with arbitrarily produced solutions. On the other way, this algorithm has its disadvantage where it may cause the humble value of local minima to be stopped. Consequently, opportunities have to be invented to increase its enactment. This category involves more algorithms such Greedy Randomized Adaptive Search Procedure (GRASP) [28],Tabu Search (TS) [29], Variable Neighborhood Search (VNS) [30], Simulated Annealing (SA) [31], and Iterated Local Search (ILS) [32]. The meaning of the term "swarms" is "a set of agents possessing independent individual dynamics but exhibiting intimately coupled behaviors and collectively performing some task". Swarm Intelligence (SI) algorithms are these algorithms which are nature-inspired such as Ant Colony Optimization (ACO) [33], Artificial Bee Colony (ABC) [34], Particle Swarm Optimization (PSO) [35], Cuckoo Search (CS) [36], and Bat Algorithm (BA) [37]. SI algorithms try to mimic the biological behavior of some creatures such as ants colonies or bees, flocks of birds, and schools of fish. Swarm intelligence algorithms have inspired the field of computing study, specifically the optimization field. In terms of the computational model, swarm intelligence models are considered as computing algorithms that are useful for solving distributed optimization problems. The principles of the swarm intelligence algorithm are proximity, quality, diverse response, stability, and adaptability.

Evolutionary algorithms (EAs) are population-based metaheuristic optimization algorithms. The individuals in the population represent the potential solutions to an optimization problem. The individuals collaborate with and compete against each other to get the optimal solution in the pursuit space. The intimate of EAs encompasses a lot of algorithms, for instance, Differential Evolution, Genetic Algorithms (GAs) and Evolution Strategy (ES) accompanied by most important change amongst them the ways wherein original results are produced. Of EAs, DE has extended acceptance to unravel unceasing optimization issues. On the other hand, assurance is not extended for a DE algorithm, that implements or will work with certain types of problems in different manners, this is because of the inconsistency with the fundamental mathematical possessions of optimization issues. The feature of EAs includes, it is conceptually simple and flexible, and it utilizes prior information. Can also be combined with more traditional optimization techniques. The operators in this method include crossover, mutation, and selection to evolve a population of candidate solutions toward an optimal solution.

B) The Second Classification

Dividing the methods of approximate based on their technique to improving, hybrid, and parallel was presented during the period from 2008- 2018. In this study rule out the exact algorithms and focus on the approximate algorithms. The approximate algorithms are the very significant decision from the NP-hardness for several combinatorial optimization issues which we may not believe a particular algorithm to solve the certain case to optimality in polynomially circumscribed calculation time [38]. In Approximation algorithms field, meta-heuristics have occurred as an innovative category of algorithms in the previous two decades for optimal solutions and for these problems to get solved. Meta-Heuristic as a term emerges from the Greek language and exactly from these two words "Meta" and

"Heuristic". Meta actually associated to the meaning of "higher level" or "beyond" whereas Heuristic refers to "to find", "to know", "to discover", or "to guide an investigation" [39]. The method used in this classification included a review of the approximate algorithms used metaheuristic (local search algorithms, evolutionary algorithms, and swarm intelligence algorithms). In this classification, most research has been used to improve local search algorithms, the surrounding environment is the main factor for obtaining a better solution which the local search depends on within the present solution otherwise the existence of better solution cannot be projected.

Improvement involves the proposal to extend the concentric search in many algorithms in the quadratic task issue to include more permissible moves. And proposed an innovative effectual alteration for such technique for a tough optimization problem that is the project of the quadratic problem. The heuristic class that studied supremely leads to the augmentation methods as the local study regarded as the best prevalent methods of perfection for improvement. The goal of improvement is to develop algorithm performance to reach a near optimal solution with reasonable time.

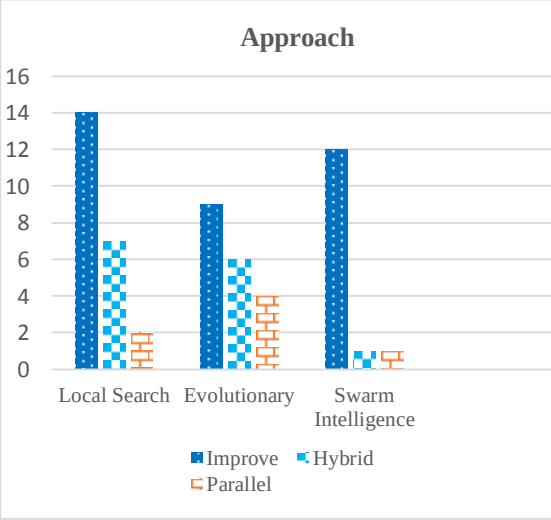

Figure 6: Approach used to solve QAP model

Fig. 6 showing the different approach in this classification. The second classification includes Improve, Hybrid, and Parallel for each of the categories Local Search Algorithms, Evolutionary Algorithms, and Swarm Intelligence. Some reference contains improved, hybrid in the same category, and another reference contains improved, hybrid together in two categories. Some algorithm continuous optimization problems such as PSO so needed to improve to a discrete version for resolving combinatorial optimization problems like QAP [39].

Hybrid algorithms provide the greatest results for both problems of classical optimization as well as for various actual life [40].

The best solution or near optimal solution to solve large and complex of QAP have been got it by used the metaheuristic and hybrid methods [14].

5. Conclusion

The Quadratic Assignment Problem (QAP) is a famous NP-hard combinatorial optimization problem which established scores of consideration from the unrestricted investigation from the time when it consumes various applied presentations, which facilities allocation, backboard wiring, hospital layout, and campus planning are included. In this paper, this study presents a review of the QAP and the methods which used to solve it. The results of this work as follows:

•The algorithms which used to solve the problems of combinatorial optimization include two categories, the first one is an Exact and the second is an Approximate algorithm.

•From the viewpoint of computational complexity, the Exact methods have been used to solve QAP with small instances of optimal results, however, in middle or large cases optimal solutions cannot be found.

•Hybrid algorithms provide the greatest results for both problems of classical optimization as well as for various actual life.

•The local search is the best prevalent methods of improvement; the goal of improvement is to develop algorithm performance to reach a near optimal solution with reasonable time.

•The best solution or near optimal solution to solve large and complex of QAP have been got it by used the metaheuristic and hybrid algorithms.

Based on the above mentioned, can find a new algorithm of solving QAP by depending on the hybrid algorithms between two or three different categories of the approximate algorithms discussed in this research, the idea of combining two or more different algorithms into a single hybrid algorithm is inspired by the possibility of the performance of this new algorithm better than any of their individual performance.

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