Faculty of Electronic and Computer Engineering

FILTER AND OBSERVER DESIGN FOR POLYNOMIAL DISCRETE-TIME SYSTEMS: A SUM OF SQUARES BASED APPROACH

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DECLARATION

I declare that this thesis entitled "Filter and Observer Design for Polynomial Discrete-time Systems: A Sum of Squares Based Approach" is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature	:	....
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Date	:	…………………………….

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Electronic Engineering.

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Supervisor Name	:	ASSOC. PROF. DR. MOHD SHAKIR BIN MD SAAT
Date	:	…………………………….

DEDICATION

To my beloved husband, mother and father.

ABSTRACT

The polynomial discrete-time systems are the type of systems where the dynamics of the systems are described in polynomial forms. This system is classified as an important class of nonlinear systems due to the fact that many nonlinear systems can be modelled as, transformed into, or approximated by polynomial systems. The focus of this thesis is to address the problem of filter and observer design for polynomial discrete-time systems. The main reason for focusing on this area is because the filter and observer design for such polynomial discrete-time systems is categorised as a difficult problem. This is due to the fact that the relation between the Lyapunov matrix and the filter and observer gain is not jointly convex when the parameter-dependent or state-dependent Lyapunov function is under consideration. Therefore the problem cannot possibly be solved via semidefinite programming (SDP). In light of the aforementioned problem, we establish novel methodologies of designing filters for estimating the state of the systems both with and without $H\infty$ performance and also designing an observer for state estimation and also as a controller. We show that through our proposed methodologies, a less conservative design procedure can be rendered for the filter and observer design. In particular, a so-called integrator method is proposed in this research work where an integrator is incorporated into the filter and observer structures. In doing so, the original systems can be transformed into augmented systems. Furthermore, the state-dependent function is selected in a way that its matrix is dependent only upon the original system state. Through this selection, a convex solution to the filter and observer design can be obtained efficiently. The existence of such filter and observer are given in terms of the solvability of polynomial matrix inequalities (PMIs). The problem is then formulated as sum of squares (SOS) constraints, therefore it can be solved by any SOS solvers. In this research work, SOSTOOLS is used as a SOS solver. Finally, to demonstrate the effectiveness and advantages of the proposed design methodologies in this thesis, numerical examples are given in filter design system. The simulation results show that the proposed design methodologies can estimate and stabilise the systems and achieve the prescribed performance requirements.

ABSTRAK

Sistem diskret masa polinomial adalah jenis sistem di mana dinamik sistem diberikan dalam bentuk polinomial. Sistem ini diklasifikasikan sebagai satu kelas penting dalam sistem tak lelurus kerana ianya boleh dimodelkan sebagai, dijelmakan, atau dianggarkan oleh sistem polinomial. Penyelidikan ini difokuskan untuk menangani masalah rekabentuk penapis dan reka bentuk pemerhati bagi sistem diskret masa polinomial. Tujuan utama untuk memberi tumpuan kepada bidang ini adalah kerana rekabentuk penapis dan pemerhati untuk sistem diskret masa polinomial dikategorikan sebagai masalah yang sukar. Ini adalah disebabkan oleh hubungan antara matrik Lyapunov dan penapis dan pemerhati tidak convex apabila kebergantungan-parameternya atau kebergantungan-keadaan fungsi Lyapunov digunakan. Oleh itu, masalah ini tidak mungkin dapat diselesaikan melalui pengaturcaraan separu-tentu (SDP). Memandangkan masalah yang dinyatakan di atas, kami menghasilkan cara baru bagi merekabentuk penapis untuk menganggarkan keadaan sistem tersebut dengan memasukkan prestasi H∞ dan juga tiada prestasi H∞ dan juga merekabentuk pemerhati untuk membuat anggaran keadaan dan juga sebagai pengawal. Kami menunjukkan bahawa melalui kaedah yang dicadangkan, prosedur reka bentuk yang kurang konservatif boleh dihasilkan dalam rekabentuk penapis dan pemerhati. Secara khususnya, kaedah pengamiran dicadangkan dalam kajian ini di mana pengamir dimasukkan ke dalam struktur penapis dan pemerhati. Dengan berbuat demikian, sistem asal boleh diubah menjadi sistem matrik imbuhan. Tambahan lagi, fungsi kebergantungan-keadaan dipilih dengan cara matriknya hanya bergantung kepada pembolehubah-keadaan sistem asal. Melalui pemilihan ini, penyelesaian convex untuk rekabentuk penapis dan pemerhati diperolehi dengan cekap. Kewujudan penapis dan pemerhati diberikan dalam bentuk ketidaksamaan matrik polinomial (PMI). Masalah ini kemudian dirumuskan sebagai kekangan jumlah kuasa dua (SOS), oleh itu ia boleh diselesaikan oleh mana-mana penyelesai SOS. Dalam penyelidikan ini, SOSTOOLS digunakan sebagai penyelesai SOS. Akhir sekali, untuk menunjukkan keberkesanan dan kelebihan metodologi yang dicadangkan di dalam tesis ini, contoh simulasi diberikan dalam sistem reka bentuk penapis. Keputusan simulasi menunjukkan bahawa kaedah reka bentuk yang dicadangkan boleh menganggarkan pembolehubah-keadaan dan menstabilkan sistem dan mencapai keperluan prestasi yang ditetapkan.

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LIST OF ABBREVIATIONS

FKEKK	-	Fakulti Kejuruteraan Elektronik dan Kejuruteraan Komputer
RAGS	-	Research Acculturation Grant Scheme
SDP	-	Semidefinite Programming
NP	-	Nondeterministic polynomial
PMIs	-	Polynomial Matrix Inequalities
SOS	-	Sum of Squares
CSDP	-	Common Security and Defence Policy
SDPA	-	Semi-Definite Programming Algorithm
A-D	-	Analog to Digital
D-A	-	Digital to Analog
ZOH	-	Zero Order Hold
LPV	-	Linear Parameter Varying
TS	-	Takagi-Sugeno
LMIs	-	Linear Matrix Inequalities
HJIs	-	Hamilton Jacobi Inequalities
UAV	-	Unmanned Aerial Vehicle

LIST OF SYMBOLS

θ	-	Angle in radians
ϕ	-	Magnetic flux
γ	-	Radioactive
τ	-	Pulse of response
C	-	Capacitance in F
E	-	Energy in Watt
I	-	Current in Ampere
L	-	Inductance in mH
R	-	Resistance in Ohm
V	-	Voltage in Volt

LIST OF PUBLICATIONS

1. Azman, N., Saat, S. and Nguang, S.K., Nonlinear filter design for a class of polynomial discrete-time systems: An integrator approach. *Int. J. Innov. Comput. Inf. Control, 11(3)*, pp.1011-1019, 2015.

2. Noorazma Azman, Shakir Saat, Sing Kiong Nguang, "Nonlinear observer design with integrator for a class of polynomial discrete-time systems", *International Conference on Computer Communications and Control Technology (I4CT) 2015*, pp. 422-426, 2015.

3. S. Saat, N. Azman and S. K. Nguang, "Nonlinear filter design with integrator for a class of polynomial discrete-time systems," 2014 *International Symposium on Technology Management and Emerging Technologies*, Bandung, 2014, pp. 311-315.

4. Shakir Saat, Sing Kiong Nguang, A.M. Darsono and Noorazma Azman, Nonlinear Hinfinity feedback control with integrator for polynomial discrete-time systems, *Journal of the Franklin Institute, vol.351*, no. 8, pp. 4023-4038, 2014.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The focus of this thesis is to design filter and observer for a class of polynomial discrete-time systems. The scope of this work is of designing filter and H_∞ filter and observer for polynomial discrete-time systems. The H_∞ performance objective is considered in filter design. In this work, an integral method is proposed. In doing so, a convex solution to the polynomial discrete-time systems state estimation with the utilisation of a state-dependent Lyapunov function can be obtained in a less conservative way than the available approaches. The effectiveness of the proposed method is validated using numerical example. This chapter is concluded with the outline of the thesis, highlighting the summary of each chapter.

1.2 Motivation

Nonlinear systems play a vital role in the control systems engineering point of view due to the fact that in practice all plants are nonlinear in nature. That is why the research on nonlinear systems is very important. In specific, in this research, we consider the polynomial system because its ability to approach any analytical of nonlinear systems. That is why the polynomial systems constitute in this research for an important class of nonlinear systems and attract considerable attention from researchers to involve themselves in this area, especially on the stability analysis and discrete-time of polynomial systems. Since many years ago, the problem of stabilization of nonlinear systems received a great deal of attention and several methods have been proposed in the works before. The failure in previous works, this research proposed remain restrictive to particular classes of nonlinear models, and to date no general method for the analysis of general nonlinear systems. There is the reason of continuing research on studying and focus in nonlinear systems. On the other hand, the polynomial systems constitute an important class of nonlinear systems which has the advantages.

The advantages are to describe the dynamical behavior of a large set of processes as electrical machines and robot manipulators and the ability to approach any analytical nonlinear system, since any analytical nonlinear function can be approximated by a polynomial expansion. Therefore, the problem of state estimation of polynomial system will provide a better solution in the framework of general nonlinear system estimation.

1.3 Problem statement

By studying polynomial system, discrete-time polynomial system in particular, the main issue is the nonconvexity. This is because the filter and observer design for polynomial discrete-time systems is a nonconvex problem. This problem is due to the relationship between the Lyapunov function and the filter gain is not jointly convex. The problem is related to the issue of bilinear matrix inequalities in linear systems, where the decision variables are not jointly convex. The nonconvex problem cannot be possible to solve via existing computational algorithms, semidefinite programming (SDP) in (L.Vandenberge and S. Boyd, 1996). This problem is also known as NP(Nondeterministic Polynomial)-hard problem. Due to this, the thesis aims to propose a method that is able to solve the problem and therefore the problem can be able to be solved via SDP algorithm.

1.4 Objective

The main objectives of this research is to design the filter and observer for polynomial discrete-time system. More specifically, the objectives of this research are as follows:

1. To propose a less conservative design method of designing nonlinear $H\infty$ filter for polynomial discrete-time systems.
2. To design a less conservative nonlinear observer for polynomial discrete-time systems.
3. To validate and analyze the effectiveness of the proposed method through numerical examples.

1.5 Scope of works

The scope of this work is of designing a nonlinear filter, nonlinear $H\infty$ filter and nonlinear observer for polynomial discrete-time systems. The $H\infty$ performance objective is considered in filter design only.

1.6 Significant of study

This research is predicted to be of benefit for designed on filter and observer of polynomial discrete-time systems. A global filter design method for polynomial discrete-time systems by using SOS-SDP based is established without any assumptions about nonlinear terms of the error dynamics. In this study, to ensure a convex solution to the filter design problem can be obtained, an integrator is incorporated into the filter structures. To compute the filter gains, SOS techniques have been used to reduce the problems to SDP. By having this, the proposed method will provide a less conservative method in solving the filter and observer design problems.

1.7 Contribution of the research

The contributions of this research can be summarized as follows:

1. The nonlinear filter and observer have been designed for polynomial discrete-time systems.
2. A convex solution to the filter and observer design problem is obtained in a less conservative way than the available approaches by introducing an integrator into the filter and observer structures.
3. To compute the filter and observer gains, SOS techniques have been used to reduce the problems to SDP and therefore can solve via existing toolboxes, i.e SOSTOOLS.
4. A sufficient conditions for the existence of a nonlinear $H\infty$ filter and observer have been given in terms of solvability of the polynomial matrix inequalities (PMIs) which formulated as SOS constraints.

1.8 Thesis Outline

The contents of the thesis are as follows:

Chapter 2 presents an overview of linear and nonlinear systems and their theoretical background. Previous works on nonlinear discrete-time systems and nonlinear continous-time systems are discussed. This chapter also reviews some important theorical about nonlinear discrete-time systems, including the methods of nonlinear system study and the available methods in the discretization framework. The method to solve nonconvex problem for filter and observer design also discuss in this chapter. The SOSTOOLS current version like SeDuMi or SDPT3 also highlighted in this chapter, both of that are SDP solver.

Chapter 3 deals with the problem of filtering design for a class of polynomial discrete-time systems. By utilising the integrator method, a possible solution to the filter design problem is presented. Solutions to the filter design have been derived in terms of PMIs, which are formulated as SOS constraints. A numerical example is given along with the theoretical presentation.

Chapter 4 presents a nonlinear H_∞ filter for polynomial discrete-time systems based on purposed method in Chapter 3. The H_∞ performance objective is considered here. The objective in this is to design a dynamic filter such that the ratio between the energy of the estimation errors and the energy of the exogenous inputs is minimized or guaranteed to be less or equal to a prescribed value.

Chapter 5 deals with the problem of observer design for a class of polynomial discrete-time systems. The observer is to control and at the same time estimate the state of polynomial discrete-time systems. By utilising the integrator method, a possible solution to the observer design problem is presented. Solutions to the observer design have been derived in terms of PMIs, which are formulated as SOS constraints.

Chapter 6 discuss the summary of thesis and suggestions for future research work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The polynomial discrete-time systems are the type of systems where the dynamics of the systems are described in polynomial forms. This system is classified as an important class of nonlinear systems due to the fact that many nonlinear systems can be modelled as, transformed into, or approximated by polynomial systems. This chapter provides background studies on the filter and observer design using several methods.

2.2 Linear systems

A system whose performance obeys the principle of superposition is defined as a linear system. This principle states that if the input $r_1(t)$ produces the response $c_1(t)$ and the input $r_2(t)$ yields the response $c_2(t)$, then for all a and b the response to the input $ar_l(t) + br_2(t)$ will be $ac_1(t) + bc_2(t)$; and this must be true for all inputs. A system is defined as time-invariant if the input $r(t+\tau)$ produces the response $c(t+\tau)$ for all input functions $r(t)$ and all choices for τ. The forced response for linear systems in Figure 2.1 is based on the principle of superposition and the application of convolution is one of the example of linear system. The simplest class of systems to deal with analytically is of course the class of linear invariant systems. For such systems the choice of time origin is of no consequence since

Figure 2.1: The forced response for linear systems example.

responses to more complex input forms. This permits one in principle to generalizs from the response for any one input to the responses for all other inputs. The elementary input function most commonly used as the basis for this generalization is the The simplest class of systems to deal with analytically is of course the class of linear time-invariant systems. For such systems represent in (G. Becker and A. Packard, 1994) the choice of time origin is of no consequence since any translation in time of the input simply translates the output through the same interval of time, and the responses to simple input forms can be superimposed to determine the responses to more complex input forms. This permits one in principle to generalizs from the response for any one input to the responses for all other inputs. The elementary input function most commonly used as the basis for this generalization is the unit-impulse function; the response to this input is often called the system weighting function. All possible modes of behavior of a linear invariant system are represented in its weighting function. Having once determined this function, which is just the response to one particular input, the performance of such a system can hold no surprises. A linear variable system, although it still obeys the principle of superposition, is appreciably more difficult to deal with analytically. The response to a single input function in this case does not suffice to define the responses to all inputs; rather, a one-parameter infinity of such responses is needed. A single elementary form of input such as the unit-impulse function is adequate, but this input must be introduced, and the response determined, with all translations in time. Furthermore, the calculation of such responses very often cannot be done analytically. For invariant systems, the calculation of the weighting function requires the solution of a linear invariant differential equation, and there is a well-established procedure for finding the homogeneous solution to all such equations. There is no comparable general solution procedure for linear variable differential equations, and so the determination of the time-variable weighting function for linear variable systems usually eludes analytic attack.

2.3 Nonlinear systems

Nonlinear system is any system which is the superposition principle does not hold. Then, for any class of inputs to the response for any other input is no option of simplifying from the responses in this system. Thus, need the quite specific work for nonlinear to solve problem which is a fundamental and important difficulty. One of the solution is calculate the

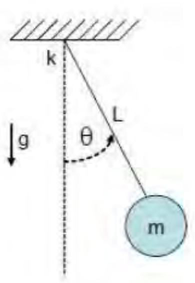

Figure 2.2: The pendulum for nonlinear systems example (F. Forni and R. Sepulchre, 2014).

response for a precise example of initial conditions and input, but this result about response characteristics in other cases are make very little corollary against it. However, In spite of the analytic difficulties, one of them try to attempt to deal in some way with nonlinear systems because no choice but, because they occupy very important places in anyone's list of practical systems. In fact, approximations to real systems can be thought as nonlinear systems. The pendulum in (F. Forni and R. Sepulchre, 2014) is one of the example for nonlinear systems in Figure 2.2. In (Vander Velde and Wallace E, 1968) finally end of their range of practical linearity if acceptable to take large values but most physical variables and the estimation is very good. Limiting is almost universally present in control systems since most instrumented signals can take values only in a bounded range. The values only in a limited range can take because the limitation almost generally existing in control systems since most instrumented signals. For example, a resolver or synchrony differential, have a restricted range of linearity because of many error detectors.

Then, the electrical and hydraulic actuators are example of drive systems can be assumed of as linear over only small ranges. The gas jets is one of the example have no linear range at all. Then, in (Willsky, 1976) the control systems certainly includes signal quantization because of using digital data processing. The system designer would choose to escape, but cannot are instances of nonlinear effects. To design some nonlinear a result into his system is some of good causes why potency also choose. The switching the power supply directly into the actuator by using two- or three-level switch as a controller.

In this situation, to drive the actuator ending with a power amplifier by using level switch more saving weight and space compared with using high-gain chain of linear amplification. Also, like optimal or original-value controllers are need nonlinear behavior for controllers of sophisticated design. The increasing of utility of digital operations, the trend toward lighter weight and smaller systems and demand for higher-performance systems are few examples illustrate in (Vander and Wallace, 1968) to broaden the place that nonlinear systems occupy. Therefore, the system designer continues to grow by apply useful ways which is need for analytic tools which can deal. In (Vander and Wallace, 1968) studied some of the performance characteristics of a broad class of nonlinear invariant systems. The methods existing here can be, and have been, stretched to some special cases of nonlinear variable systems, and the potentials for doing so are comparatively clear, once the basic analytic tool is well agreed.

2.3.1 Nonlinear system representation

In the control systems engineering, the nonlinear systems play a vital role. The main reason why we consider the nonlinear systems in our work because due to the fact that in practice all plants are nonlinear in nature, see (D. Zhao and J. Wang, 2009; R. Mtar et al., 2009; M. M. Belhaouane et al., 2010). In mathematic, one whose output is not directly proportional to its input or a nonlinear system is one that does not satisfy the superposition principle is the best example to explain nonlinearity is obviously a saturation. Then, to deliver an infinite amount of energy to any real-world system is existence of this condition. In general, the nonlinear systems may be written as (2.1) for the state equations and output equations:

$$\begin{aligned} \dot{x}(t) &= f[x(t), x(t)] \\ y(t) &= g[x(t), u(t)] \end{aligned} \tag{2.1}$$

The Stochastic system is one of the example of nonlinear systems in the Brusselator model (S.S.Ge and C.Wang, 2002) which is described as below:

$$\begin{aligned} \dot{x_1}(t) &= C_0 - (D_0 + 1)x_1(t) + x_1^2 x_2(t) + 0.5x_1 x_2(t) \\ \dot{x_2}(t) &= D_0 x_1(t) + u(t) - x_1^2 x_2(t) + 0.2x_1^2(t) \end{aligned} \tag{2.2}$$

which is x_1 and x_2 represent the density of immediate reaction. $C_0, D_0 > 0$ are the parameters to depict the provision of reservoir chemicals. The response of a linear invariant system to any forcing input can be expressed as a convolution of that input with the system weighting function. The response for any initial conditions of whatever magnitude can always be decomposed into the same set of normal modes which are properties of the system. The normal modes of all such systems are expressible as complex exponential functions of time, or as real exponentials and exponentially increasing or decreasing sinusoids. A special case of the latter is the undamped sinusoid, a singular situation. If a system displays an undamped sinusoidal normal mode, the amplitude of that mode in the response for a given set of initial conditions is, as for all other normal modes, dependent on the initial conditions.

2.3.2 Behavior of nonlinear systems

The response characteristics of nonlinear systems, on the other hand, cannot be summarized in a way such as this. These systems display a most interesting variety of behavioral patterns which must be described and studied quite specific. The most obvious departure from linear behavior is the dependence of the response on the amplitude of the excitation-either forcing input or initial conditions. Even in the absence of input, nonlinear systems have an important variety of response characteristics. A fairly common practical situation is that in which the system responds to small initial conditions by returning in a well-behaved manner to rest, whereas it responds to large initial conditions by diverging in an unstable manner. In other cases, the response to certain initial conditions may lead to a continuing oscillation, the characteristics of which are a property of the system, and not dependent on initial conditions. Such an oscillation may be viewed as a trajectory in the state space of the system which closes on itself and thus repeats; it is called a limit cycle. Nonlinear systems may have more than one limit cycle, and the one which is established will depend on the initial conditions, but the characteristics of each member of this discrete set of possible limit cycles are not dependent on initial conditions-they are properties of the system. This phenomenon is not possible in linear systems. Since a limit cycle, once excited, will continue in the absence of further excitation, it constitutes a point of very practical concern for the system designer, and an analytic tool to study limit cycles is of evident importance. A simple example of a limit cycling system is the Van der Pol oscillator in (Parlitz and Lauterborn,