ASM Sci. J., Special Issue 6, 2019 for SKSM26, 167-173

Interaction Between Two Inclined Cracks in Bonded Dissimilar Materials

K.B. Hamzah[1, 4], N.M.A. Nik Long[1, 2*], N. Senu[1, 2] and Z.K. Eshkuvatov[3]

[1]*Laboratory of Computational Sciences and Mathematical Physics, Institute for Mathematical Research, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia*

[2]*Mathematics Department, Faculty of Science, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia*

[3]*Faculty of Science and Technology, Universiti Sains Islam Malaysia, 71800 Negeri Sembilan, Malaysia*

[4]*Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia*

The interaction between two inclined cracks subjected to remote tension lying in the upper half of bonded dissimilar materials is considered. The hypersingular integral equation for the problem is formulated using the complex variable function method with the crack opening displacement function as the unknown and the tractions along the crack as the right-hand term. The appropriate quadrature formulas are applied in solving the hypersingular integral equation for the unknown function. Numerical results showed that the nondimensional stress intensity factor depends on the position of the cracks and the elastic constants ratio.

Keywords: incline crack; bonded dissimilar materials; complex variable function; hypersingular integral equation; stress intensity factors

I. INTRODUCTION

A number of papers have been published to analyze the stability and safety of the materials which contains cracks in an infinite plane (Murakami *et al.*, 1987, Chen, 1993), finite plane (Chen, 1987, Lai & Schijve, 1990), half plane (Chen *et al.*, 2009; Elfakhakhre *et al.*, 2017). For crack problems in bonded dissimilar materials, Fredholm integral equations with density distributions as undetermined functions were used to calculate the nondimensional stress intensity factor (SIF) (Chen, 1986). The nondimensional SIF for a circular arc crack problem embedded in one of two bonded dissimilar materials were solved using logarithmic singular integral equation (Chen & Hasebe, 1992). The nondimensional SIF of a perpendicular crack to the interface of bonded dissimilar materials were calculated by utilising the combinations of Chebyshev polynomials and collocation methods (Yang & Wang, 2018). The body force method with continuous distributions along cracks were used to find the nondimensional SIF of the crack problems in bonded dissimilar materials but excluded cracks at the interface (Isida & Noguchi, 1993). The combination of direct boundary integral method and displacement discontinuity method was used in solving the crack problems in bonded dissimilar materials (Long & Xu, 2016). The nondimensional SIF for two-dimensional interface cracks, three-dimensional penny-shaped cracks and circumferential surface cracks in bonded dissimilar materials were calculated by using the proportional crack opening displacements (Lan *et al.*, 2017). The nondimensional SIF was calculated for the collinear interface cracks in bonded dissimilar materials by combining the solution of the inner and outer collinear cracks (Itou, 2016). The linear elastic fracture analysis for interface crack problems in bonded dissimilar materials was proposed by an extended finite element method (Wang & Waisman, 2017). The edge cracks, semi-infinite interface cracks and substrate cracks in bonded dissimilar materials were analyzed using

*Corresponding author's e-mail: nmasri@upm.edu.my

the randomly oriented inclusions and networks (Birman, 2018).

The aim of this paper is to investigate the interaction between two cracks lie in the upper half of bonded dissimilar materials subjected to the remote stress from the x-axis direction, $\sigma_x = p$ for any $p \in \mathbb{R}$ by using the modified complex variable function method.

II. PROBLEM FORMULATION

Complex variable function method introduced by Muskhelishvili (1953) is used to formulate the hypersingular integral equations (HSIE) for the interaction between two cracks lie in the upper half of bonded dissimilar materials. The stress components $(\sigma_x, \sigma_y, \sigma_{xy})$, the resultant force function $f(X,Y)$ and the displacements (u,v) can be described by two complex potential functions $\phi(\omega)$ and $\psi(\omega)$ as follows

$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2\left[\bar{\omega}\phi''(\omega) + \psi'(\omega)\right] \tag{1}$$

$$f = -Y + iX = \phi(\omega) + \omega\overline{\phi'(\omega)} + \overline{\psi(\omega)} \tag{2}$$

$$2G(u+iv) = \kappa\phi(\omega) - \omega\overline{\phi'(\omega)} - \overline{\psi(\omega)} \tag{3}$$

where G is shear modulus of elasticity, $\kappa = 3-4v$ for plane strain, $\kappa = (3-v)/(1+v)$ for plane stress and v is Poisson's ratio and a bar over a function denotes the conjugated value. A derivative in a specified direction of Eq. (2) denotes the traction along the crack segment $\overline{\omega, \omega + d\omega}$ with its normal and tangential components are N and T, respectively, as follows

$$\begin{aligned}\frac{d}{d\omega}\{-Y+iX\} &= \phi'(\omega) + \overline{\phi'(\omega)} \\ &+ \frac{d\bar{\omega}}{d\omega}\left[\omega\overline{\phi''(\omega)} + \overline{\psi'(\omega)}\right] = N + iT\end{aligned} \tag{4}$$

where $N + iT$ depends on the positions of point $\omega = x + iy$ and the direction of the segment $d\bar{\omega}/d\omega$.

Nik Long & Eshkuvatov (2009) expressed the complex potentials for a crack L in an infinite plane as follows

$$\begin{aligned}\phi(\omega) &= \frac{1}{2\pi}\int_L \frac{g(\varsigma)d\varsigma}{\varsigma - \omega}, \\ \psi(\omega) &= \frac{1}{2\pi}\int_L \frac{\overline{g(\varsigma)}d\bar{\varsigma}}{\varsigma - \omega} - \frac{1}{2\pi}\int_L \frac{\bar{\varsigma}g(\varsigma)d\varsigma}{(\varsigma - \omega)^2} \\ &+ \frac{1}{2\pi}\int_L \frac{\overline{g(\varsigma)}d\varsigma}{\varsigma - \omega}\end{aligned} \tag{5}$$

where $g(\varsigma)$ is COD function defined by

$$g(\varsigma) = \frac{2G}{i(\kappa+1)}\left[\left(u(\varsigma)+iv(\varsigma)\right)^+ - \left(u(\varsigma)+iv(\varsigma)\right)^-\right]$$

$\left(u(\varsigma)+iv(\varsigma)\right)^+$ and $\left(u(\varsigma)+iv(\varsigma)\right)^-$ denote the displacement at point ς of the upper and lower crack faces, respectively.

Modified complex potentials for the crack lie in bonded dissimilar materials are defined as

$$\begin{aligned}\phi_1(\omega) &= \phi_{1p}(\omega) + \phi_{1c}(\omega), \\ \psi_1(\omega) &= \psi_{1p}(\omega) + \psi_{1c}(\omega)\end{aligned} \tag{6}$$

where $\phi_{1p}(\omega)$ and $\psi_{1p}(\omega)$ are the principle parts and $\phi_{1c}(\omega)$ and $\psi_{1c}(\omega)$ are complementary parts of the complex potentials at the upper plane. These complex potentials are defined as follows

$$\begin{aligned}\phi_{1c}(\omega) &= \Lambda_1\left[\omega\overline{\phi'_{1p}}(\omega) + \overline{\psi_{1p}}(\omega)\right] \\ \psi_{1c}(\omega) &= \Lambda_2\overline{\phi_{1p}}(\omega) - \Lambda_1\left[\omega\overline{\phi'_{1p}}(\omega)\right. \\ &\left. + \omega^2\overline{\phi''_{1p}}(\omega) + \omega\overline{\psi'_{1p}}(\omega)\right]\end{aligned} \tag{7}$$

where the principle parts of complex potentials are referred to an isotropic homogeneous materials or infinite plane and $\overline{\phi_{1p}}(\omega) = \overline{\phi_{1p}(\bar{\omega})}$. The complex potentials for the lower plane $\phi_2(\omega)$ and $\psi_2(\omega)$ are defined as follows

$$\begin{aligned}\phi_2(\omega) &= (1+\Lambda_2)\phi_{1p}(\omega) \\ \psi_2(\omega) &= (\Lambda_1 - \Lambda_2)\omega\phi'_{1p}(\omega) + (1+\Lambda_1)\psi_{1p}(\omega).\end{aligned} \tag{8}$$

The bi-elastic constants Λ_1 and Λ_2 are defined as

$$\Lambda_1 = \frac{G_2 - G_1}{G_1 + \kappa_1 G_2}, \quad \Lambda_2 = \frac{\kappa_1 G_2 - \kappa_2 G_1}{G_2 + \kappa_2 G_1}.$$

The HSIE for a crack lies in the upper half of bonded dissimilar materials can be obtained by substituting Eq. (6) into (4) and applying Eqs. (5) and(7), then letting point ω

approaches ς_0 on the crack and changing $d\bar{\omega}/d\omega$ into $d\bar{\varsigma}_0/d\varsigma_0$, yields

$$N(\varsigma_0)+iT(\varsigma_0)=\frac{1}{\pi}\int_L \frac{g(\varsigma)d\varsigma}{(\varsigma-\varsigma_0)^2}+\frac{1}{2\pi}\int_L A_1(\varsigma,\varsigma_0)g(\varsigma)d\varsigma+\frac{1}{2\pi}\int_L A_2(\varsigma,\varsigma_0)\overline{g(\varsigma)}d\varsigma \tag{9}$$

where

$$\begin{aligned}A_1(\varsigma,\varsigma_0)=&\frac{1}{(\varsigma-\varsigma_0)^2}\left[\frac{(\varsigma-\varsigma_0)^2}{(\bar{\varsigma}-\bar{\varsigma}_0)^2}\frac{d\bar{\varsigma}}{d\varsigma}\frac{d\bar{\varsigma}_0}{d\varsigma_0}-1\right]\\&+\Lambda_1\left[\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}+\frac{2(\bar{\varsigma}_0-\bar{\varsigma})}{(\varsigma-\bar{\varsigma}_0)^3}\right.\\&+\frac{d\bar{\varsigma}_0}{d\varsigma_0}\left(\frac{2(2\varsigma_0-3\bar{\varsigma}_0+\bar{\varsigma})}{(\varsigma-\bar{\varsigma}_0)^3}-\frac{6(\bar{\varsigma}_0-\bar{\varsigma})(\bar{\varsigma}_0-\varsigma_0)}{(\varsigma-\bar{\varsigma}_0)^4}\right.\\&\left.\left.-\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}\right)\right]+\Lambda_2\frac{d\bar{\varsigma}_0}{d\varsigma_0}\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}\\&+\Lambda_1\left[\frac{1}{(\bar{\varsigma}-\varsigma_0)^2}+\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}-\frac{d\bar{\varsigma}_0}{d\varsigma_0}\left(\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}\right.\right.\\&\left.\left.+\frac{2(\bar{\varsigma}_0-\varsigma_0)}{(\varsigma-\bar{\varsigma}_0)^3}\right)\right]\frac{d\bar{\varsigma}}{d\varsigma}\end{aligned}$$

$$\begin{aligned}A_2(\varsigma,\varsigma_0)=&\frac{\varsigma-\varsigma_0}{(\bar{\varsigma}-\bar{\varsigma}_0)^3}\left[\frac{(\bar{\varsigma}-\bar{\varsigma}_0)}{(\varsigma-\varsigma_0)}\left(\frac{d\bar{\varsigma}}{d\varsigma}+\frac{d\bar{\varsigma}_0}{d\varsigma_0}\right)\right.\\&\left.-2\frac{d\bar{\varsigma}}{d\varsigma}\frac{d\bar{\varsigma}_0}{d\varsigma_0}\right]+\Lambda_1\left[\frac{1}{(\bar{\varsigma}-\varsigma_0)^2}+\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}\right.\\&\left.-\frac{d\bar{\varsigma}_0}{d\varsigma_0}\left(\frac{1}{(\varsigma-\bar{\varsigma}_0)^2}+\frac{2(\bar{\varsigma}_0-\varsigma_0)}{(\varsigma-\bar{\varsigma}_0)^3}\right)\right]\\&+\Lambda_1\left(\frac{1}{(\bar{\varsigma}-\varsigma_0)^2}+\frac{2(\varsigma_0-\varsigma)}{(\bar{\varsigma}-\varsigma_0)^3}\right)\frac{d\bar{\varsigma}}{d\varsigma}.\end{aligned}$$

In Eq. (9), first integral on the right of the equation is hypersingular, others are regular.

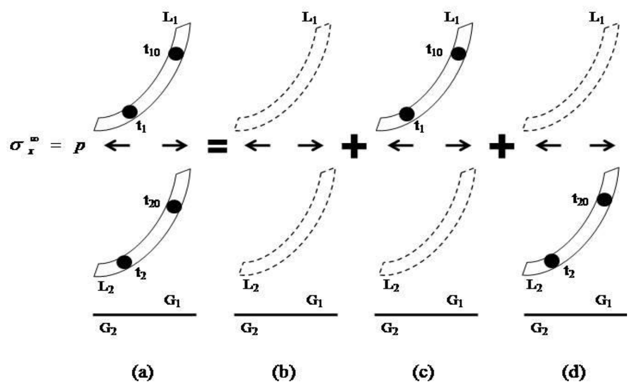


Figure 1. Superposition principle for the two cracks problem

Superposition principle can be applied for solving two cracks L_1 and L_2 lie in the upper half of bonded dissimilar materials (Fig. 1(a)). Summation of an elastic bonded dissimilar materials with remote tension $\sigma_x^\infty = p$ (Fig. 1(b)), crack problems with loading on the crack faces of L_1 (Fig. 1(c)) and L_2 (Fig. 1(d)) yields the HSIE for the two cracks as follows

$$\begin{aligned}\left(N(\varsigma_{j0})+iT(\varsigma_{j0})\right)_j=&\frac{1}{\pi}\int_{L_j}\frac{g_j(\varsigma_j)d\varsigma_j}{(\varsigma_j-\varsigma_{j0})^2}+\frac{1}{2\pi}\int_{L_j}A_1(\varsigma_j,\varsigma_{j0})g_j(\varsigma_j)d\varsigma_j\\&+\frac{1}{2\pi}\int_{L_j}A_2(\varsigma_j,\varsigma_{j0})\overline{g_j(\varsigma_j)}d\varsigma_j+\frac{1}{\pi}\int_{L_k}\frac{g_k(\varsigma_k)d\varsigma_k}{(\varsigma_k-\varsigma_{j0})^2}\\&+\frac{1}{2\pi}\int_{L_k}A_1(\varsigma_k,\varsigma_{j0})g_k(\varsigma_k)d\varsigma_k+\frac{1}{2\pi}\int_{L_k}A_2(\varsigma_k,\varsigma_{j0})\overline{g_k(\varsigma_k)}d\varsigma_k\end{aligned} \tag{10}$$

where $k,j=1,2(k\neq j)$. In Eq. (10), the first three integrals on the right hand side represent the traction influence on crack L_1 caused by COD $g_1(\varsigma_1)$ on crack L_1. The next three integrals represent the traction influence on crack L_1 caused by COD $g_2(\varsigma_2)$ on crack L_2.

If $G_2=0$, then $\Lambda_1=\Lambda_2=-1$, Eqns. (9) and (10) reduce to the HSIE for a single and multiple cracks in a half plane elasticity, respectively (Chen *et al.*, 2009). If $G_1=G_2$, then $\Lambda_1=\Lambda_2=0$ Eqns. (9) and (10) reduce to the HSIE for a single and multiple cracks in an infinite plane, respectively (Nik Long & Eshkuvatov, 2009).

In order to solve the HSIEs (9) and (10), we map the function $g_j(\varsigma_j)$ on a real axiss with an interval $2a$ as follows

$$g_j(\varsigma_j)\Big|_{t_j=s_j} = \sqrt{a_j^2 - s_j^2}\,H_j(s_j),\; j=1,2. \quad (11)$$

The following quadrature formulas are used to find the numerical solutions of HSIEs (10) and (11)

$$\frac{1}{\pi}\int_{-a_j}^{a_j} \frac{\sqrt{a_j^2 - s_j^2}\,H_j(s_j)\,ds_j}{(s_j - s_{j0})^2} = \sum_{j=1}^{M+1} W_j(s_{j0})H_j(s_j) \quad (12)$$

$$\frac{1}{\pi}\int_{-a_j}^{a_j} \sqrt{a_j^2 - s_j^2}\,H_j(s_j)\,ds_j = \frac{1}{M+2}\sum_{j=1}^{M+1}\left(a_j^2 - s_{j0}^2\right)H_j(s_j) \quad (13)$$

where $H_j(s_j) = H_{j1}(s_j) + iH_{j2}(s_j), M \in \mathbb{Z}^+$,

$$s_j = a\cos\left(\frac{j\pi}{M+2}\right),\; j=1,2,\ldots,M+1$$

and

$$W_j(s_{j0}) = -\frac{2}{M+2}\sum_{n=0}^{M}(n+1)\sin\left(\frac{j\pi}{M+2}\right) \bullet \sin\left(\frac{(n+1)j\pi}{M+2}\right)U_n\left(\frac{s_{j0}}{a}\right)$$

and the observation points

$$s_{j0} = s_{j0,k} = a\cos\left(\frac{k\pi}{M+2}\right), k=1,2,\ldots,M+1.$$

Here $U_n(t)$ is a Chebyshev polynomial of the second kind, defined by

$$U_n(t) = \frac{\sin((n+1)\theta)}{\sin\theta}, \text{where } t=\cos\theta.$$

III. NUMERICAL RESULTS

The stress intensity factor (SIF) at the tips A_j and B_j of crack L_j are defined as

$$(K_1 - iK_2)_{A_j} = \sqrt{2\pi}\lim_{t\to t_{A_j}}\sqrt{\left|t - t_{A_j}\right|}\,g'_j(\varsigma)$$

$$(K_1 - iK_2)_{B_j} = \sqrt{2\pi}\lim_{t\to t_{B_j}}\sqrt{\left|t - t_{B_j}\right|}\,g'_j(\varsigma)$$

where $j=1,2$.

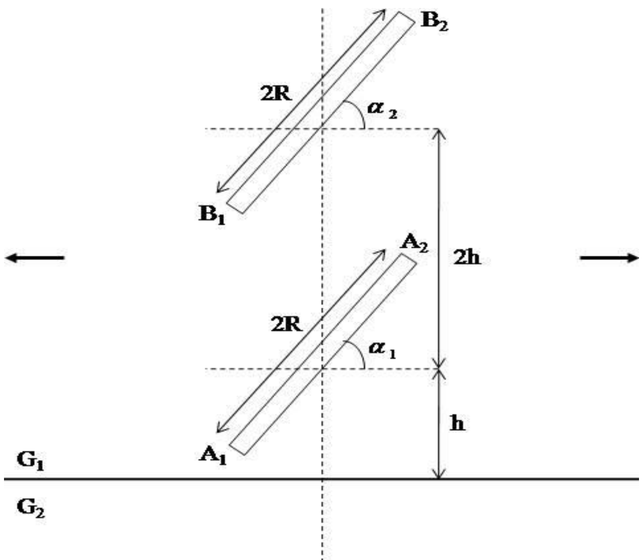


Figure 2. Two inclined cracks in the upper half of bonded dissimilar materials

Table 1. SIF for two inclined cracks when $\alpha_1 = \alpha_2 = 90^o$

G_2/G_1	SIF	R/h				
		0.1	0.3	0.5	0.7	0.9
0.0	F_{1A1}^a	1.0042	1.0411	1.1320	1.3345	1.9925
	F_{1A1}^b	1.0042	1.0411	1.1320	1.3345	1.9925
	F_{1A2}^a	1.0040	1.0380	1.1160	1.2811	1.8144
	F_{1A2}^b	1.0041	1.0380	1.1160	1.2811	1.8145
	F_{1B1}^a	1.0019	1.0209	1.0727	1.2043	1.7106
	F_{1B1}^b	1.0017	1.0209	1.0729	1.2042	1.7107
	F_{1B2}^a	1.0017	1.0160	1.0455	1.0992	1.2200
	F_{1B2}^b	1.0016	1.0160	1.0457	1.0991	1.2200
1.0	F_{1A1}^a	1.0011	1.0101	1.0280	1.0579	1.1174
	F_{1A1}^c	1.0012	1.0102	1.0280	1.0579	1.1174
	F_{1A2}^a	1.0013	1.0138	1.0479	1.1332	1.4538
	F_{1A2}^c	1.0013	1.0138	1.0480	1.1333	1.4539

[a] Present study

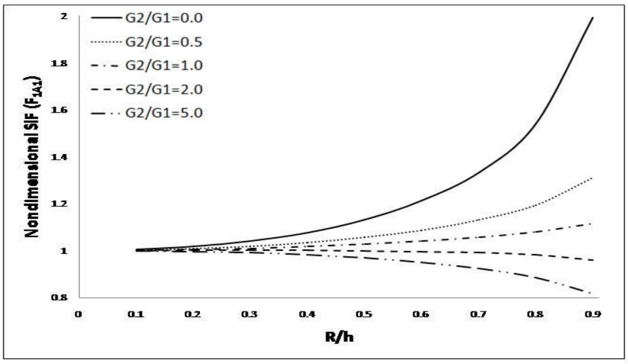
[b] Elfakhakhre *et al.* (2017)

[c] Murakami *et al.* (1987)

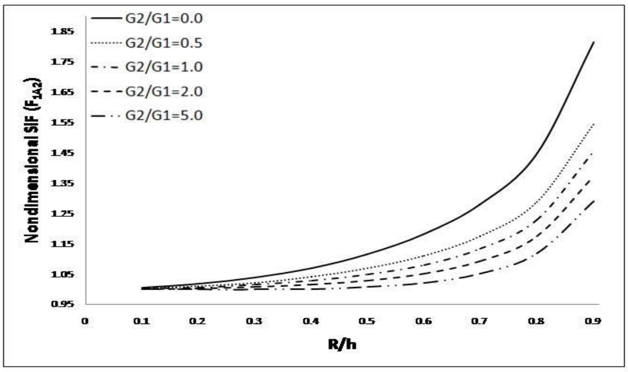
The nondimensional SIF for two inclined cracks lies in the upper half of bonded dissimilar materials with $\alpha_1 = \alpha_2 = 90^o$ and R/h varies are presented in Table 1. Our results agree well with those of Elfakhakhre *et al.* (2017) for $G_2/G_1 = 0.0$. For $G_2/G_1 = 1.0$, our results are in good agreement with those of Murakami *et al.* (1987). For $G_2/G_1 = 1.0$, the nondimensional SIF at crack tips A_1 and A_2 are equal to SIF at tips B_2 and B_1, respectively.

Fig. 3 shows the nondimensional SIF against R/h for different values of G_2/G_1. It is found that as the ratio G_2/G_1 increases, the nondimensional SIF decreases. At cracks tips A_2, B_1 and B_2 the nondimensional SIF increases as R/h increases whereas at crack tip A_1 nondimensional SIF increases for $G_2/G_1 \leq 1.0$ and decreases for $G_2/G_1 > 1.0$.

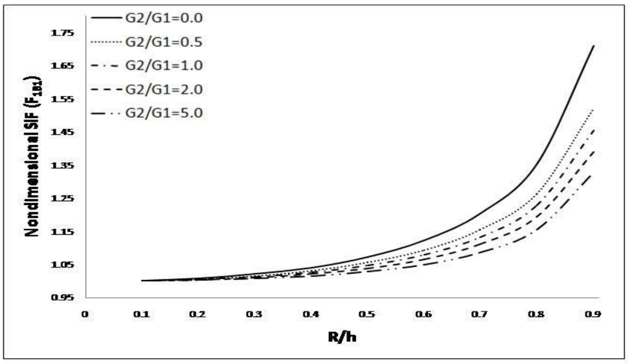
Table 2 shows the nondimensional SIF for two inclined cracks lie in the upper half of bonded dissimilar materials for different values of α_1, $\alpha_2 = 90^o$ and $R/h = 0.9$. Our results are comparable with those of Chen (1993) for $G_2/G_1 = 1.0$. For the different values of G_2/G_1, the nondimensional SIF against α_1 are presented in Fig. 4. It is found that as the angle α_1 increases the nondimensional SIF increases. However, the nondimensional SIF decreases as the ratio G_2/G_1 increases at all crack's tips.



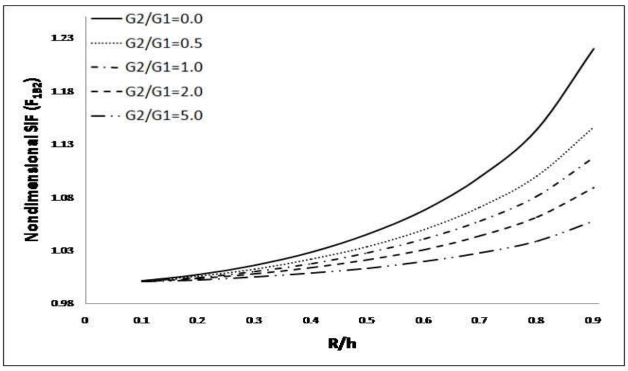
(a) SIF at the crack tip A_1



(b) SIF at the crack tip A_2

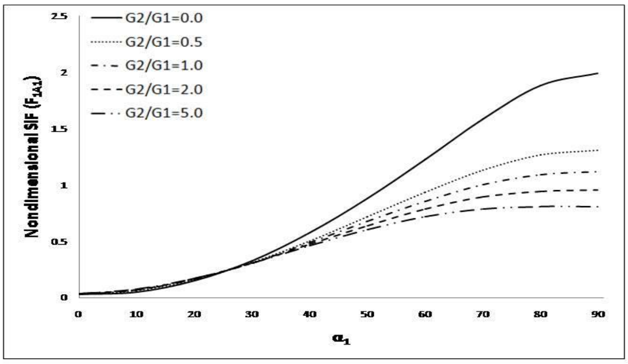


(c) SIF at the crack tip B_1

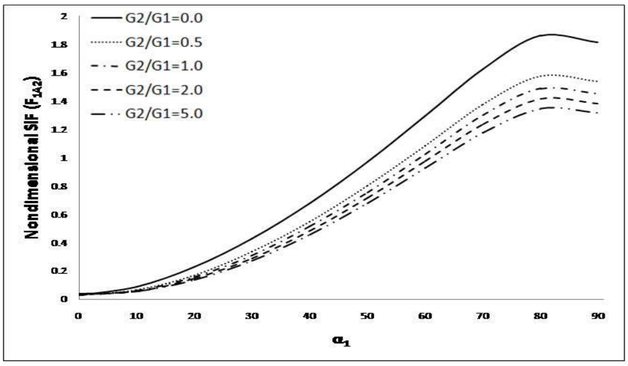


(d) SIF at the crack tip B_2

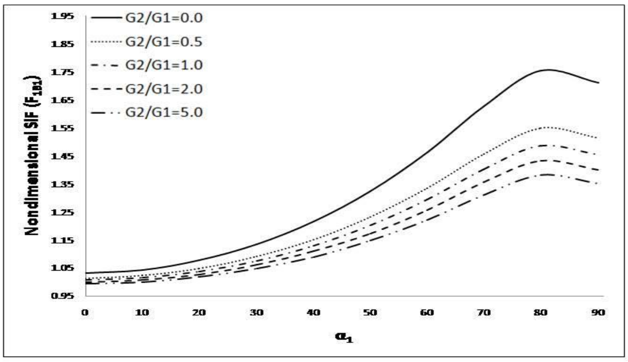
Figure 3. SIF for two inclined cracks when $\alpha_1 = \alpha_2 = 90^o$



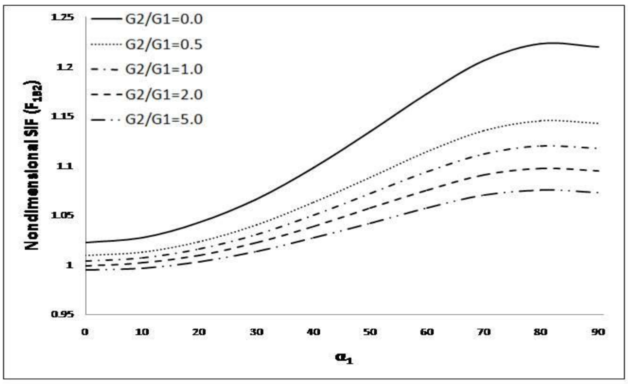
(a) SIF at the crack tip A_1



(b) SIF at the crack tip A_2



(c) SIF at the crack tip B_1



(d) SIF at the crack tip B_2

Figure 4. SIF for two inclined cracks when α_1 is changing, $\alpha_2 = 90^o$ and $R/h = 0.9$

Table 2. SIF for two inclined cracks for $G_2/G_1 = 1.0$, $\alpha_2 = 90^o$, $R/h = 0.9$ and different value of α_1

SIF	α_1			
	0°	30°	60°	90°
$F_{1A1}{}^a$	0.0305	0.3086	0.8566	1.1174
$F_{1A1}{}^b$	0.0305	0.3086	0.8566	1.1174
$F_{1A2}{}^a$	0.0305	0.3101	1.0252	1.4538
$F_{1A2}{}^b$	0.0305	0.3101	1.0252	1.4539
$F_{1B1}{}^a$	1.0070	1.0756	1.2932	1.4538
$F_{1B1}{}^b$	1.0071	1.0757	1.2933	1.4539
$F_{1B2}{}^a$	1.0040	1.0310	1.0938	1.1174
$F_{1B2}{}^b$	1.0040	1.0310	1.0939	1.1174

[a] Present study

[b] Chen (1993)

IV. CONCLUSION

In this paper the modified complex variable function method were used to formulate the hypersingular integral equations for two inclined cracks lie in the upper half of bonded dissimilar materials with different elastic constants G_1 and G_2. Numerical results showed the behavior of the nondimensional SIF at the cracks tips. For $G_2 = 0$ and $G_1 = G_2$ the nondimensional SIF at all crack tips are equal to the SIF at cracks tips in half plane and infinite plane elasticity problems, respectively. The nondimensional SIF increases for $G_2/G_1 \leq 1.0$ and decreases for $G_2/G_1 > 1.0$ as the distance between the cracks and the boundary decreases at crack tip A_1. However, for the constant distance between the cracks and the boundary, the nondimensional SIF at all cracks tips decreases as the elastic constant ratio G_2/G_1 increases.

V. ACKNOWLEDGEMENT

The first author would like to thanks the Ministry of Education Malaysia, Universiti Putra Malaysia and Universiti Teknikal Malaysia Melaka for the financial support. The second author would like to thanks the Universiti Putra Malaysia for Putra Grant, project no:9567900 and FRGS project no:01-01-16-18685R5524975.

VI. REFERENCES

Birman, V. (2018). Control of fracture at the interface of dissimilar materials using randomly oriented inclusions and networks. International Journal of Engineering Science, 130, 157-174.

Chen, Y.H. (1987). Solution of stiffened problems for a finite internally cracked plate by using complex potentials and the generalized variational method. Computer Methods in Applied Mechanics and Engineering, 62, 1-16.

Chen, Y.Z. (1986). Multiple crack problems for two bonded half planes in plane and antiplane elasticity. Engineering Fracture Mechanics, 25(1), 1-9.

Chen, Y.Z. (1993). New Fredholm integral equation for multiple crack problem in plane elasticity and antiplane elasticity. International Journal of Fracture, 64, 63-77.

Chen, Y.Z. & Hasebe, N. (1992) Stress-intensity factors for curved circular crack in bonded dissimilar materials. Theoretical and Applied Fracture Mechanics, 17, 189-196.

Chen, Y.Z., Lin, X.Y. & Wang, X.Z. (2009). Numerical solution for curved crack problem in elastic half-plane using hypersingular integral equation. Philosophical Magazine, 89(26), 2239-2253.

Elfakhakhre, N.R.F., Nik Long, N.M.A. & Eshkuvatov, Z.K. (2017). Stress intensity factor for multiple cracks in half plane elasticity. AIP Conference Proceedings, 1795(1), 1-8.

Isida, M. & Noguchi, H. (1993). Arbitrary array of cracks in bonded half planes subjected to various loadings. Engineering Fracture Mechanics, 46(3), 365-380.

Itou, S. (2016). Stress intensity factors for four interface-close cracks between a nonhomogeneous bonding layer and one of two dissimilar elastic half-planes. European Journal of Mechanics A/Solids, 59, 242-251.

Lai, J. & Schijve, J. (1990). The stress intensity factor and stress concentration for a finite plate with a single crack emanating from a hole'. Engineering Fracture Mechanics, 36(4), 619-630.

Lan, X., Ji, S., Noda, N.A. & Cheng, Y. (2017). Stress intensity factor solutions for several crack problems using the proportional crack opening displacements. Engineering Fracture Mechanics, 171, 35-49.

Long, G. & Xu, G. (2016). A combined boundary integral method for analysis of crack problems in multilayered elastic media. International journal of Applied Mechanics, 8(5), 1-21.

Murakami, Y. (1987). Handbook of Stress Intensity Factors. Pergamon Press, Oxford.

Muskhelishvili, N.I. (1953). Some Basic Problems of the Mathematical Theory of Elasticity. Noordhoff International Publishing, Leyden.

Nik Long, N.M.A. & Eshkuvatov, Z.K. (2009). Hypersingular integral equation for multiple curved cracks problem in plane elasticity. International Journal of Solids and Structures, 46, 2611-2617.

Wang, Y. & Waisman, H. (2017). Material dependent crack-tip enrichment functions in XFEM for modeling interfacial cracks in biomaterials. International Journal for Numerical Methods in Engineering, 112(11), 1495-1518.

Yang, M. & Wang, X. (2018). A bridged crack perpendicular to a bimaterial interface. ActaMechanica, 229(5), 2063-2078.