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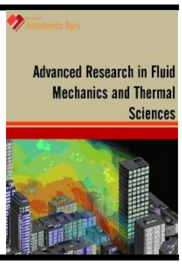

Thermally Stratified Flow of Cu-Al_2O_3/water Hybrid Nanofluid past a Permeable Stretching/Shrinking Circular Cylinder

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ABSTRACT

The present study emphasizes the thermally stratified hybrid nanofluid flow due to a permeable stretching/shrinking cylinder. Thermal buoyancy force is also taken into consideration to incorporate with the thermal stratification process. An improved hybrid nanofluid (dual nanoparticles) may offer a better heat transfer performance in many engineering applications. In the present work, the combination of copper (Cu) and alumina (Al_2O_3) nanoparticles with water as the working fluid is analytically modeled using the extended form of Tiwari and Das nanofluid model. A suitable transformation is adopted to simplify the boundary layer and energy equations into a nonlinear system of ODEs. A boundary value problem solver with fourth order accuracy (bvp4c) in the MATLAB software is utilized to solve the transformed system. The change in velocity and temperature as well as the heat transfer rate and skin friction coefficient are deliberated and graphically manifested for appropriate values of the dimensionless stretching/shrinking, nanoparticles volume fraction, and thermal stratification parameters. The presence of dual solutions is seen on all the profiles within the range of selected parameters.

1. Introduction

Hybrid nanofluid is an invention of fluids which may improve the heat transfer performance as compared to the traditional coolant (i.e. water, ethylene glycol) and nanofluid (coolant with single nanoparticle) in many engineering and technological applications such as manufacturing, thermal power plants and microelectronics. The combination of water and ethylene glycol is extensively applied as coolant in the automobile cooling system. Hong *et al.,* [1] briefly studied the heat transfer performance of hybrid nanofluid as automobile radiator coolant. Sajid and Ali [2] reviewed on the preparation and thermal conductivity of the hybrid nanofluids and concluded that the stability of the hybrid nanofluids might depend on the proper selection of the hybrid nanoparticles. The

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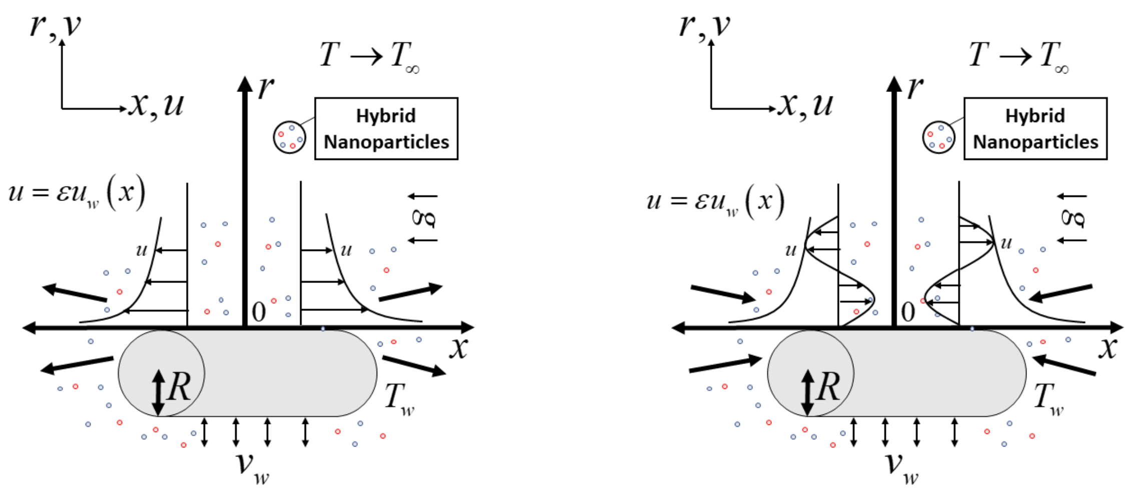
nanoparticles that are frequently used are metals (i.e. Cu, Ag, Ni, Au), metal oxides (i.e. Al_2O_3, CuO, SiO_2, Fe_2O_3), carbon materials (CNTs, MWCNTs, graphite, diamond), metal carbide and metal nitride. Types of nanoparticles and base fluid, size, shape and concentration of the nanoparticles are among the factors that affect the thermal conductivity of the hybrid nanofluid. Sarkar *et al.,* [3] discussed the importance of proper hybridization technique which beneficial in the heat transfer enhancement. A comprehensive review by Sidik *et al.,* [4] also concluded that the thermophysical properties of the hybrid nanofluid was higher as compared to the nanofluid and traditional heat transfer (base) fluid. A few studies related to the hybrid nanofluid were also examined by these researchers [5]–[11].

Manufacturing processes i.e. metal, and polymer sheets are among the engineering applications which related to the study of boundary layer flow past a stretching/shrinking surface. The viscous flow problem was initiated by Crane [12] and Miklavčič and Wang [13] towards a stretching plate and shrinking plate, respectively. According to Miklavčič and Wang [13], the shrinking sheet might distract the velocity away from the sheet, hence an adequate wall mass suction was necessary to enhance the flow near to the surface. Devi and Devi [14] applied the thermophysical properties for hybrid nanofluid to the flow past a stretching sheet. It was observed that the Nusselt number of Cu-water nanofluid was smaller than the hybrid Cu-Al_2O_3/water nanofluid. Rostami *et al.,* [15] solved the problem of mixed convective stagnation point flow over a static flat plate using a hybrid nanofluid and attained two solutions. They found that the dual solutions appeared in both assisting and opposing flow parts. Many further investigations were also conducted on the boundary layer flow using different geometries such as cylinder, cone and wedge with various physical parameters (wall mass suction, magnetic, thermal radiation, buoyancy). Mukhopadhyay [16] studied an axisymmetric mixed convective flow of a viscous fluid towards a stretching cylinder in a porous medium. He found that the rate of heat transfer for a cylindrical surface was larger as compared to the flat plate/sheet. Najib *et al.,* [17] investigated the stagnation point flow of a viscous fluid over a stretching/shrinking cylinder with the imposition of chemical reaction. The study indicated that two solutions were possible for the shrinking cylinder while stretching cylinder produced unique solution. The rate of mass transfer and the surface shear stress increased as the curvature (cylinder) parameter enhanced. Omar *et al.,* [18] extended the previous study [17] by using copper-water nanofluid and found that both shear stress and heat transfer rate were greater for cylinder as compared to the flat plate. Dhanai *et al.,* [19] considered the coupled effects of velocity and thermal slip on Magnetohydrodynamics (MHD) mixed convection nanofluid over an inclined cylinder. In their work, Buongiorno's model of nanofluid was used to incorporate the Brownian motion and thermophoresis effects. Pandey and Kumar [20] scrutinized the Cu-water nanofluid flow with the combined effects of velocity, thermal slip and thermal radiation parameters towards a stretching cylinder. In their work, they considered 0-6% of volume fraction nanoparticles with an increment of 2%. Poply *et al.,* [21] also obtained dual solutions and concluded that the heat transfer rate intensified with the curvature parameter while reduced with the magnetic parameter. Three dimensional stagnation point flow of hybrid nanofluid past a static cylinder was studied by Nadeem *et al.,* [22] and they observed that the heat transfer rate of the hybrid nanofluid was higher as compared to the nanofluid. In their work, water was considered with the combination of copper (Cu) and alumina (Al_2O_3). Rehman *et al.,* [23] accentuated the combined effects of double stratification and mixed convection in Casson fluid flow over an inclined stretching cylinder. The study showed that the Casson fluid temperature and concentration diminished with the intensification of the thermal and solutal stratifications, respectively. Bakar *et al.,* [24] performed a stability analysis due to the existence of two solutions in their problem. The inclusion of suction parameter was believed to produce the non-unique solutions. Recently, the copper-water nanofluid flow over a permeable shrinking cylinder was analyzed by Soomro *et al.,* [25]. Soomro *et al.,* [25] pointed out that the opposing flow might generate dual solutions whereas the assisting flow produce unique solution.

Inspired by the above literatures, the present work deals with the thermally stratified flow towards a permeable stretching/shrinking cylinder using an application of hybrid nanoparticles. The hybrid Cu-Al_2O_3/water nanofluid is modeled using the thermophysical properties proposed by Devi and Devi [14]. The finite difference method which is programmed in the MATLAB software as the bvp4c solver are used to compute the transformed similarity (ordinary) differential equations. The numerical results (local Nusselt number and skin friction coefficient) are graphically presented in conjunction with the velocity and temperature profiles for the selected values of the control parameters. The authors are confident that no other studies are published regarding the thermally stratified flow over stretching/shrinking cylinder using hybrid nanofluid.

2. Methodology

Consider a two-dimensional, steady viscous boundary layer flow with hybrid Cu-Al_2O_3 nanoparticles towards a cylinder with radius, R and (u,v) is the velocity components measured in the (x,r) directions as displayed in Figure 1. The stretching/shrinking cylinder is moved with linear velocity, $u_w(x)=a(x/L)$ such that a and L (characteristic length of the cylinder) are a constant. The wall surface of the cylinder is permeable to allow the possible wall fluid suction in the physical problem. Thermal buoyancy is considered to mix with the thermal stratification phenomenon. The wall temperature is assumed to be given by $T_w(x)=T_0+A(x/L)$ where T_0 is the beginning temperature of the ambient hybrid nanofluid and A is the characteristic temperature of the circular cylinder. The ambient temperature is assumed in the linear stratified form of $T_\infty(x)=T_0+B(x/L)$ such that B is a positive constant.



(a) Stretching surface $(\varepsilon>0)$ (b) Shrinking surface $(\varepsilon<0)$

Fig. 1. Coordinate system of the physical model

Under all these considerations, the boundary layer with energy equations are

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\frac{(\rho\beta_T)_{hnf}\, g\left(T-T_\infty\right)}{\rho_{hnf}}, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\left(\frac{k}{\rho C_p}\right)_{hnf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \quad (3)$$

with a set of initial and boundary conditions

$$u(x,R)=\varepsilon u_w(x),\quad v(x,R)=v_w,\quad T(x,R)=T_w(x), \quad (4)$$

$$u(x,r)\to 0,\quad T(x,r)=T_\infty(x)\quad \text{as}\quad r\to\infty. \quad (5)$$

where $v_w=-\sqrt{a\nu_f/L}\,(R/r)S$ is the constant mass flux velocity and ν_f is the kinematic viscosity of the base fluid. The problem is enforced with mass suction if $v_w<0$ while mass injection for $v_w>0$. Furthermore, T is the hybrid nanofluid temperature, g is the gravitational acceleration, μ_{hnf}, ρ_{hnf}, $(\beta_T)_{hnf}$, k_{hnf} and $(\rho C_p)_{hnf}$ are the dynamic viscosity, density, thermal expansion, thermal conductivity and specific heat of hybrid nanofluid at constant pressure, accordingly. The analytical model of thermophysical properties for both nanofluids (traditional and hybrid) are exemplified in Table 1 while Table 2 provides the thermophysical properties of the copper and alumina nanoparticles with pure water (25°C).

Table 1
The thermophysical properties between hybrid and traditional nanofluids (see Devi and Devi [14] and Rostami *et al.,* [15])

Properties	Hybrid Nanofluid	Traditional Nanofluid
Density	$\rho_{hnf}=(1-\phi_2)\left[(1-\phi_1)\rho_f+\phi_1\rho_{s1}\right]+\phi_2\rho_{s2}$	$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$
Heat Capacity	$(\rho C_p)_{hnf}=(1-\phi_2)\left[(1-\phi_1)(\rho c_p)_f+\phi_1(\rho c_p)_{s1}\right]+\phi_2(\rho c_p)_{s2}$	$(\rho C_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s$
Dynamic Viscosity	$\frac{\mu_{hnf}}{\mu_f}=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$	$\frac{\mu_{nf}}{\mu_f}=\frac{1}{(1-\phi_1)^{2.5}}$
Thermal Conductivity	$k_{hnf}=\left[\frac{k_{s2}+2k_{bf}-2\phi_2(k_{bf}-k_{s2})}{k_{s2}+2k_{bf}+\phi_2(k_{bf}-k_{s2})}\right]k_{bf}$ where $k_{bf}=\left[\frac{k_{s1}+2k_f-2\phi_1(k_f-k_{s1})}{k_{s1}+2k_f+\phi_1(k_f-k_{s1})}\right]k_f$	$k_{nf}=\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right]k_f$
Thermal Expansion	$(\rho\beta_T)_{hnf}=(1-\phi_2)\left[(1-\phi_1)(\rho\beta_T)_f+\phi_1(\rho\beta_T)_{s1}\right]+\phi_2(\rho\beta_T)_{s2}$	$(\rho\beta_T)_{nf}=(1-\phi)(\rho\beta_T)_f+\phi(\rho\beta_T)_s$

Table 2
Thermophysical properties of the alumina, copper and water (see Rostami *et al.*, [15])

Physical properties	$\rho\left(\mathrm{kg/m^3}\right)$	$C_p\left(\mathrm{J/kgK}\right)$	$k\left(\mathrm{W/mK}\right)$	$\beta_T\left(\mathrm{K^{-1}}\right)$
Alumina	3970	765	40	0.85 x 10^{-5}
Copper	8933	385	400	1.67 x 10^{-5}
Water	997.1	4179	0.6130	21 x 10^{-5}

The following similarity transformations are introduced

$$\eta\left(\frac{a}{\nu_f L}\right)^{-1/2}=\frac{r^2-R^2}{2R},\quad \psi=\left(\frac{a\nu_f x^2}{L}\right)^{1/2}Rf\left(\eta\right),\quad \theta\left(\eta\right)=\frac{T-T_\infty\left(x\right)}{T_w\left(x\right)-T_0},\tag{6}$$

where $\frac{\partial\psi}{\partial r}=ru$ and $\frac{\partial\psi}{\partial x}=-rv$ which satisfies Eq. (1). A set of nonlinear similarity equations are obtained by adopting the transformations into Eq. (2) and (3) subject to the boundary condition (4) and (5) such that

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left[\left(1+2\gamma\eta\right)f'''+2\gamma f''\right]=f'^2-ff''-\frac{\left(\rho\beta_T\right)_{hnf}/\left(\rho\beta_T\right)_f}{\rho_{hnf}/\rho_f}\lambda\theta,\tag{7}$$

$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\left[\left(1+2\gamma\eta\right)\theta''+2\gamma\theta'\right]=\left(\theta+\delta\right)f'-f\theta',\tag{8}$$

align with the transformed condition

$$\begin{aligned}&f\left(0\right)=S,\quad f'\left(0\right)=\varepsilon,\quad \theta\left(0\right)=1-\delta,\\&f'\left(\eta\right)\to 0,\quad \theta\left(\eta\right)\to 0\quad \text{as}\quad \eta\to\infty,\end{aligned}\tag{9}$$

It is worth to mention that ϕ_1 and ϕ_2 are the first and second nanoparticles volume fraction, accordingly while s_1 and s_2 symbolize the first and second types of the nanoparticles. Further, the curvature, thermal buoyancy and thermal stratification, are

$$\gamma=\sqrt{L\nu_f/aR^2},\quad \lambda=\frac{Gr_x}{\mathrm{Re}_x^{\,2}}=\frac{g\left(\beta_T\right)_f\left(T_w\left(x\right)-T_0\right)x^3/\nu_f^{\,2}}{\left(ax^2/L\nu_f\right)^2},\quad \delta=B/A,\tag{10}$$

respectively. $S>0$ and $S<0$ represents the suction and injection parameters, correspondingly whereas $\varepsilon>0$ and $\varepsilon<0$ corresponds to the stretching and shrinking circular cylinder, proportionately for $\gamma>0$. It is noticed that for $\gamma=0$ the present problem is reduced to the stretching/shrinking flat surface. It is also worth to mention here that $\lambda>0$, $\lambda<0$ and $\lambda=0$ are referred to the assisting flow, opposing flow and pure forced convection flow cases, respectively.

Eq. (11) and (12) are the dimensionless skin friction coefficient and local Nusselt number which symbolize the wall drag and heat transfer rate, respectively.

$$C_f = \left\{ \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} f''(0) \right\} \mathrm{Re}_x^{-1/2}, \tag{11}$$

$$Nu_x = \left\{ -\frac{k_{hnf}}{k_f} \theta'(0) \right\} \mathrm{Re}_x^{1/2}, \tag{12}$$

where $\mathrm{Re}_x = u_w(x)x/\nu_f$ is the local Reynolds number.

3. Results and Discussion

An efficient bvp4c solver is successfully utilized to the ODEs (see Eq. (7) and (8)) controlled by the boundary condition (9). The solver is originated from a finite difference code (3-stage Lobatto IIIa), which is programmed in the MATLAB software. Table 3 shows a comparison value of $-\theta'(0)$ between present study and results by Devi and Devi [14] and Khan and Pop [26] in the absence of nanoparticles volume fraction $(\phi_1 = \phi_2 = 0)$ with $\varepsilon = 1,\ \gamma = \lambda = \delta = 0$ and various values of Prandtl number. It is validated from Table 3 that the bvp4c solver can generate numerical results which is in accordance with the other methods.

Table 3
The comparison values of $-\theta'(0)$ with $\varepsilon = 1$, $\gamma = \lambda = \delta = \phi_1 = \phi_2 = 0$ and various values of Prandtl number

Pr	Present	Devi and Devi [14]	Khan and Pop [26]
2	0.91135	0.91135	0.9113
6.13	1.75968	1.75968	-
7	1.89540	1.89540	1.8954
20	3.35390	3.35390	3.3539

In the present work, hybrid Cu-Al_2O_3/water nanofluid is formed by dispersed Cu (ϕ_2) nanoparticles into the mixture of Al_2O_3 (ϕ_1) and water-based fluid. Al_2O_3 nanoparticles $(\phi_1 = 10\%)$ is fixed in the entire analysis with $\mathrm{Pr} = 6.2$ to signify the water as the working fluid. Furthermore, the nanoparticles volume fraction for Cu is varied, $5\% \le \phi_2 \le 9\%$ throughout the problem. The numerical results are demonstrated in the graphical form for various values of the control parameters, namely stretching/shrinking ε, suction S, nanoparticles (Cu) volume fraction ϕ_2 and thermal stratification δ to clearly get the physical insight of the problem. First (physical) and second solution which are denoted by the straight and dashed lines, respectively, exist within the specific values of the governing parameters as exhibited in Figure 2-7. Generally, first solution is assigned to the solution which initially converges. Suction is physically used in certain engineering processes to enhance the flow near to the surface. It is justified that an application of higher suction $(S > 2)$, may induce dual or multiple solutions. Figure 2 and 3 manifest the fluid velocity and temperature with an expansion of suction parameter. The velocity profile inflates when S increases while the temperature profile lessens. As expected, second solution which is not a physical solution shows a contradictory flow behavior. This is as a result of the suction parameter which cause a reduction in both momentum and thermal boundary layer thicknesses.

The fluid velocity and temperature profiles with the imposition of thermal stratification parameter δ are demonstrated in Figure 4 and 5. The velocity profile only slightly affected and diminishes while the inclining values of δ enhance the fluid temperature. Figure 6 and 7 illustrate the variations of the $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ with the intensification of ϕ_2 towards stretching/shrinking parameter ε. It is worth to mention again that $\varepsilon < 0$ and $\varepsilon > 0$ correspond to the shrinking and stretching cylinder, accordingly whereas $\varepsilon = 0$ represents static cylinder. $\mathrm{Re}_x^{1/2} C_f$ (skin friction coefficient) increases as $\varepsilon > 0$ and $\mathrm{Re}_x^{1/2} C_f = 0$ for all values of ϕ_2 when $\varepsilon = 0$ as exhibited in Figure 6. However, there is a contradictory result when the shrinking parameter starts to develop. Figure 7 displays the impact of ϕ_2 on heat transfer rate for both surfaces (stretching/shrinking cylinder). An upsurge of ϕ_2 seems to deteriorate the heat transfer rate for both stretching and shrinking cylinders. However, this result may differ if higher magnitude of stretching parameter $(\varepsilon > 1)$ is used. Figure 8 presents the $\mathrm{Re}_x^{-1/2} Nu_x$ towards ε for selected values of δ. The thermal stratification process may reduce the heat transfer rate $\left(\mathrm{Re}_x^{-1/2} Nu_x\right)$ for both surfaces.

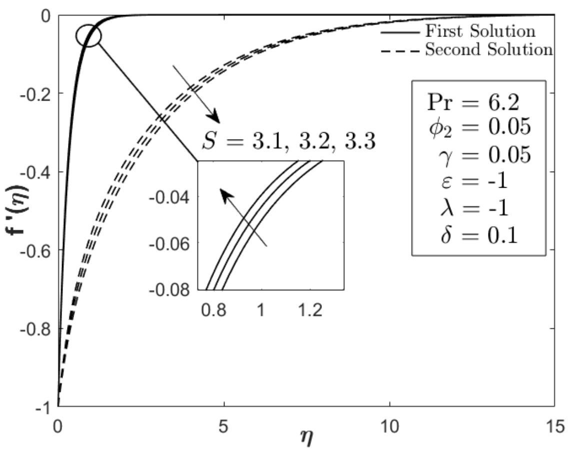


Fig. 2. Velocity profile with the addition of S.

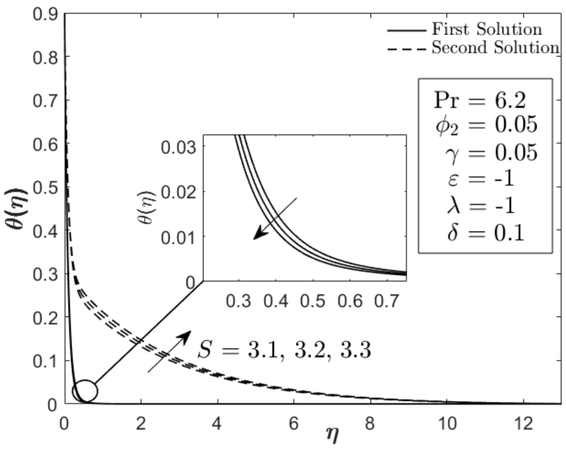


Fig. 3. Temperature profile with the addition of S.

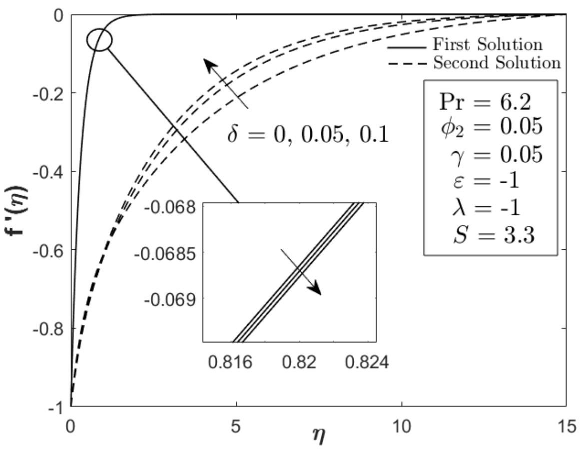


Fig. 4. Velocity profile with the addition of δ.

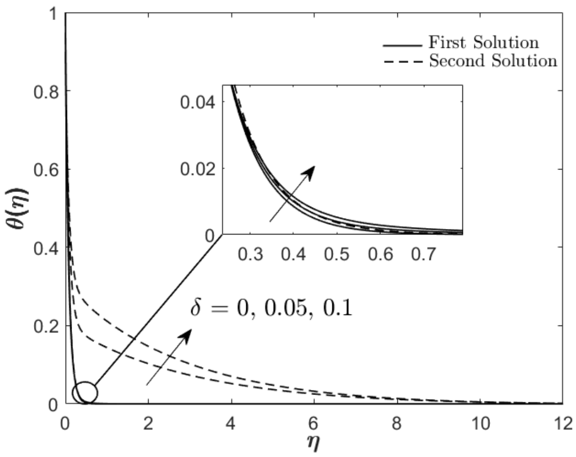


Fig. 5. Temperature profile with the addition of δ.

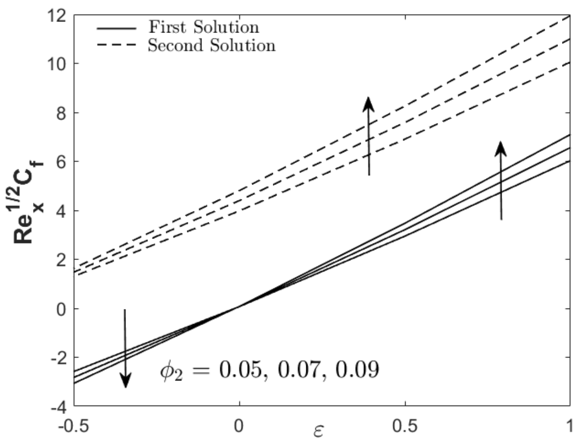


Fig. 6. Variation of $Re_x^{1/2} C_f$ for pertinent values of ϕ_2 when $S = 3.3, \lambda = -1$ and $\delta = \gamma = 0.05$.

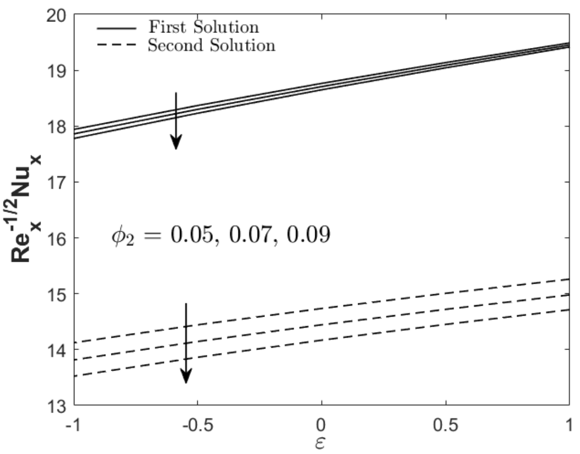


Fig. 7. Variation of $Re_x^{-1/2} Nu_x$ for pertinent values of ϕ_2 when $S = 3.3, \lambda = -1$ and $\delta = \gamma = 0.05$.

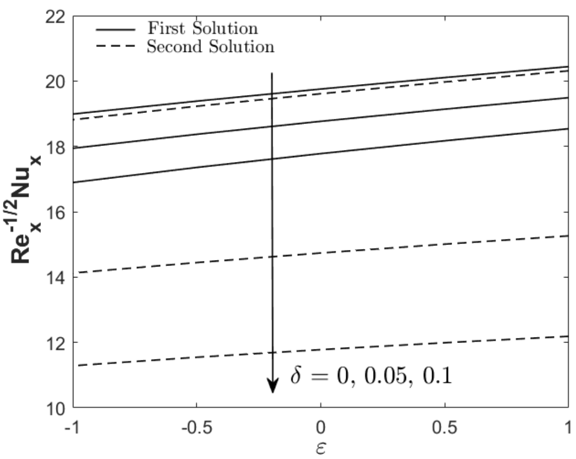


Fig. 8. Variation of $Re_x^{-1/2} Nu_x$ for pertinent values of δ when $S = 3.3, \lambda = -1$ and $\phi_2 = \gamma = 0.05$.

4. Conclusions

The focus of the study is to examine the flow of a hybrid Cu-Al_2O_3/water nanofluid with heat transfer over a stretching/shrinking cylinder under the influence of the suction, buoyancy and thermal stratification parameters. The effect of the pertinent parameters on the temperature and velocity profiles is graphically exemplified inclusive of the dimensionless Nusselt number and skin friction coefficient. The appearance of two solutions are within the specific values of the control parameters. In addition, the heat transfer rate for the shrinking cylinder tends to downturn while incline for the stretching cylinder with the enhancement of the Cu volume fraction (ϕ_2). Thermal stratification process also affects the heat transfer rate for both stretching and shrinking cylinder. Hence, further investigations are needed in the future time to study the main parameter that responsible to diminish the Nusselt number for the shrinking/stretching cylinder case.

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