

FURTHER EXPERIMENTS ON BUCKLING OF CONES WITH IMPERFECT LENGTH SUBJECTED TO AXIAL COMPRESSION

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ABSTRACT

The paper presents further experiments into the buckling of mild steel truncated cones having imperfect length subjected to axial compression. Two types of wave form imperfections with different number of waves along the compressed edge are considered in the current paper, and they are: (i) triangular waves, and (ii) square waves. Twenty-two (22) conical models (two perfect and twenty imperfect cones with axial imperfection amplitude, A , of 0.28) were manufactured in pairs from mild steel. Test results reveal good repeatability of experimental data. The errors within each pair vary from 0% to 4%. The buckling load of the cone is differently affected by wave imperfections shape. The results indicate that the wave shape strongly affect the buckling strength of conical shells. Also, it is shown that the buckling loads of tested cones are less sensitive when imperfection shape is triangular compared to their sensitivity under other imperfection shape.

KEYWORDS: Axial Compression, Buckling, Imperfection Sensitivity, Steel Cones & Wave Profile

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1. INTRODUCTION

Conical shells, find applications as adapters between two cylinders of different diameter. When used for such applications especially under axial compression, the contact interaction between two neighbouring shell structures becomes very important – since the safe performance of such structures are generally believed to be sensitive to imperfection such as waviness on the structures due to manufactured induced imperfection. This imperfection can lead to buckling of one or both structures. This is not a new problem but it is far from being solved. Past literatures on imperfection sensitivity of conical shells can be found in Refs [1 – 4].

Traditionally, it is a common practice to represent this wavy shape on the structures in the form of sinusoidal waves as can be found in [5 – 10] for cylinders and [11, 12] for cones. Whilst, Refs [5 – 7] were devoted to influence of waves on the compressed edge of the cylinder, Refs [8 – 10] on the other hand examine the effect of waves on the cross-sections of the cylinders (i.e., around the cylinder circumference). In Ref [5], the effect of non-uniform length having sinusoidal profile towards the buckling behaviour of axially compressed cylinders was discussed. The sinusoidal shape was located along the compressed edge of the cylinder. Numerical results obtained in this paper show rapid drop of buckling strength for small axial imperfection amplitude as compared to the perfect cylinder. A careful research on the imperfection sensitivity of axially compressed mild steel cylinders having variable length at one end in the form of sinusoidal wave can be found in Ref [6]. The

results of geometrically imperfect cylinders were benchmarked with Eigen mode shape of the same imperfection amplitude. It is found that buckling strength of axially compressed cylindrical shells is severely reduced in the case of cylinders which have variable length, and that the load bearing capacity of 'axially imperfect' cylinders can be up to five times smaller than that of Eigen mode imperfection. In Ref. [7] the analysis on axially compressed mild steel cylindrical shells with non-uniform axial length having sinusoidal and triangular waves were conducted through experimental tests and numerical computation using ABAQUS FE code. Non-uniform axial length introduction to cylindrical structures are proven to decrease the collapse load of the perfect cylinders. Also, it was reported that axially compressed cylinders having triangular waves at the compressed edge are less sensitive to imperfection when compared to cylinders having sinusoidal waves. Moreover, in Refs [8-10] the imperfection sensitivity of composite cylinders having wavy cross section subjected to axial compression were discussed. An alternative approach to improve the imperfection sensitivity of cylindrical shells has been proposed in Ref [8] which was based on symmetry-breaking wavy cylindrical shells. This design was formulated by NURBS interpolation on control points whose positions are optimized by evolutionary algorithms. The optimization design is found to be imperfection-insensitive and based on the mass efficiency studies, the said structural design was more efficient than even a perfect circular cylindrical shell and most stiffened cylindrical shells. In 2015, a numerical analysis was carried out in Ref [9] using the previously stated design criteria of composite cylindrical shells in [8]. It appeared that cylindrical shells manufactured with this design approach can achieve higher critical stresses and knockdown factors value than any other known cylindrical shells, while also being practically insensitive to geometric imperfections. In Ref [10], tests data alongside with their numerical simulations on axially compressed cylinder with wavy cross section were presented. Both the numerical analyses and experimental results proved that the wavy shells are not sensitive towards imperfection. The load carrying capability of cylindrical shells can significantly improve with the introduction of optimal symmetry-breaking wavy cross-sections on the structure. In general, results showed that axially compressed cylindrical shells having wave imperfection at the compressed edge are more sensitive, in comparison to cylindrical shells having wave imperfection at the cylinder's cross-sections.

Furthermore, very few studies have been reported on the influence of imperfect length on the buckling behaviour of cones. They can be found in Refs [11, 12]. In ref. [11], the influence of imperfect length on the buckling behaviour of truncated mild steel cones was discussed. The sinusoidal waves were located at the compressed edge of the axially compressed cone. Experimental results suggested that the number of waves have a strong impact on the buckling load of axially compressed conical shells. In addition, the presence of sinusoidal waves greatly reduced the buckling load of perfect cones. Numerical validation of the aforementioned study has been extended in Ref [12]. ABAQUS FE code was used to model the axially compressed mild steel cones. Results demonstrated that the buckling load of conical shells was strongly affected by the presence of sinusoidal waves.

However, from Ref [7], it was argued that for axially compressed cylinder with imperfect length, different wave shape will affect the buckling behaviour of the structures differently. For instance, it was concluded in Ref. [7], that cylinder with triangular waves are less sensitive as compared to cylinders having sinusoidal waves. Although, it is not quite clear if this is applicable to other geometry and if sinusoidal wave will produce the most conservative results. This paper seeks to provide clarity to the above issues by providing further experiments into influence of different waves shape (i.e., triangular and square) on the buckling behaviour of conical with imperfect length under axial compression. The current paper aims to complement the earlier results presented in Ref. [12] on cones with imperfect length having

sinusoidal waves. The effect of (i) increasing the number of waves, and (ii) wave shape (triangular and square waves) on the buckling load of axially compressed conical shell will be investigated.

2. MATERIAL AND METHODS

Twenty-two laboratory scale (two perfect and twenty imperfect) conical shells were manufactured from 1 mm mild steel plate and tested under axial compression. The geometric parameters of the specimens were assumed to be: big radius-to-small radius ratio, $r_2/r_1 = 2.0$; small radius-to-thickness ratio, $r_1/t = 25$; axial length-to-big radius ratio, $L/r_2 = 2.24$; nominal wall thickness, $t = 1$ mm and cone angle, $\beta = 12.6^\circ$ as shown in Figure 1. The cones have imperfection amplitude, $A = 0.28$. To manufacture the specimens, several steps were followed. First of all, the samples were cut out to the desired dimension from the flat plate using laser cutting machine. Then, the cut out samples are rolled to conical shape using the conventional rolling machine. After the rolling process, the seam between the two neighbouring meridional free edges of the cone is welded together using Metal Inert Gas (MIG) welding process. During the manufacturing process, initial geometric imperfection having triangular and square waves along the compressed edge of the imperfect cones were introduced. Figure 2 provides photograph exemplifying manufactured specimens having triangular waves (Figure 2a) and square waves (Figure 2b). To ensure repeatability of experimental data, all cones were manufactured in pairs. The choice of introducing uneven length at the small radius end of the cone can be attributed to the fact that for conical shells the spread of plastic strain is concentrated within the small radius end of the cone [13, 14]. Next, six flat tensile coupons (three in the axial direction - V1, V2, V3 and another three in the lateral direction - H1, H2, H3) were cut from the same material from which the conical model are made. The design of the tensile coupon is according to British standard, [15]. All tensile specimens were tested, until they fail using Instron machine at the rate of 1 mm/min (see Ref. [11] for further details). The average material data obtained from experiment are Young's Modulus $E = 168.791$ GPa, yield stress, $\sigma_{yp} = 229.774$ MPa and Poisson ratio, $\nu = 0.3$. The Poisson's ratio of the material was assumed to be 0.3 (data taken from material data sheet).

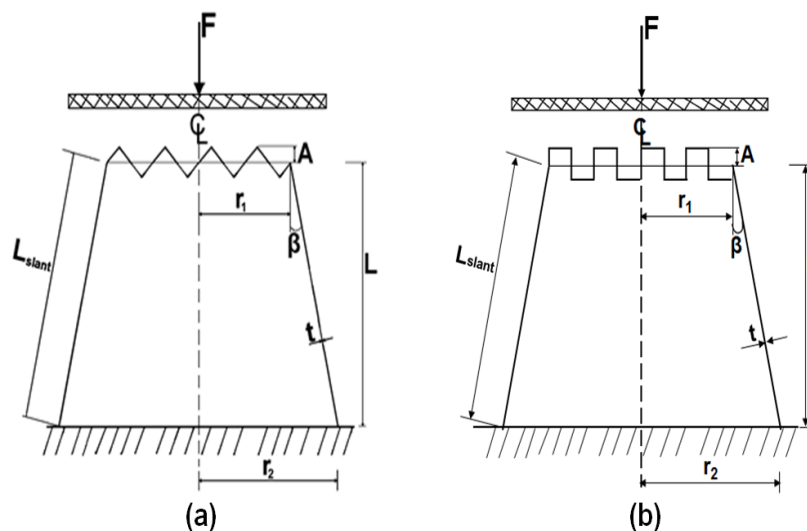


Figure 1: Geometry of the Analyzed Cone having non-uniform Length with Number of Waves, $N = 4$, in the form of Triangular wave (Figure 1a), and Square waves (Figure 1b).

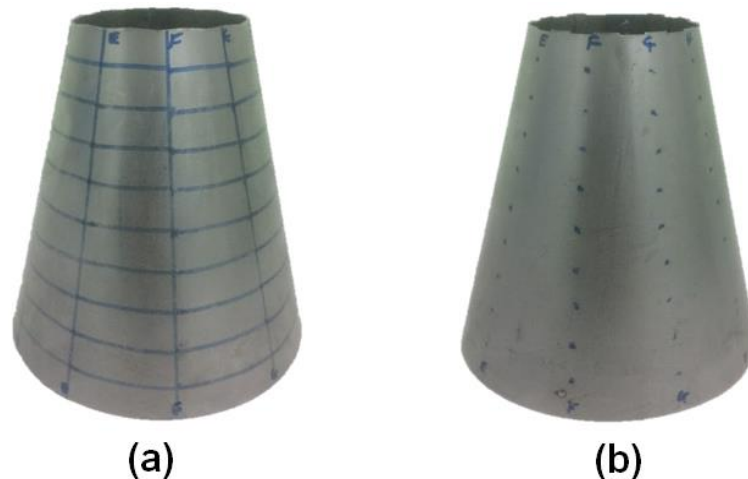


Figure 2: Photograph of Manufactured non-Uniform Length Cones with Number of Waves, $N = 12$, in the Form of Triangular Wave (Figure 2a), and Square Waves (Figure 2b).

Before testing, manufacture-induced imperfections were taken into consideration. A number of measurements (i.e., wall thickness, diameter, axial length and slant length) of all the conical models were taken. Firstly, the wall thickness of the cones was measured at eleven (11) equal points along the axial meridian using micrometer screw gage. This was then repeated along the circumference of the cone at 36° apart, resulting in $11 \times 10 = 110$ measuring points. The minimum thickness t_{\min} , maximum thickness t_{\max} , average thickness t_{ave} and the standard deviation t_{std} for all the manufactured cones are provided in Table 1.

Table 1: Measurements of all Tested Cones

Model	N	t_{\min}	t_{\max}	t_{ave}	$\overline{2r_1}$	$\overline{2r_2}$	\overline{L}	$\overline{L_{\text{slant}}}$	t_{std}	Wave shape
1	0	0.95	0.96	0.956	49.71	99.02	112.99	114.77	0.00497	Perfect
2	0	0.95	0.96	0.955	49.53	98.78	112.71	114.73	0.00500	Perfect
3	4	0.95	0.97	0.960	51.16	99.80	111.89	114.55	0.00573	Triangular
4	4	0.95	0.97	0.959	50.60	99.52	111.81	114.45	0.00536	Triangular
5	6	0.95	0.97	0.959	50.58	99.81	111.85	114.40	0.00471	Triangular
6	6	0.95	0.97	0.960	50.82	100.67	111.86	114.48	0.00531	Triangular
7	8	0.95	0.97	0.960	50.03	99.11	111.78	114.42	0.00483	Triangular
8	8	0.95	0.97	0.959	50.55	99.48	111.76	114.35	0.00523	Triangular
9	10	0.95	0.97	0.959	50.15	99.62	111.68	114.49	0.00395	Triangular
10	10	0.95	0.97	0.959	50.37	100.04	112.00	114.29	0.00554	Triangular
11	12	0.95	0.97	0.959	50.26	99.45	111.48	113.99	0.00428	Triangular
12	12	0.95	0.97	0.959	50.28	99.26	111.68	114.31	0.00518	Triangular
13	4	0.95	0.97	0.959	50.19	99.25	111.64	113.92	0.00509	Square
14	4	0.95	0.97	0.959	50.11	98.99	111.82	114.29	0.00489	Square
15	6	0.95	0.97	0.958	50.90	99.62	111.77	114.36	0.00530	Square
16	6	0.95	0.97	0.959	50.42	99.65	111.29	114.37	0.00489	Square
17	8	0.95	0.97	0.959	50.23	99.40	111.79	114.53	0.00509	Square
18	8	0.95	0.97	0.959	49.99	99.52	112.05	114.43	0.00535	Square
19	10	0.95	0.97	0.960	49.93	99.21	111.29	114.58	0.00447	Square
20	10	0.95	0.97	0.960	50.57	99.90	112.07	114.50	0.00391	Square
21	12	0.95	0.97	0.959	50.27	99.42	112.09	114.49	0.00412	Square
22	12	0.95	0.97	0.959	50.28	99.68	111.94	114.54	0.00489	Square

Secondly, the inner and outer diameters of the specimens were measured using digital Vernier caliper.

Measurements were taken at five equally spaced diameters at the top and bottom ends respectively. The average measured mid-surface diameter for all specimens are also given in Table 1. Thirdly, digital Vernier caliper was used to measure the axial length and slant length of the cones at eleven equal points. The measured average axial length and slant length can be found respectively in columns 8 and 9 in Table 1. These shape measurements were assumed to represent the most obvious characteristic of the specimens. Finally, all the twenty-two conical shells were subjected to axial compressive load using Instron machine. Prior to loading, the specimen was covered with top and bottom plate. It is expected that this will provide the same boundary condition previously employed for numerical prediction, i.e., cone is assumed to be fixed at the big radius ends, while the same condition was employed at the top ends except movement in the axial direction. An incremental load is applied to the cone at the rate of 1 mm/min (the same loading rate employed for the material testing). During the experiment, the axial load and its corresponding compression extension of the cones were recorded using the machine controller.

3. RESULTS AND DISCUSSIONS

Experimental results for twenty-two axially compressed perfect and imperfect conical models subjected to axial compression are presented in this section. Figures 3 and 4 depict the plot of average collapse load against number of waves for cones with triangular and square waves, respectively. The magnitude of the collapse load for all the tested cones is given in Table 2. From Figures 3 and 4, it is apparent that nominally identical conical shell failed with similar collapse load. Hence, confirming repeatability of experimental data. The errors in each pairs are indicated with error bar in Figures 3 and 4. The percentage error in collapse load within each pair were: 2% (model 1 vs 2), 4% (model 3 vs 4), 2% (model 5 vs 6), 1% (model 7 vs 8), 2% (model 9 vs 10), 3% (model 11 vs 12), 0% (model 13 vs 14), 0% (model 15 vs 16), 1% (model 17 vs 18), 1% (model 19 vs 20), and 1% (model 21 vs 22).

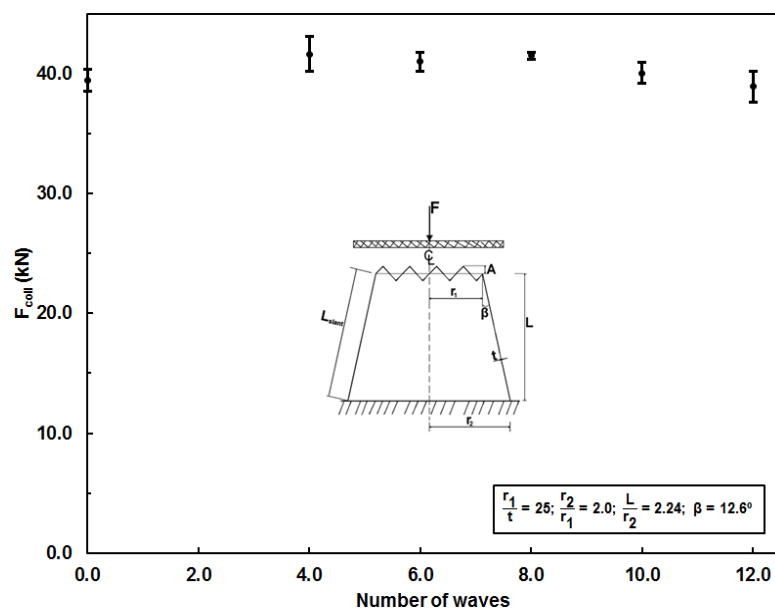


Figure 3: Plot of Average Collapse Force Versus Number of Waves for Cones with Triangular waves with Imperfection Amplitude, $A = 0.28$.

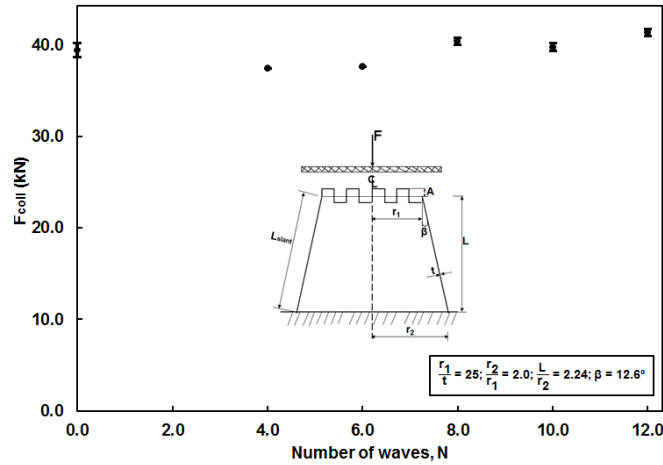


Figure 4: Plot of Average Collapse Force Versus Number of Waves for Cones with Square waves with Imperfection Amplitude, A = 0.28.

Table 2: Measurements of All Tested Cones

Model	N	Buckling Load of Cone (kN)	Wave shape
1	0	38.9944	Perfect
2	0	39.9061	Perfect
3	4	40.9108	Triangular
4	4	42.3461	Triangular
5	6	41.3938	Triangular
6	6	40.6120	Triangular
7	8	41.3638	Triangular
8	8	41.6715	Triangular
9	10	40.5003	Triangular
10	10	39.6463	Triangular
11	12	39.5320	Triangular
12	12	38.3001	Triangular
13	4	37.4852	Square
14	4	37.3869	Square
15	6	37.6064	Square
16	6	37.6480	Square
17	8	40.1560	Square
18	8	40.6531	Square
19	10	39.9595	Square
20	10	39.5825	Square
21	12	41.4835	Square
22	12	41.2421	Square

It can be seen that the largest error of 4% is found in conical shells with triangular waves, N = 4 – models 3 and 4 (see Figure 3) and the smallest error of 0% is observed twice in conical shells with square waves, N = 4 – models 13 and 14, and N = 6 – models 15 and 16 (see Figure 4). Generally, it can be said that repeatability result for cones with square waves was better than that of cones with triangular waves. Plot of experimental collapse load against compression extension for two nominally identical perfect cones (models 1 and 2) follow the same collapse paths at the pre-collapse and the post-collapse region – as can be seen in Figure 5. Again, there is very good agreement for both cones in terms of collapse load and the compression extension. A similar plot for imperfect cones having triangular and square waves with N = 12 is presented in Figures 6 and 7, respectively. Again, the failure path of both cones was the same. All cones fail

through bulging in the region where the compressed load is applied as exemplified for perfect and imperfect cones (with triangular and square waves, $N = 12$) in Figure 8. Furthermore, it is obvious that the collapse load of the perfect cones reduces with the introduction of non-uniform length. This is consistent with earlier results for cylinders having uneven length [5-7] and cones having sinusoidal waves [11, 12]. From Table 2, it can be noticed that imperfect cones with square waves are more sensitive to imperfection as compared to imperfect cones with triangular waves. This can be attributed to the fact that for the triangular waves, the shape helps to spread out the load, thereby being able to sustain more load. Again, it is obvious that increasing the wave number on the cone, results in minimal influence on the collapse load of the cone. Therefore, it can be said that increasing the wave number has a secondary effect on the collapse load of the cone. This is true for the case of cylindrical shells reported in [5].

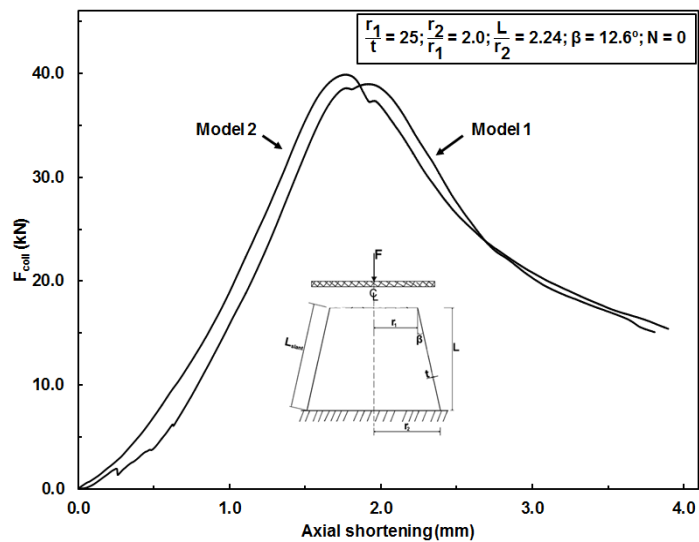


Figure 5: Plot of Axial Force Versus Compression Extension during Testing of Perfect Cones (Models 1 and 2).

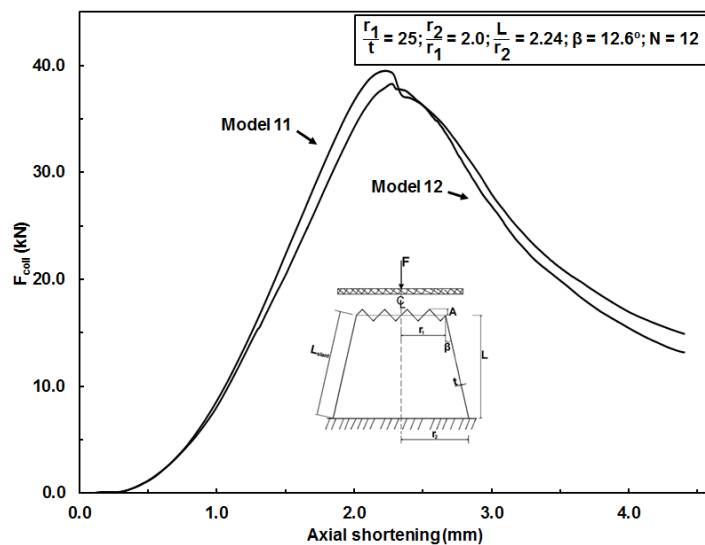


Figure 6: Plot of Axial Force versus Compression Extension during Testing of Imperfect cones with Triangular Waves, $N = 12$ (Models 11 and 12).

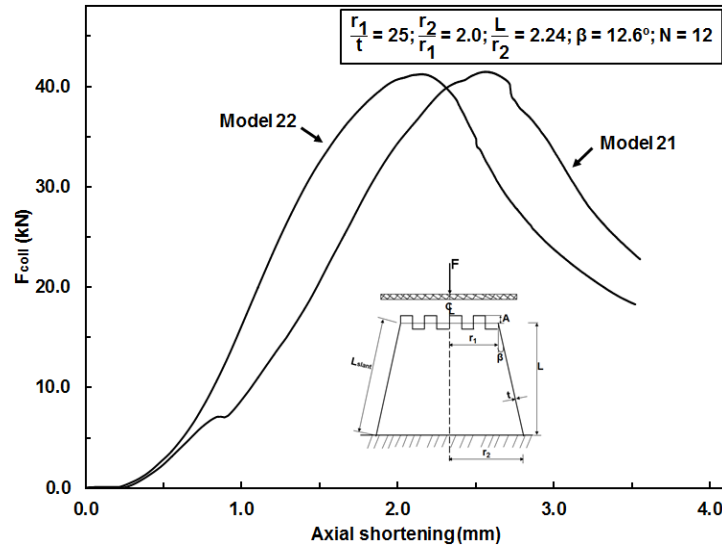


Figure 7: Plot of Axial force Versus Compression Extension during Testing of Imperfect Cones with Square Waves, N = 12 (Models 21 and 22).



Model 1, N=0

**Model 11, N=12
(Triangular)**

**Model 21, N=12
(Square)**

Figure 8: View of Collapsed Shape for Perfect and Imperfect Cones with A = 0.28 after Testing.

Finally, results of this current research are compared with the previous data on cones with imperfect axial length having sinusoidal profile [12] as shown in figure 9 (sinusoidal vs triangular) and 10 (sinusoidal vs square). It is obvious that cones having triangular waves are the most insensitive towards imperfection in comparison to other wave shapes. On the other hand, cones with square waves exhibit a different result. Two distinct regions are observed. At small wave number (i.e., $0 < N \leq 6$), cones having square profile showing more imperfection sensitivity but as the number of waves increases (i.e., $N > 6$), cones with sinusoidal waves tends to be more sensitive. This is mainly due to the ability of square waves to spread the axial loading, throughout the compressed edge of cone.

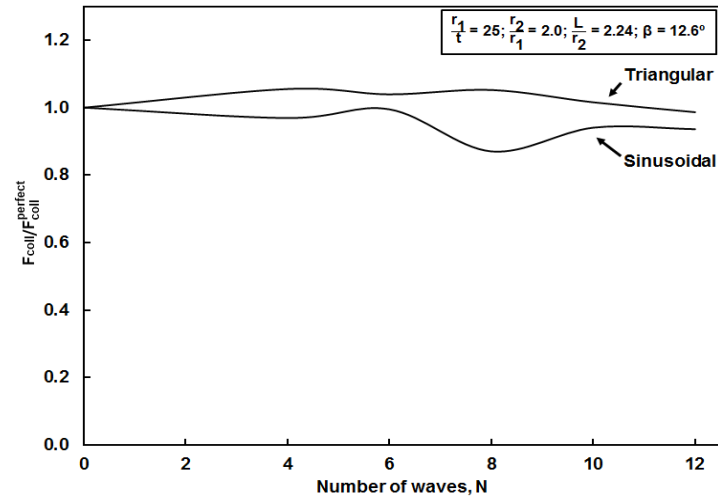


Figure 9: Plot of Average Experimental Buckling Load of Perfect and Imperfect Cones (sinusoidal and triangular waves) versus the Number of Waves.

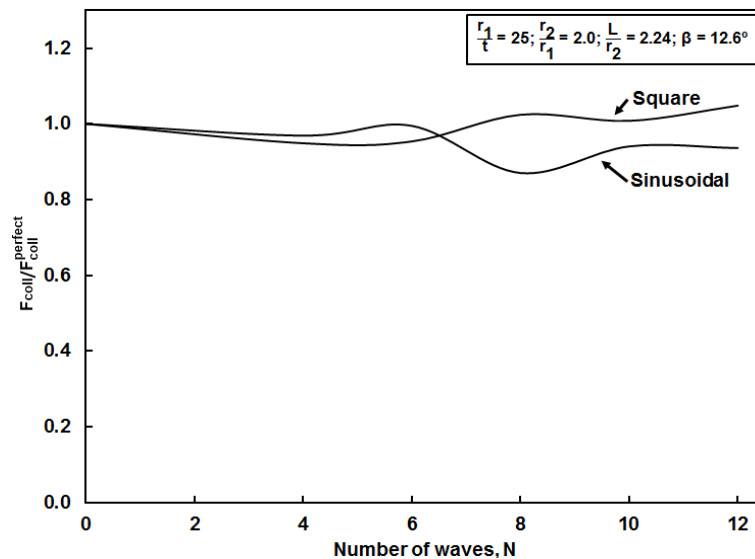


Figure 10: Plot of Average Experimental Buckling Load of Perfect and Imperfect Cones (Sinusoidal and Square Waves) versus the Number of Waves.

4. CONCLUSIONS

Results of axial compressive test on twenty-two conical model with non-uniform axial length were presented in this paper. Repeatability of experimental data was good. The results confirm the strong influence of non-uniform axial length on the load carrying capacity of the conical geometry considered. From the foregoing results, the following conclusions can be drawn: (i) the load carrying capacity of conical shell is strongly affected by the introduction of imperfect length, (ii) different wave shape and number of waves result in different load sustainability of cones; as an example, for a small number of waves, N (i.e., 0 – 6), cones with sinusoidal waves can sustain more load, however as the number of waves increases, cone with square waves can sustain more load. However, cones with triangular waves along its compressed edge can sustain more load as compared to other wave shapes.

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