

Grey wolf optimizer for identification of liquid slosh behavior using continuous-time Hammerstein model

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ABSTRACT

This paper presents the identification of liquid slosh plant using the Hammerstein model based on Grey Wolf Optimizer (GWO) method. A remote car that carrying a container of liquid is considered as the liquid slosh experimental rig. In contrast to other research works, this paper consider a piece-wise affine function in the nonlinear function of the Hammerstein model, which is more generalized function. Moreover, a continuous-time transfer function is utilized in the Hammerstein model, which is more suitable to represent a real system. The GWO method is used to tune both coefficients in the nonlinear function and transfer function of the Hammerstein model such that the error between the identified output and the real experimental output is minimized. The effectiveness of the proposed framework is assessed in terms of the convergence curve response, output response, and the stability of the identified model through the bode plot and pole zero map. The results show that the GWO based method is able to produce a Hammerstein model that yields identified output response close to the real experimental slosh output.

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1. INTRODUCTION

Nowadays, liquid slosh inside a cargo always happens in many situations. For example, ships with liquid container carriers are at high risk of generating sloshing load during operation [1]. In the metal industries, high oscillation can spill molten metal that is dangerous to the operator [2]. Meanwhile, sloshing of fuel and other liquids in moving vehicles may cause instability and undesired dynamics [3]. Hence, it is necessary to completely study the behavior of this residual slosh induced by the container motion. One may study the behavior of liquid slosh through developing the exact mathematical model of liquid slosh. So far, many researchers focus on the first principle approach to model the slosh behavior, while there are few literatures to discuss it from the perspective of nonlinear system identification approach.

On the other hand, block oriented nonlinear system identification has become a popular techniques to model a complex plants. The block oriented nonlinear model can be classified into three categories, which are Hammerstein model, Wiener model and Hammerstein Wiener model. In particular, Hammerstein model is a model that consists of a nonlinear function followed by linear dynamic sub-plant, while Wiener model consists of a linear dynamic sub-plant followed by nonlinear function, and finally, Hammerstein-Wiener model contains a linear dynamic sub-plant inserted between two or more nonlinear functions in series. Among these three block oriented models, Hammerstein model is famous due to its simple model structure

and it has been widely used for nonlinear system identification. Specifically, the Hammerstein model has been applied to model a real plant such as Solid Oxide fuel cell [4], bidirectional DC motor [5], oxygen uptake estimation [6], stretch reflex dynamics [7], turntable servo system [8], pneumatic muscle actuators [9], amplified piezoelectric actuators [10] and multi-axis piezoelectric micro positioning stages [11]. On the other hand, there are many tools that have been utilized to identify the Hammerstein model. There are the iterative method [12-14], the subspace method [15-17], the least square method [18], the blind approach [19] and the parametric instrumental variables method [20]. Moreover, many also consider the optimization tools for Hammerstein model, such as Bacterial Foraging algorithm [21], Cuckoo search algorithm [22], Particle Swarm optimization [23], and Genetic algorithm [24]. Based on the above literature, several limitations are ineluctable in their works, which are:

- a. Most of the Hammerstein models used in their study are based on discrete-time model, while many real plants can be easily represented in continuous-time model.
- b. Almost all the methods assume a known structure of nonlinear function, which consists of several basis functions.

Though, our proposed work can solve a more general class of continuous-time Hammerstein model by assuming an unknown structure of nonlinear function. In particular, a piece-wise affine function is adopted with so many basis functions. Due to the introduction of the piece-wise affine function, a high dimensional design parameter tuning is considered in this study, which make the identification problem more complex. On the other hand, Grey Wolf Optimizer (GWO) [25] has become a top notch optimization algorithm which has solved various types of engineering problems [25-27]. To the best of our knowledge, there are still few works to discuss on the GWO for identification of Hammerstein model. Thence, it motivates us to see the effectiveness of the GWO in modeling the liquid slosh plant from the real experimental data.

This paper presents the identification of liquid slosh plant using the Hammerstein model based on GWO method. A remote car carrying a container of liquid is considered as the liquid slosh experimental rig. The GWO method is used to tune both coefficients in the nonlinear function and transfer function of the Hammerstein model such that the error between the identified output and the real experimental output is minimized. The effectiveness of the proposed framework is assessed in terms of the convergence curve response, output response, and the stability of the identified model through the bode plot and pole zero map.

2. RESEARCH METHOD

This section explains on how the identification of liquid slosh based on Hammerstein model and GWO based method is performed. Firstly, the experimental setup of liquid slosh plant is explained. Here, it is shown on how the input and output data are generated. Secondly, we show the procedure to identify the model of liquid slosh plant using the Hammerstein based Grey Wolf Optimizer.

2.1. Liquid slosh experimental rig

In this study, a mobile liquid slosh plant is considered to replicate real situation of a moving container carrying liquid, as shown in Figure 1. In particular, a remote control car is used to carry a small tank filled with liquid. The tank is also equipped with four plastic wheels so that it can move smoothly as shown in Figure 1(a). Moreover, three accelerometer sensors (ADXL335) that are floated on the surface of liquid are used to measure liquid oscillation as shown in Figure 1(b). For simplicity of our study, the liquid slosh data from only one of the sensor is recorded and only z-axis output data is considered. Figure 2 shows a general schematic diagram of liquid slosh experimental rig.

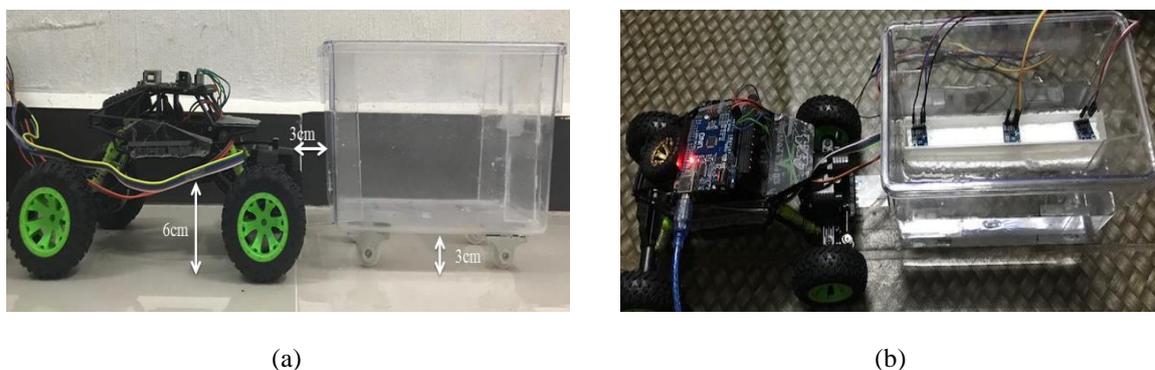


Figure 1. Liquid slosh experimental rig, (a) side view, (b) plan view

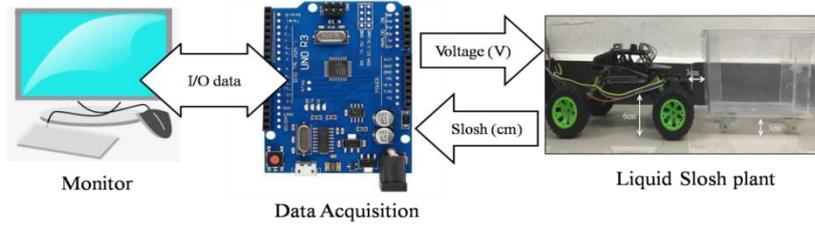


Figure 2. Schematic diagram of liquid slosh experimental rig

In particular, an Arduino UNO is used as a data acquisition platform to process the input and output data. Here, we generate a voltage from the Arduino UNO to the remote car and concurrently the Arduino UNO also will acquire the slosh data from the accelerometer. Both the input and output data can be monitored and analyzed from the personal computer using the LabView software. In order to identify the model of liquid slosh, the remote car is required to move to a certain distance and suddenly stop to generate a liquid oscillation or slosh inside the tank. Thence, we apply the input voltage as shown in Figure 3 to move the remote car. Concurrently, the liquid slosh data is recorded as shown in Figure 4. These two data are then used to develop the Hammerstein model based GWO, which is discussed in the next section.

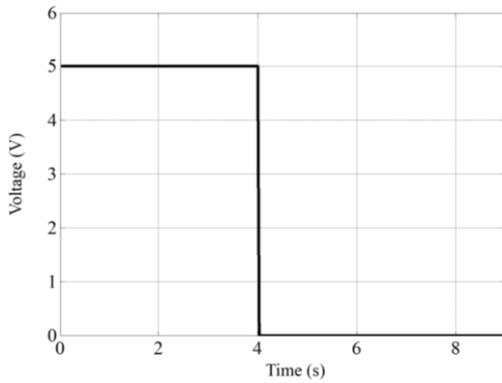


Figure 3. Input voltage applied to the remote car

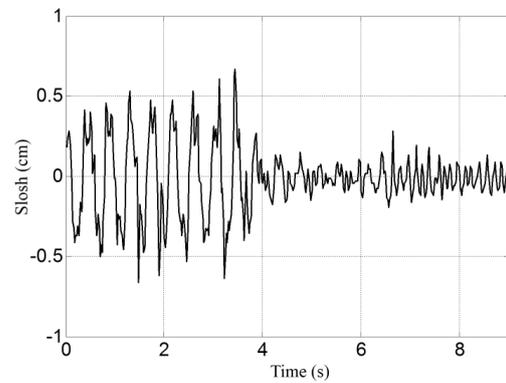


Figure 4. Output slosh from the accelerometer

2.2. Identification of liquid slosh using Hammerstein based grey wolf optimizer

In this section, the proposed Grey Wolf Optimizer (GWO) for identification of liquid slosh plant in Section 2.1 based on Hammerstein model is presented. Firstly, a problem formulation to identify the liquid slosh plant is explained. Then, it is shown on how to apply the GWO method to identify the liquid slosh based on Hammerstein model. Figure 5 shows a complete block diagram to identify the liquid slosh model in Section 2.1. The proposed Hammerstein model consists of nonlinear function $h(u)$ followed by the transfer function $G(s)$. The nonlinear function is a piece-wise affine function given by

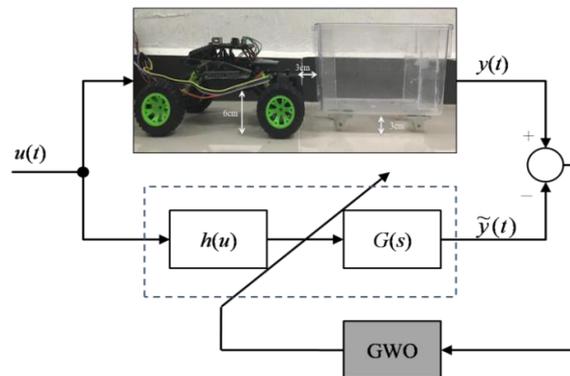


Figure 5. Block diagram of Hammerstein model based GWO

$$h(u) = \begin{cases} c_0 + m_1(u - d_0) & \text{if } d_0 \leq u < d_1 \\ c_1 + m_2(u - d_1) & \text{if } d_1 \leq u < d_2 \\ \vdots & \\ c_{\sigma-1} + m_{\sigma}(u - d_{\sigma-1}) & \text{if } d_{\sigma-1} \leq u < d_{\sigma} \end{cases} \quad (1)$$

and the transfer function $G(s)$ is given by

$$G(s) = \frac{B(s)}{A(s)} = \frac{s^m + b_{m-1}s^{m-1} + \dots + b_0}{a_m s^m + a_{m-1}s^{m-1} + \dots + a_0} \quad (2)$$

In (1), the symbol $m_i = (c_i - c_{i-1})/(d_i - d_{i-1}) (i = 1, 2, \dots, \sigma)$ are the segment slope with connecting input and output points as $d_i (i = 0, 1, \dots, \sigma)$ and $c_i (i = 0, 1, \dots, \sigma)$, respectively. For simplicity of notation, let $\mathbf{d} = [d_0, d_1, \dots, d_{\sigma}]^T$ and $\mathbf{c} = [c_0, c_1, \dots, c_{\sigma}]^T$. The input of the real liquid slosh plant and the identified model is defined by $u(t)$, while the output of the real liquid slosh plant and the identified model are denoted by $y(t)$ and $\tilde{y}(t)$, respectively. Thence, the expression of the identified output can be written as

$$\tilde{y}(t) = G(s)h(u(t)) \quad (3)$$

Moreover, several assumptions are adopted in this work, which are:

- a. The order of the polynomial $A(s)$ and $B(s)$ are assumed to be known
- b. The nonlinear function $h(u(t))$ is one-to-one map to the input $u(t)$ and the values of $d_i (i = 1, 2, \dots, \sigma)$ are pre-determined according to the response of input $u(t)$.

Next, let t_s be a sampling time for the real experimental input and output data ($u(t)$, $y(t)$) ($t=0, t_s, 2t_s, \dots, Nt_s$). Then, in order to accurately identify the liquid slosh model, the following objective function in (4) is adopted in this study:

$$E(G, h) = \sum_{\eta=0}^N (y(\eta t_s) - \tilde{y}(\eta t_s))^2 \quad (4)$$

Finally, our problem formulation can be described as follows.

Problem 1. Based on the given real experimental data ($u(t)$, $y(t)$) in Figure 1, find the nonlinear function $h(u)$ and the transfer function $G(s)$ such that the objective function in (4) is minimized.

Furthermore, it is shown on how to apply the GWO in solving **Problem 1**. For simplicity, let the design parameter of **Problem 1** is defined as $\mathbf{x} = [b_0 \ b_1 \ \dots \ b_{m-1} \ a_0 \ a_1 \ \dots \ a_m \ c_0 \ \dots \ c_{\sigma}]^T$, where the elements of the design parameter are the coefficients of both the nonlinear function and the transfer function of the continuous-time Hammerstein model. In GWO framework, let $x_i (i = 1, 2, \dots, M)$ be the design parameter of each agent i for M total number of agents. Then, consider $x_{ij} (j = 1, 2, \dots, D)$ be the j -th element of the vector $x_i (i = 1, 2, \dots, M)$, where D is the size of the design parameter. Thence, by adopting objective function in (4), a minimization problem is expressed as

$$\arg \min_{x_i(1), x_i(2), \dots} E(x_i(k)) \quad (5)$$

for iterations $k=1, 2, \dots$, until maximum iteration k_{\max} . Finally, the procedure of the GWO in solving **Problem 1** is shown below:

- **Step 1:** Determine the total number of agents M and the maximum iteration k_{\max} . Set $k=0$ and initialize the design parameter $x_i(0) (i = 1, 2, \dots, M)$ according to the upper bound \mathbf{x}_{up} and lower bound \mathbf{x}_{low} values of the design parameter.
- **Step 2:** Calculate the objective function in (4) for each search agent i .
- **Step 3:** Update the values of the best design parameter \mathbf{x}^{α} , the second best design parameter \mathbf{x}^{β} and the third best design parameter \mathbf{x}^{δ} based on the generated objective function in **Step 2**.
- **Step 4:** For each agent, update the design parameter using the following equation:

$$x_1(k+1) = \frac{x^1 + x^2 + x^3}{3} \quad (6)$$

where

$$\begin{aligned} x^1 &= x^{\alpha} - P \cdot |Q \cdot x^{\alpha} - x_i(k)|, x^2 = x^{\beta} - P \cdot |Q \cdot x^{\beta} - x_i(k)|, \\ x^3 &= x^{\delta} - P \cdot |Q \cdot x^{\delta} - x_i(k)| \end{aligned} \quad (7)$$

for $i=1, 2, \dots, M$. In (6), the vectors \mathbf{P} and \mathbf{Q} are expressed as follows:

$$\mathbf{P} = 2a \cdot \mathbf{r}_1 - a \quad (8)$$

$$\mathbf{Q} = 2 \cdot \mathbf{r}_2 \quad (9)$$

where \mathbf{r}_1 and \mathbf{r}_2 are the random vectors, where each element is generated independently from 0 to 1, and each element of a is linearly decreased from 2 to 0 over the course of iteration using the following equation:

$$a = 2 \left(1 - \frac{k}{k_{max}} \right) \quad (10)$$

Note that the vectors of \mathbf{P} and \mathbf{Q} in (8) and (9), which consists of random vectors \mathbf{r}_1 and \mathbf{r}_2 , are generated independently between \mathbf{x}^1 , \mathbf{x}^2 and \mathbf{x}^3 .

- **Step 5:** After the maximum iteration is achieved, record the best design parameter \mathbf{x}^a and obtained the continuous-time Hammerstein model in Figure 1. Otherwise, repeat **Step 2**.

3. RESULTS AND ANALYSIS

In this section, the effectiveness of the GWO based method for identifying the liquid slosh system using continuous-time Hammerstein model is demonstrated. In particular, the convergence curve response of the objective function in (4), the bode plot and pole-zero mapping of linear function and the plot of nonlinear function, will be presented and analyzed in this study.

Based on the experimental setup in Section 2.1, the input response $u(t)$ as shown in Figure 3 is applied to the liquid slosh plant, and the output response $y(t)$ is recorded as shown in Figure 4. Here, the input and output data are sampled at $t_s=0.02$ for $N=450$. In this study, the structure of $G(s)$ is selected as follows:

$$G(s) = \frac{B(s)}{A(s)} = \frac{s^3 + b_2s^2 + b_1s + b_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0} \quad (11)$$

After performing several preliminary testing on the given data ($u(t)$, $y(t)$). Meanwhile, the input points for piece-wise affine function of $h(u(t))$ are given by $\mathbf{d}=[0, 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5]^T$. The design parameter $\mathbf{x} \in R^{18}$ with its corresponding transfer function and nonlinear function is shown in Table 1. Next, the GWO algorithm is applied to tune the design parameter with initial values of design parameter are randomly selected between the upper bound \mathbf{x}_{up} and lower bound \mathbf{x}_{low} as shown in Table 1. Here, we choose the number of agents $M=40$ with maximum iterations $k_{max}=5000$.

Table 1. Design parameter of liquid slosh plant

\mathbf{x}	Coefficients	\mathbf{x}_{low}	\mathbf{x}_{up}	\mathbf{x}^a
x_1	b_2	-5	35	-4.8017
x_2	b_1	-5	35	0.2657
x_3	b_0	-5	35	-0.0243
x_4	a_4	-5	35	-0.1119
x_5	a_3	-2200	-1	-4.3214
x_6	a_2	-2200	-1	-31.6384
x_7	a_1	-2200	-1	-901.2917
x_8	a_0	-2200	-1	-1280.4483
x_9	c_0	-5	5	-1.9997
x_{10}	c_1	-5	5	0.3249
x_{11}	c_2	-5	5	1.5968
x_{12}	c_3	-5	5	-0.0757
x_{13}	c_4	-5	5	-0.0099
x_{14}	c_5	-5	5	-0.0048
x_{15}	c_6	-5	5	-0.0251
x_{16}	c_7	-5	5	0.0584
x_{17}	c_8	-5	5	0.0216
x_{18}	c_9	-5	5	-2.2603

Figure 6 shows the response of the objective function convergence after 5000 iterations to produce the best design parameter as shown in the final column of Table 1. It shows that the GWO based method is able to minimize the objective function in (4) and produce a quite close output response $y(t)$ as compared to the real output $\tilde{y}(t)$, which can be clearly seen in Figure 7. Note that the identified output response tends to yield high oscillation when input is injected to the system and it start to attenuate when the input is zero, which is quite similar to the response of real experimental output.

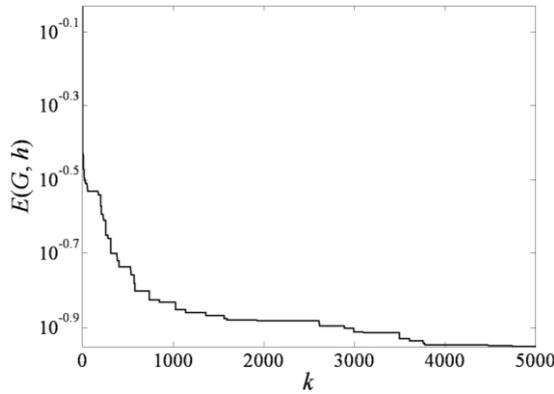


Figure 6. Convergence curve response

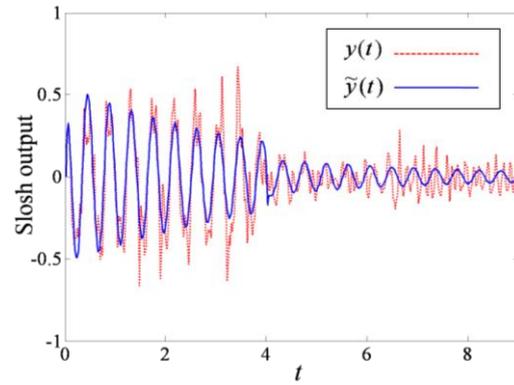


Figure 7. Response of the identified output $\tilde{y}(t)$ and real output $y(t)$

In the real experimental setup, we can say that the liquid slosh system is stable since the liquid slosh output is reduced gradually as $t \rightarrow \infty$. In order to validate our model regarding the stability, we use the bode diagram and the pole-zero map of the identified transfer function $G(s)$ as shown in Figures 8 and 9, respectively. From the bode diagram, it can be observed that a gain margin of 2.56 dB at 17.4 rad/s and phase margin of 4.18 degree at 16.5 rad/s are produced, which indicates that the linear system is stable. This is also supported by the pole-zero map where all the poles are located at the left hand side of y-axis. In particular, the obtained values of poles are -36.6623, $-0.2344 \pm j14.5053$ and -1.4828 , while the obtained values of zeros are 4.7468, $0.0274 \pm j0.0661$. On the other hand, we also can observe the feature of nonlinear function by plotting the obtained piece-wise function as depicted in Figure 10. Note that our nonlinear function is not restricted to any form of nonlinear function (i.e., quadratic), which is more generalized and provide more flexibility of searching a justifiable function.

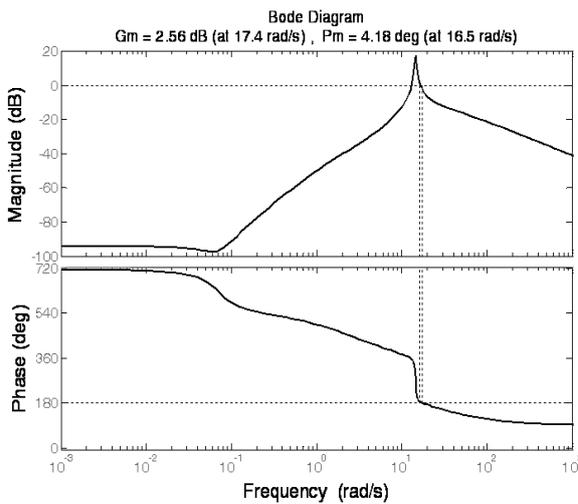


Figure 8. Bode plot of transfer function $G(s)$

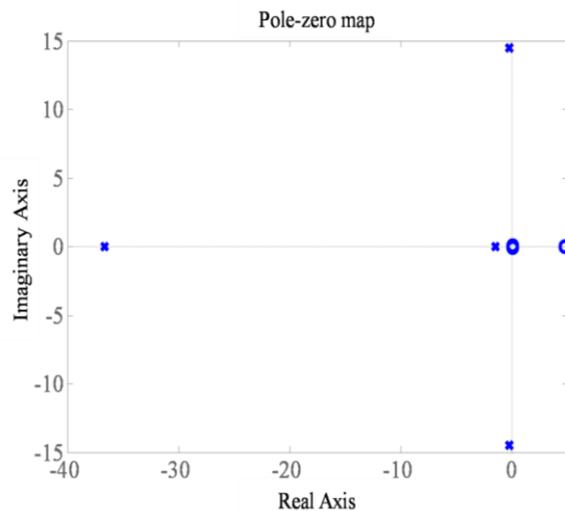


Figure 9. Pole-zero map of transfer function $G(s)$

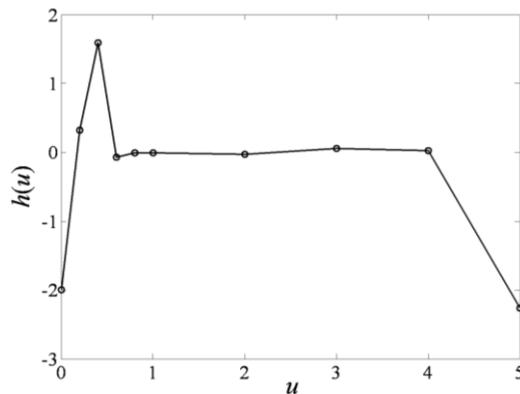


Figure 10. Resultant of piece-wise affine function $h(u)$

4. CONCLUSION

In this paper, an identification of liquid slosh plant using continuous-time Hammerstein model based on Grey Wolf Optimizer (GWO) has been presented. The results demonstrated that the proposed generic Hammerstein model based on GWO has a good potential in identifying the real liquid slosh behavior. In particular, it is shown that the proposed method is able to produce a quite close identified output with real liquid slosh output. Moreover, the resultant linear model has been proved to be stable based on the bode plot and pole-zero map. It is also shown that the used of piecewise-affine function gives more flexibility for the GWO to search more generic nonlinear function. In the future, our work can be extended to various types of nonlinear function such as continuous-time Wiener and Hammerstein-Wiener.

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