Menemui Matematik (Discovering Mathematics)
Vol. 42, No. 1: 34-44 (2020)

Mode Stresses for a Thermally Insulated Curve Crack in Bonded Two Half Planes

KB Hamzah[1], NMA Nik Long[2], N Senu[3] and ZK Eshkuvatov[4]
[1,2,3]*Laboratory of Computational Sciences and Mathematical Physics, Institute for Mathematical Research, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia*
[2,3]*Department of Mathematics, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia*
[4]*Faculty of Science and technology, Universiti sains Islam Malaysia, 71800 Negeri Sembilan, Malaysia*
[1]*Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia*
[1]khairum@utem.edu.my, [2]nmasri@upm.edu.my, [3]norazak@upm.edu.my, [4]zainidin@usim.edu.my

ABSTRACT

Modifified complex potential (MCP) function was applied to formulate the new hypersingular integral equations (HSIEs) for a thermally insulated curve crack in the upper side of bonded two half planes subjected to various mode stresses by the assist from conditions of continuity for the displacement, resultant force and heat conduction functions. This new HSIEs is solve numerically by using curve length coordinate function and quadrature formulas for the unknown crack opening displacement (COD) function and the right hand side is the traction along the crack. The obtained COD function is then used to compute the stress intensity factors (SIF) in order to analyze the stability behavior of the materials containing cracks or flaws. Numerical results presented the behavior of nondimensional SIF at tips subjected to four mode stresses such as Mode I (Normal), Mode II (Shear), Mode III (Tearing) and Mix Mode stresses.

Keywords: Thermally insulated crack, Bonded two half planes, Hypersingular integral equation, Stress intensity factor

INTRODUCTION

The emergence of crack in the engineering structure effecst their stability, strength and life cycle. This situation worsen when the structures are exposed to the thermal. Many researchers have interested and published their works on the formulation and investigation of the thermally insulated crack problems in an elastic half plane (El, 2008; Sourki, 2016), infinite plane (Chen, 2003; Deng, 2019) and bonded half planes (Lee, 1991; Chao, 1995; Wang, 2000) subjected to remote mode stress.

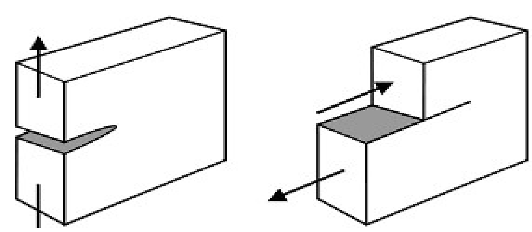


Figure 1: Three types of mode stresses in fracture mechanics

There are three types of mode stresses in fracture mechanics such as Mode I (Normal), Mode II (Shear) and Mode III (Tearing) stresses. Mode I stress is a normal opening mode, whereas Mode II and Mode III stresses are shear sliding modes as shows in Fig. 1 (Shields et al., 1992). A crack faces in the plane can be exposed in any one of these three modes stresses or a mix of these three modes.

Nondimensional thermal SIF for a crack lies in the boundary of bonded two half planes with vertical uniform heat flow and traction free conditions was calculated by using complex variable approach (Lee, 1991). The singular integral equations were derived and obtained numerical results by using the appropriate interpolation formulas for two dimensional thermally insulated crack problems in bonded two half planes (Chao, 1995). The system of multiple thermally cracks problems in bonded two half planes was solved by utilized a laminated composite plate model based on Fourier and Laplace transforms (Wang, 2000). The interaction between internal defects and a crack lies in the boundary of bonded two half planes exposed to a thermal loading and a particular heat source was analyzed by using singular integral equations (Petrova, 2004). Nondimensional SIF for an edge crack straight-up to the boundary of bonded two half planes subjected to a convective cooling on the surface was calculated by using singular integral equation method (Rizk, 2008). The rational mapping function and a complex variable approach were used to determine the SIF for a crack lies in the boundary of bonded two half planes subjected to the temperature and described the relationship between stress and temperature (Hasebe, 2014). The Fredholm integral equations was used to calculate nondimensional SIF at tips of crack for a crack lies in the boundary of bonded two thermoelastic half planes subjected to a uniform heat flux (Mishra, 2018).

This paper analyze the behavior of nondimensional SIF for a thermally insulated curve crack in the upper side of bonded two half planes subjected to the various mode stresses; Mode I (Normal), Mode II (Shear), Mode III (Tearing) and Mix Mode stresses.

MATHEMATICAL FORMULATION

Muskhelishvili (1953) introduced the complex potentials for heat conduction problem in terms of temperature distribution $T(x,y)=\mathrm{Re}[\omega(\eta)]$ and resultant heat flux function $Q(x,y)=-k\,\mathrm{Im}[\omega(\eta)]$ in terms of heat conductivity k as follows

$$\omega(\eta)=T(x,y)+iQ(x,y). \tag{1}$$

The stress components $(\sigma_x,\sigma_y,\sigma_{xy})$, resultant force (X,Y), and displacements functions (u,v) for a thermally insulated crack in an elastic infinite plane are denoted in terms of three complex potential functions $\varphi(\eta)$, $\psi(\eta)$ and $\omega(\eta)$ as follows

$$\sigma_y-\sigma_x+2i\sigma_{xy}=2\left[\overline{\eta}\varphi''(\eta)+\psi'(\eta)\right], \tag{2}$$

$$i(X+iY)=\varphi(\eta)+\eta\overline{\varphi'(\eta)}+\overline{\psi(\eta)}, \tag{3}$$

$$2G(u+iv)=\kappa\varphi(\eta)-\eta\overline{\varphi'(\eta)}-\overline{\psi(\eta)}+(1+\kappa)G\beta\int\omega(\eta)d\eta\,, \tag{4}$$

where G is shear modulus, $\kappa=(3-v)/(1+v)$ and $\beta=(1+v)\mu/2$ for plane stress, $\kappa=3-4v$ and $\beta=(1+v)\mu/2(1-v)$ for plane strain, v is Poisson's ratio and μ is the thermal expansion coefficient. The overbar stands for complex conjugation. The derivation of resultant force function (3) with respect to η gives

$$\frac{d}{d\eta}\{i(X+iY)\}=\varphi'(\eta)+\overline{\varphi'(\eta)}+\frac{d\overline{\eta}}{d\eta}\left[\eta\overline{\varphi''(\eta)}+\overline{\psi'(\eta)}\right]=N+iT\,, \tag{5}$$

where the traction along the segment $\overline{\eta,\eta+d\eta}$ is in terms of normal (*N*) and tangential (*T*) components.

Complex potentials for a crack *L* in an infinite thermoelastic modeled by the distribution of COD function, $g(\tau)$, and the temperature jump along the crack faces, $\gamma(\tau)$ are denoted by (Nik Long and Eshkuvatov, 2009; Chen et al., 2003)

$$\varphi(\eta)=\frac{1}{2\pi}\int_L \frac{1}{\tau-\eta}g(\tau)d\tau\,, \tag{6}$$

$$\psi(\eta)=\frac{1}{2\pi}\int_L \frac{1}{\tau-\eta}\overline{g(\tau)}d\tau+\frac{1}{2\pi}\int_L g(\tau)\left(\frac{1}{\tau-\eta}d\bar{\tau}-\frac{\bar{\tau}}{(\tau-\eta)^2}d\tau\right), \tag{7}$$

$$\omega(\eta)=\frac{1}{i\pi}\int_L \frac{1}{\tau-\eta}\gamma(\tau)d\tau\,, \tag{8}$$

where $g(\tau)$ is interpreted by

$$g(\tau)i(\kappa+1)=2G\left[(u(\tau)+iv(\tau))^+-(u(\tau)+iv(\tau))^-\right],\quad \tau\in L \tag{9}$$

$(u(\tau)+iv(\tau))^+$ and $(u(\tau)+iv(\tau))^-$ denote the displacements at point τ of the upper and lower crack faces, respectively. The dislocation distribution $g'(\tau)$ is related to the temperature jump and COD as follows (Chen et al., 2003)

$$g'(\tau)=g(\tau)-2i\beta\gamma(\tau). \tag{10}$$

In addition, the single-valuedness condition for the displacement as follows

$$\int_L g'(\tau)d\tau=0\,. \tag{11}$$

Apply Eq. (11) into (10), yields

$$\int_L \gamma(\tau)d\tau=\frac{-i}{2\beta}\int_L g(\tau)d\tau\,. \tag{12}$$

Substituting Eq. (12) into (8) to represent the complex potential of the temperature jump in terms of COD as follows

$$\omega(\eta)=\frac{-1}{2\pi\beta}\int_L \frac{1}{\tau-\eta}g(\tau)d\tau\,. \tag{13}$$

The condition for the stress components in the upper, ε_{x_1}, and lower sides, ε_{x_2}, of bonded two half planes in terms of Young's modulus of elasticity in the upper, $E_1=2G_1(1+\nu_1)$ and lower sides, $E_2 = 2G_2(1+v_2)$ by consider only one mode stress and all other stresses do not exist, then Mode I stress component is reduced to

$$\begin{aligned} \varepsilon_{y_1} &= \varepsilon_{y_2} \\ \frac{1}{E_1}\sigma_{y_1} &= \frac{1}{E_2}\sigma_{y_2} \end{aligned} \tag{14}$$

for Mode II stress component is reduced to

$$\begin{aligned} \varepsilon_{x_1} &= \varepsilon_{x_2} \\ \frac{1}{E_1}\sigma_{x_1} &= \frac{1}{E_2}\sigma_{x_2} \end{aligned} \tag{15}$$

for Mode III stress component is reduced to

$$\begin{aligned} \varepsilon_{x_1 y_1} &= \varepsilon_{x_1 y_2} \\ \frac{1+v_1}{E_1}\sigma_{x_1 y_1} &= \frac{1+v_2}{E_2}\sigma_{x_2 y_2} \end{aligned} \tag{16}$$

whereas for Mix Mode stress component is reduced to

$$\begin{aligned} \varepsilon_{x_1} = \varepsilon_{x_2} &= \varepsilon_{y_1} = \varepsilon_{y_2} \\ \frac{1-v_1}{E_1}\sigma_{x_1} = \frac{1-v_2}{E_2}\sigma_{x_2} &= \frac{1-v_1}{E_1}\sigma_{y_1} = \frac{1-v_2}{E_2}\sigma_{y_2}. \end{aligned} \tag{17}$$

The MCP method for the crack in the upper side of bonded two half planes involve principal $(\omega_{1p}(\eta), \varphi_{1p}(\eta), \psi_{1p}(\eta))$ and complementary parts $(\omega_{1c}(\eta), \varphi_{1c}(\eta), \psi_{1c}(\eta))$ of the complex potentials defined as

$$\omega_1(\eta) = \omega_{1p}(\eta) + \omega_{1c}(\eta), \tag{18}$$

$$\varphi_1(\eta) = \varphi_{1p}(\eta) + \varphi_{1c}(\eta), \tag{19}$$

$$\psi_1(\eta) = \psi_{1p}(\eta) + \psi_{1c}(\eta). \tag{20}$$

Whereas for a crack in the lower side, the complex potentials are denoted by ω_2, φ_2 and ψ_2. Note that the principal parts of complex potentials are equal to the complex potentials for the thermally insulated cracks in an elastic infinite plane.

Since the resultant heat flux and temperature are continuous across the boundary, the heat conduction problem (1), yields

$$\left[\omega_1(\tau) + \overline{\omega_1(\tau)}\right]^+ = \left[\omega_2(\tau) + \overline{\omega_2(\tau)}\right]^-, \quad \tau \in L, \tag{21}$$

$$k_1\left[\omega_1(\tau) - \overline{\omega_1(\tau)}\right]^+ = k_2\left[\omega_2(\tau) - \overline{\omega_2(\tau)}\right]^-, \quad \tau \in L. \tag{22}$$

Applying Eq. (18) into Eqs. (21) and (22), the following expressions are obtained

$$\omega_{1c}(\eta)=\frac{k_1-k_2}{k_1+k_2}\overline{\omega_{1p}}(\eta), \quad \omega_2(\eta)=\frac{2k_1}{k_1+k_2}\omega_{1p}(\eta), \tag{23}$$

where $\overline{\omega_{1p}}(\eta)=\overline{\omega_{1p}(\bar{\eta})}$, L_b is boundary of bonded two half planes, S_1 and S_2 are upper and lower sides of bonded two half planes, respectively. Whereas the conditions of continuity for resultant force (3) and displacement (4) are defined as

$$\left[\varphi_1(\tau)+\tau\overline{\varphi'_1(\tau)}+\overline{\psi_1(\tau)}\right]^+=\left[\varphi_2(\tau)+\tau\overline{\varphi'_2(\tau)}+\overline{\psi_2(\tau)}\right]^-, \tag{24}$$

$$\begin{aligned} &G_2\left[\kappa_1\varphi_1(\tau)-\tau\overline{\varphi_1'(\tau)}-\overline{\psi_1(\tau)}+(1+\kappa_1)G_1\beta_1\int\omega_1(\tau)d\tau\right]^+ \\ &\quad=G_1\left[\kappa_2\varphi_2(\tau)-\tau\overline{\varphi_2'(\tau)}-\overline{\psi_2(\tau)}+(1+\kappa_2)G_2\beta_2\int\omega_2(\tau)d\tau\right]^- \end{aligned} \tag{25}$$

where $\tau\in L$. Applying Eqs. (18), (19), (20) and (23) into Eqs. (24) and (25), the following expressions are obtained

$$\varphi_{1c}(\eta)=\Gamma_1\left[\eta\overline{\varphi'_{1p}}(\eta)+\overline{\psi_{1p}}(\eta)\right]+\Gamma_2\int_{L_1}\overline{\omega_{1p}}(\eta)d\eta, \tag{26}$$

$$\psi_{1c}(\eta)=\Gamma_3\overline{\varphi_{1p}}(\eta)-\eta\varphi'_{1c}(\eta)+\Gamma_4\left[(1+\kappa_1)\beta_1\int_{L_1}\overline{\omega_{1p}}(\eta)d\eta-\frac{2(1+\kappa_2)k_1\beta_2}{k_1+k_2}\int_{L_2}\overline{\omega_{1p}}(\eta)d\eta\right], \tag{27}$$

$$\varphi_2(\eta)=\Gamma_5\varphi_{1p}(\eta)+\Gamma_4\left[(1+\kappa_1)\beta_1\int_{L_1}\omega_{1p}(\eta)d\eta-\frac{2(1+\kappa_2)k_1\beta_2}{k_1+k_2}\int_{L_2}\omega_{1p}(\eta)d\eta\right], \tag{28}$$

$$\psi_2(\eta)=\Gamma_6\left[\eta\varphi'_{1p}(\eta)+\psi_{1p}(\eta)\right]-\eta\varphi'_2(\eta)+\Gamma_2\int_{L_1}\omega_{1p}(\eta)d\eta, \tag{29}$$

where Γ_1, Γ_2, Γ_3, Γ_4, Γ_5, Γ_6 are bi-elastic constants denoted as

$$\Gamma_1=\frac{G_2-G_1}{G_1+\kappa_1G_2}, \Gamma_2=\frac{(1+\kappa_1)G_1G_2\beta_1}{G_1+\kappa_1G_2}\frac{k_2-k_1}{k_1+k_2}, \Gamma_3=\frac{\kappa_1G_2-\kappa_2G_1}{G_2+\kappa_2G_1},$$
$$\Gamma_4=\frac{G_1G_2}{G_2+\kappa_2G_1}, \Gamma_5=\frac{(1+\kappa_1)G_2}{G_2+\kappa_2G_1}, \Gamma_6=\frac{(1+\kappa_1)G_2}{G_1+\kappa_1G_2}.$$

The principal part of the traction for a thermally insulated crack in the upper side of bonded two half planes can be obtained by substituting Eqs. (6) and (7) into (5). Then by imposing point η closer to τ_o on the crack and setting $d\bar{\eta}/d\eta$ into $d\bar{\tau}_o/d\tau_o$ gives

$$\{N(\tau_o)+iT(\tau_o)\}_{1p}=\frac{1}{\pi}\int_L\frac{1}{(\tau-\tau_o)^2}g(\tau)d\tau+\frac{1}{2\pi}\int_L B_1(\tau,\tau_o)g(\tau)d\tau+\frac{1}{2\pi}\int_L B_2(\tau,\tau_o)\overline{g(\tau)}d\bar{\tau}, \tag{30}$$

where

$$B_1(\tau,\tau_o)=\left[\frac{(\tau-\tau_o)^2}{(\bar{\tau}-\bar{\tau}_o)^2}\frac{d\bar{\tau}}{d\tau}\frac{d\bar{\tau}_o}{d\tau_o}-1\right]\frac{1}{(\tau-\tau_o)^2}, B_2(\tau,\tau_o)=\left[\frac{(\bar{\tau}-\bar{\tau}_o)}{(\tau-\tau_o)}\left(\frac{d\bar{\tau}}{d\tau}+\frac{d\bar{\tau}_o}{d\tau_o}\right)-2\frac{d\bar{\tau}}{d\tau}\frac{d\bar{\tau}_o}{d\tau_o}\right]\frac{(\tau-\tau_o)}{(\bar{\tau}-\bar{\tau}_o)^3}$$

For the complementary part, substitutes Eqs. (26) and (27) into (5) and applying (6), (7) and (13). Then by imposing point η closer to τ_o on the crack and setting $d\bar{\eta}/d\eta$ into $d\bar{\tau}_o/d\tau_o$ gives

$$\{N(\tau_o)+iT(\tau_o)\}_{1c}=\frac{1}{2\pi}\int_L E_1(\tau,\tau_o)g(\tau)d\tau+\frac{1}{2\pi}\int_L E_2(\tau,\tau_o)\overline{g(\tau)}d\bar{\tau}\,, \qquad (31)$$

where

$$E_1(\tau,\tau_o)=\Gamma_1\frac{1}{(\tau-\bar{\tau}_o)^4}\Bigg[(\tau-\bar{\tau}_o)^2+2(\bar{\tau}_o-\bar{\tau})(\tau-\bar{\tau}_o)+(\tau-\bar{\tau}_o)^2\frac{d\bar{\tau}}{d\tau}-\left(\left((\tau-\bar{\tau}_o)^2+2(\bar{\tau}_o-\tau_0)(\tau-\bar{\tau}_o)\right)\frac{d\bar{\tau}}{d\tau}\right.$$
$$\left.-2(2\tau_o-3\bar{\tau}_o+\bar{\tau})(\tau-\bar{\tau}_o)+6(\bar{\tau}_o-\bar{\tau})(\bar{\tau}_o-\tau_0)+(\tau-\bar{\tau}_o)^2\right)\frac{d\bar{\tau}_o}{d\tau_o}\Bigg]+\Gamma_1\frac{1}{(\bar{\tau}-\tau_0)^2}\frac{d\bar{\tau}}{d\tau}+\Gamma_3\frac{d\bar{\tau}_o}{d\tau_o}\frac{1}{(\tau-\bar{\tau}_o)^2}$$
$$-\frac{1}{\beta_1}\frac{1}{(\tau-\bar{\tau}_o)^2}\left[\Gamma_2\left((\tau-\bar{\tau}_o)-(\tau-\tau_o)\frac{d\bar{\tau}_o}{d\tau_o}\right)+\Gamma_4\left(\beta_1(1+\kappa_1)(\tau-\bar{\tau}_o)-\frac{2(1+\kappa_2)k_1\beta_2}{k_1+k_2}(\tau-\bar{\tau}_o)\right)\frac{d\bar{\tau}_o}{d\tau_o}\right]$$

$$E_2(\tau,\tau_o)=\Gamma_1\left[\frac{1}{(\bar{\tau}-\tau_0)^3}\left((\bar{\tau}-\tau_0)+(\bar{\tau}+\tau_0-2\tau)\frac{d\bar{\tau}}{d\tau}\right)+\frac{1}{(\tau-\bar{\tau}_o)^3}\left((\tau-\bar{\tau}_o)-(\tau-2\tau_0+\bar{\tau}_o)\frac{d\bar{\tau}_o}{d\tau_o}\right)\right]$$
$$+\Gamma_2\frac{1}{\beta_1}\frac{1}{\bar{\tau}-\tau_0}\frac{d\bar{\tau}}{d\tau}$$

Summing Eqs. (30) and (31) yields the following HSIEs for a thermally insulated crack in the upper side of bonded two half planes

$$\{N(\tau_o)+iT(\tau_o)\}=\{N(\tau_o)+iT(\tau_o)\}_{1p}+\{N(\tau_o)+iT(\tau_o)\}_{1c}$$
$$=\frac{1}{\pi}\int_L\frac{1}{(\tau-\tau_o)^2}g(\tau)d\tau+\frac{1}{2\pi}\int_L[B_1(\tau,\tau_o)+E_1(\tau,\tau_o)]g(\tau)d\tau\,, \qquad (32)$$
$$+\frac{1}{2\pi}\int_L[B_2(\tau,\tau_o)+E_2(\tau,\tau_o)]\overline{g(\tau)}d\bar{\tau}.$$

Note that the first integral in Eq. (32) with the equal sign represents the hypersingular integral and must be denoted as a finite part integral (Nik Long and Eshkuvatov, 2009). If $k_2=0$ and $G_2=0$, Eq. (32) reduces to the equation for a crack in an elastic half plane (Chen et al., 2009). If $k_1=k_2$, $\mu_1=\mu_2$ and $G_1=G_2$, Eq. (32) reduces to the equation for a crack in an elastic infinite plane (Rafar et al., 2017).

For solving the HSIEs (32), used the curved length coordinate function to map the COD function $g(\tau)$ on a real axis *s* with an interval 2*a* as below

$$g(\tau)\big|_{\tau=\tau(s)}=\sqrt{a^2-s^2}H(s),\quad (H(s)=H_1(s)+iH_2(s)). \qquad (33)$$

Then we apply the quadrature formulas introduced by Mayrhofer and Fischer (1992).

NUMERICAL RESULTS

SIF at the tips A_j is interpreted as

$$K_{A_j} = \left(K_1 - iK_2\right)_{A_j} = \sqrt{2\pi} \lim_{\tau \to \tau_{A_j}} \sqrt{\left|\tau - \tau_{A_j}\right|} g'(\tau) = \sqrt{a\pi} F_{A_j} \tag{34}$$

where $j = 1, 2$ and $F_{A_j} = F_{1A_j} + iF_{2A_j}$ is the nondimensional SIF at tips A_j.

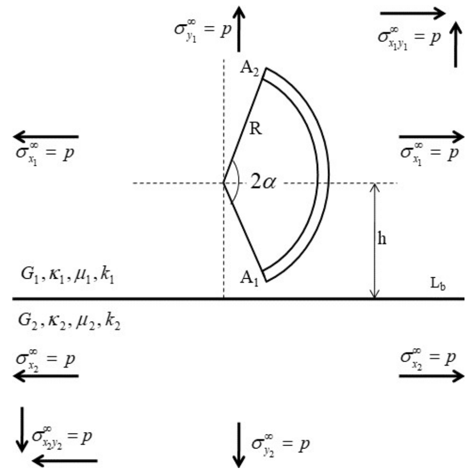


Figure 2: A thermally insulated curve crack in the upper side of bonded two half planes

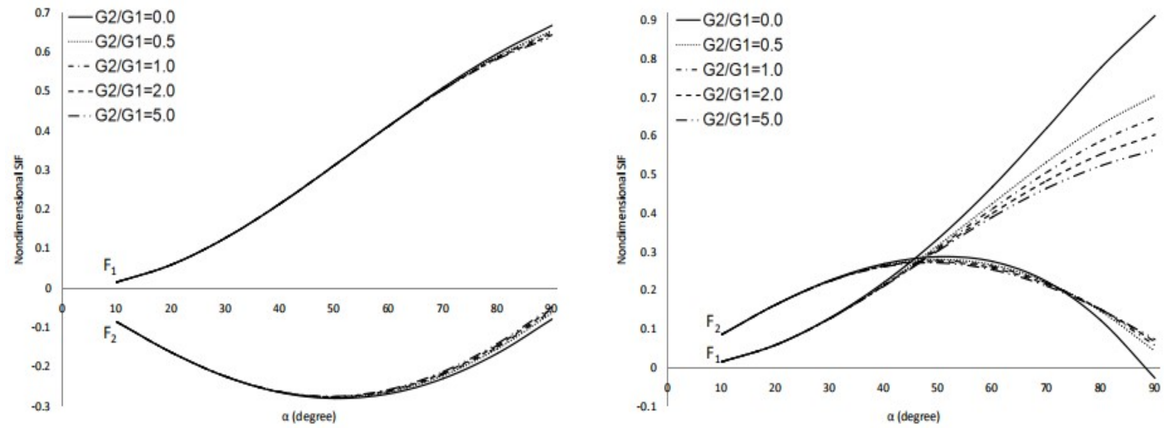
Consider a thermally insulated curve crack in the upper side of bonded two half planes subjected to various mode stresses as denoted in Fig. 2. Table 1 presents the nondimensional SIF for $h = 1.5R$, $G_2/G_1 = 1.0$, $k_2/k_1 = 1.0$, $\mu_2/\mu_1 = 1.0$ and α varies subjected to Mode II stress, $\sigma_{x_1} = \sigma_{x_2} = p$. It is observed that the value of F_1 at tips A_1 and A_2 are equals, while F_2 at tip A_1 and A_2 are opposite. Our results are totally agree with those of Chen and Hasebe (2003).

Table 1: Nondimensional SIF for a thermally insulated curve crack when $h = 1.5R$, $G_2/G_1 = 1.0$, $k_2/k_1 = 1.0$, $\mu_2/\mu_1 = 1.0$ and α varies subjected to Mode II stress.

SIF	10°	20°	30°	40°	50°	60°	70°	80°	90°
F_{1A1}*	0.9735	0.8970	0.7783	0.6278	0.4583	0.2823	0.1115	-0.0441	-0.1764
F_{1A1}**	0.9736	0.8970	0.7779	0.6272	0.4575	0.2815	0.1107	-0.0447	-0.1768
F_{2A1}*	0.1727	0.3317	0.4672	0.5704	0.6362	0.6630	0.6516	0.6057	0.5306
F_{2A1}**	0.1723	0.3318	0.4673	0.5703	0.6359	0.6625	0.6511	0.6053	0.5303
F_{1A2}*	0.9735	0.8970	0.7783	0.6278	0.4583	0.2823	0.1115	-0.0441	-0.1764
F_{2A2}*	-0.1727	-0.3317	-0.4672	-0.5704	-0.6362	-0.6630	-0.6516	-0.6057	-0.5306

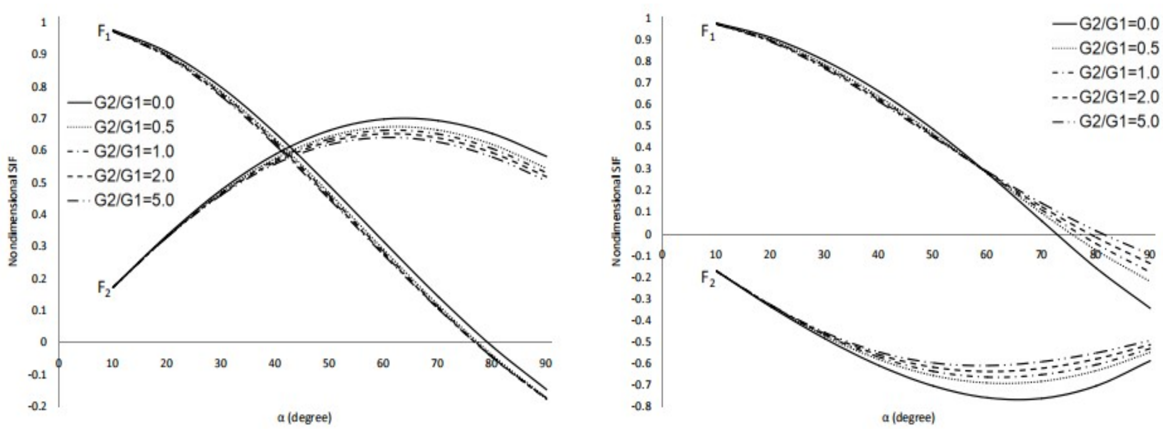
*Current study
**Chen and Hasebe (2003)

Figs. 3, 4, 5 and 6 present the nondimensional SIF for $h = 1.5R$, $G_2/G_1 = 1.0$, $k_2/k_1 = 1.0$, $\mu_2/\mu_1 = 1.0$ and α varies subjected to various mode stresses. Fig. 3 shows the nondimensional



(a) Tip A_1 (b) Tip A_2

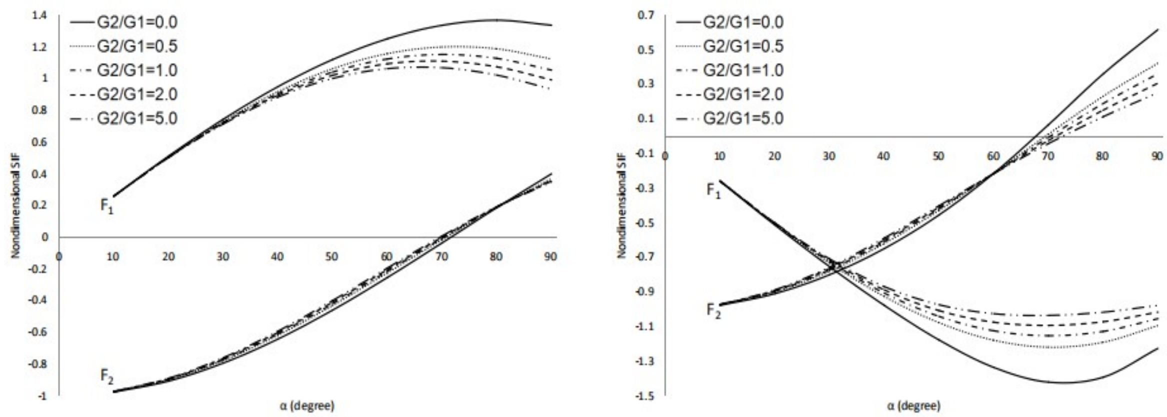
Figure 3: Nondimensional SIF versus α subjected to Mode I stress at crack tips



(a) Tip A_1 (b) Tip A_2

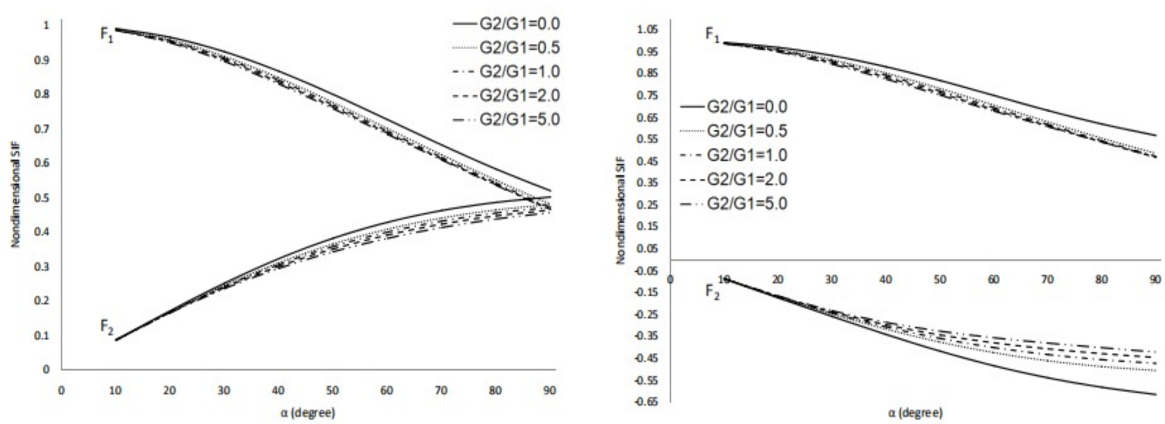
Figure 4: Nondimensional SIF versus α subjected to Mode II stress at crack tips

SIF subjected to Mode I stress, $\sigma_{y_1} = \sigma_{y_2} = p$. At tip A_1, it is obtained that F_1 increases with increasing of α and F_2 decreases for $\alpha < 50^o$, and F_1 and F_2 no significant differences with increasing of G_2/G_1. Whereas at tip A_2, F_1 increases with increasing of α and F_2 decreases for $\alpha > 50^o$, and F_1 decreases and F_2 increases for $\alpha > 75^o$ with increasing of G_2/G_1. Fig. 4 shows the nondimensional SIF subjected to Mode II stress, $\sigma_{x_1} = \sigma_{x_2} = p$. At tip A_1, it is found that F_1 decreases with increasing of α and F_2 increases for $\alpha < 75^o$, and F_1 and F_2 decrease with increasing of G_2/G_1. Whereas at tip A_2, F_1 decreases with increasing of α and F_2 decreases for $\alpha < 65^o$, and F_1 increases for $\alpha > 60^o$ and F_2 increases with increasing of G_2/G_1. Fig. 5 shows the nondimensional SIF subjected to Mode III stress, $\sigma_{x_1y_1} = \sigma_{x_2y_2} = p$. At tip A_1, it is found that F_1 and F_2 increase with increasing of α, and F_1 decreases and F_2 increases for $\alpha < 80^o$ with increasing of G_2/G_1. Whereas at tip A_2, F_1 decreases for $\alpha < 70^o$ and F_2 increases with increasing of α , and F_1 increases and F_2 decreases for $\alpha > 60^o$ with increasing of G_2/G_1 . Fig. 6 shows the nondimensional SIF subjected to Mix Mode stress, $\sigma_{x_1} = \sigma_{x_2} = \sigma_{y_1} = \sigma_{y_2} = p$. At tip A_1, it is found that F_1 decreases and F_2 increases with increasing of α , and F_1 and F_2 decrease with increasing of G_2/G_1 . Whereas at tip A_2, F_1 behaves as F_1 at tip A_1, while F_2 decreases as α increases and increases with increasing of G_2/G_1 .



(a) Tip A_1 (b) Tip A_2

Figure 5: Nondimensional SIF versus α subjected to Mode III stress at crack tips



(a) Tip A_1 (b) Tip A_2

Figure 6: Nondimensional SIF versus α subjected to Mix Mode stress at crack tips

CONCLUSION

In this paper, the new HSIEs for a thermally insulated curve crack in the upper side of bonded two half planes subjected to various mode stresses such as Mode I (Normal), Mode II (Shear), Mode III (Tearing) and Mix Mode stresses is formulated by using the MCP function with the COD function as the unknown. The new HSIEs reduces to a crack problem in an elastic infinite plane when $k_1 = k_2$, $\mu_1 = \mu_2$ and $G_1 = G_2$, and for $k_2 = 0$ and $G_2 = 0$ the new HSIEs reduces to a crack problem in an elastic half plane. Nondimensional SIF at the tips depends on the bi-elastic constant ratio G_2/G_1 , various mode stresses and cracks geometries.

ACKNOWLEDGMENTS

This paper was supported by Universiti Putra Malaysia (UPM) for Putra Grant (9567900) and Fundamental Research Grant Scheme (FRGS) (01-01-16-18685R5524975). The first author also would like to express his great appreciation to the Universiti Teknikal Malaysia Melaka (UTeM) and Ministry of Education Malaysia (MoE) for the financial support (SLAB).

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