



Faculty of Information and Communication Technology

AN S-CURVE EFFICIENT FRONTIER FOR EVALUATION MODEL

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AN S-CURVE EFFICIENT FRONTIER FOR EVALUATION MODEL

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**A thesis submitted
in fulfillment of the requirements for the degree of Doctor of Philosophy**

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DECLARATION

I declare that this thesis entitled “An S-curve Efficient Frontier For Evaluation Model” is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in term of scope and quality for the award of Doctor of Philosophy.

Signature : 

Supervisor Name : Associate Professor Dr. Nor Azman Bin Abu

Date : 7.7.2021

DEDICATION

Bismillahirrahmanirrahim

To Shahrin, Azam, Ida and little Aysar, Umar, Syifa and little Khalis, Amirah,

Aiman and Afnan ... Nothing is impossible if Allah wills it ...

ABSTRACT

A normal distribution has been an ideal model in providing mental picture on a distribution of an object. A normal distribution projects a global view on a variation of an object which is expected to spread. This mental picture signifies an ideal view on statistic motion in the past three centuries. A simple linear regression has been a practical predictive model following a normal distribution along an independent variable. While a linear model provides a compact support at a central mean, a distribution of a predicted value on both ends of a simple linear regression widens as an independent variable moves further away from its central mean. A simple linear regression gives a good fit on average for an expected predicted value. This present research study is looking at feasible maximum output along an independent variable called an efficient frontier. At the same time, a simple linear regression gives a more accurate prediction near a central mean. This research study focuses on a particular attention on both far ends of an efficient frontier curve. A non-linear model is presumed to perform better than a linear model. Upon overcoming the challenge of producing a practical non-linear model, a new S-curve efficient frontier for evaluation model is proposed in this study for better and practical estimation on an optimal output. An efficient frontier denotes an optimal curve in a non-linear model. This new S-curve efficient frontier reflects a non-linear model from a single independent variable to a prediction valuation that has a positive first derivative throughout its progression. This non-linear S-curve model is proposed as an ideal projection in a scientific valuation model. Two types of quantitative secondary data have been collected, namely second-hand car prices and market equity share prices, to test on this model generation and validation. An S-curve efficient frontier model gives a better forecast along dynamic progress on an efficient frontier projection from a simple linear regression. Hence, this novel non-linear S-curve efficient frontier predictive for evaluation model may serve as an ideal projection to many real scenarios within positive derivative progression. More importantly, this S-curve model prescribes an ideal view on a statistical motion for future endeavours. An S-curve efficient frontier for evaluation model will provide and add a dynamic mental picture in addition to a normal distribution.

AMBANG EFISIEN LINGKUK S SEBAGAI MODEL PENILAIAN

ABSTRAK

Taburan normal telah menjadi model yang ideal dalam memberikan gambaran mental mengenai taburan objek. Taburan normal berfungsi untuk mengunjurkan pandangan global mengenai variasi objek yang diharap dapat disebarkan. Gambaran mental ini memberikan pandangan yang ideal mengenai tebaran statistik pada tiga abad yang lalu. Regresi linear mudah telah menjadi model ramalan praktikal berikutan taburan normal di sepanjang pemboleh ubah bebas. Walaupun model linear memberikan sokongan padat pada min pusat, taburan nilai yang diramalkan pada kedua-dua hujung regresi linear mudah melebar ketika pemboleh ubah bebas bergerak lebih jauh dari min pusatnya. Regresi linear mudah memberikan kesesuaian yang rata-rata untuk nilai ramalan yang diharapkan. Kajian penyelidikan ini sedang melihat output maksimum yang layak di sepanjang pemboleh ubah bebas yang disebut sempadan efisien. Pada masa yang sama, regresi linear mudah memberikan ramalan yang lebih tepat berhampiran min pusat. Kajian penyelidikan ini menumpukan perhatian khusus pada kedua hujung lengkung sempadan yang efisien. Model tidak linear dianggap berprestasi lebih baik daripada model linear. Setelah mengatasi cabaran menghasilkan model praktikal bukan linear, model ambang efisien lengkung-S sebagai model penilaian yang baru dicadangkan dalam kajian ini untuk anggaran yang lebih baik dan praktikal pada output yang optimum. Batas yang efisien menunjukkan lengkung yang optimum dalam model yang tidak linear. Frontier efisien lengkung S baru ini mencerminkan model bukan linear dari pemboleh ubah bebas tunggal hingga penilaian ramalan yang mempunyai pembezaan pertama positif sepanjang perkembangannya. Model lengkung S tidak linear ini dicadangkan sebagai unjuran ideal dalam model penilaian saintifik. Dua jenis data sekunder kuantitatif telah dikumpulkan, iaitu harga kereta terpakai dan harga saham ekuiti pasaran, untuk menguji penjanaan dan pengesahan model ini. Model ambang efisien lengkung S memberikan ramalan yang lebih baik sepanjang kemajuan dinamik pada unjuran perbatasan yang efisien dari regresi linear mudah. Oleh itu, model ramalan ambang efisien lengkung S sebagai model penilaian tidak linear baru ini akan berfungsi sebagai unjuran yang ideal untuk banyak senario sebenar dalam perkembangan terbitan positif. Lebih penting lagi, model lengkung S ini menetapkan pandangan yang ideal mengenai gerakan statistik pada masa hadapan. Model ambang efisien lengkung S sebagai model penilaian akan menyediakan satu lagi gambaran mental yang dinamik sebagai tambahan kepada taburan normal.

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LIST OF ABBREVIATIONS

ANN	-	Artificial Neural Network
BLR	-	Base Lending Rate
CAPM	-	Capital Asset Pricing Model
CSR	-	Cross-sectional Regression
DEA	-	Data Envelopment Analysis
ERP	-	Electronic Road Pricing
FD	-	Fixed Deposit
GMM	-	Generalised Method of Moments
HCV	-	Hepatitis C Virus
ICAPM	-	Intertemporal Capital Asset Pricing Model
LAD	-	Least Absolute Deviation
LACP	-	Last Adjusted Closing Price
KLSE	-	Kuala Lumpur Stock Exchange
NOCMP	-	No Output Constraint Maximum Profit
NPV	-	Net Present Value
PSS	-	Product Service System
RCAPM	-	Robust Capital Asset Pricing Model
SMF	-	S-shaped Membership Function
SVM	-	Support Vector Machines
SVR	-	Sustained Viral Response

SVSI-J	-	Stochastic Volatility Stochastic Interest rate random Jump
YTD	-	Year-To-Date
ZIP	-	Zero-Inflated Poisson

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