

# Time to Collision for Emergency Obstacle Avoidance

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ORIGINAL ARTICLE

Open Access

## Article History:

Received  
 22 Aug 2020

Accepted  
 25 Jan 2021

Available online  
 1 May 2021

**Abstract** – Time to collision is one of the indices to decide if a collision is avoidable. This paper presents the study on emergency obstacle avoidance from the viewpoint of time to collision. The times to collision at which the autonomous emergency steering has to be initiated to avoid a collision are derived for the five functions which are used for the generation of the emergency lane change paths. These functions are circular arcs, polynomial, ramp sinusoidal, sigmoid, and trapezoidal acceleration profiles. The times to collision at which the steering avoidance by using these functions has to be initiated are computed and compared. For a given lane change maneuver width of 3.5 m and coefficients of friction of 0.9, 0.5, 0.2, circular arcs give the shortest, and sigmoid gives the longest time to collision.

**Keywords:** Autonomous Emergency Braking (AEB), Autonomous Emergency Steering (AES), braking maneuver, lane change maneuver, time to collision

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 Journal homepage: [www.jsaem.saemalaysia.org.my](http://www.jsaem.saemalaysia.org.my)

## 1.0 INTRODUCTION

Autonomous collision systems offer the capability to automatically avoid a collision without requiring input from the human driver. A collision may be avoided by automatic braking or automatic steering. While braking is effective at low vehicle speeds, the distance required to avoid a collision increases significantly at high vehicle speeds. At higher vehicle speeds, the steering requires a shorter longitudinal distance to avoid collision compared to braking.

Numerous measures for threat assessment of traffic scenarios are considered in the literature. These include time to collision, the inverse of time to collision, time headway, constant longitudinal deceleration required to avoid a collision, and minimum distance during collision avoidance (Zhang et al., 2006). Time to collision is a time-based measure used to assess if a collision is about to occur and it is defined as the time required for a collision between two vehicles to occur if the vehicles continue to travel at the current speed and on the same trajectory (Hayward, 1972). Time headway is defined as the ratio of the distance between the

two vehicles to the speed of the host vehicle (Fuller, 1981). These measures are important to decide if a collision is avoidable and which of either braking avoidance or steering avoidance is more effective.

In this paper, the times to collision for the five functions which are used to generate the emergency lane change maneuver are discussed. These functions are circular arcs, polynomial, ramp sinusoidal, sigmoid, and trapezoidal acceleration profiles. At low vehicle speed, the time to collision at which braking maneuver has to be initiated is shorter than that of lane-change maneuver. At high vehicle speed, the time to collision at which the braking maneuver has to be initiated is longer than that of the lane change maneuver. The times to collision at which the lane change maneuvers have to be initiated by using different functions are compared and discussed in this paper.

The remaining part of the paper proceeds as follows. The scenario and strategies for obstacle avoidance are described in Section 2. Section 3 presents the times to collision at which braking avoidance and steering avoidance have to be initiated. Numerical examples are given in Section 4. Section 5 presents the conclusions.

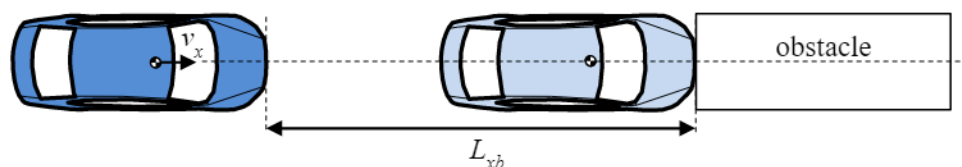
## 2.0 OBSTACLE AVOIDANCE SCENARIO AND STRATEGIES

### 2.1 Obstacle Avoidance Scenario

In this paper, a collision-avoidance scenario in which a vehicle is required to avoid an obstacle that is blocking the forward path of the vehicle. The obstacle is assumed to be stationary. It is assumed that no vehicles are traveling in the neighboring lane and it is safe to execute a lane-change maneuver.

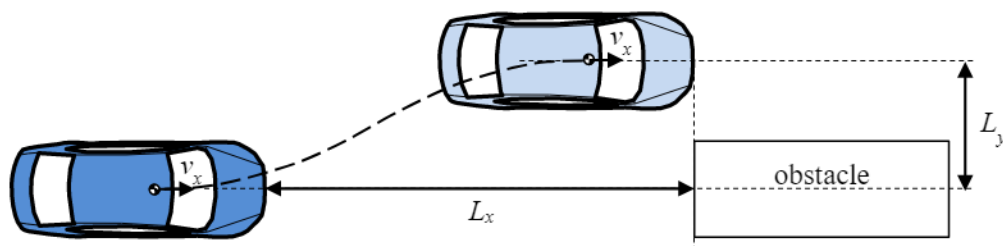
### 2.2 Obstacle Avoidance Strategies

The collision with the obstacle may be avoided by either applying full braking to stop the vehicle before the obstacle or by executing a lane change maneuver. Figure 1 shows a braking maneuver in a vehicle applies maximum braking force to stop the vehicle just before the obstacle. In Figure 1,  $L_{xb}$  is the longitudinal distance required to stop the vehicle by applying maximum braking and  $v_x$  is the initial vehicle speed.



**Figure 1:** Braking avoidance

Steering avoidance as shown in Figure 2 is a strategy in which a vehicle executes a lane-change maneuver. In Figure 2,  $L_x$  and  $L_y$  are the length and width of the lane change maneuver, respectively, and  $v_x$  is the constant vehicle speed during the maneuver.



**Figure 2:** Steering avoidance

### 3.0 TIME TO COLLISION

In this paper, time to collision is defined as the ratio of the distance between the vehicle to the obstacle divided by the speed of the vehicle. In this section, the time to collision at which the braking avoidance or steering avoidance has to be initiated is described.

#### 3.1 Braking Avoidance

For braking avoidance, the longitudinal distance required to stop the vehicle is given by

$$L_{xb} = \frac{v_x^2}{2d_{\max}} \quad (1)$$

where  $d_{\max}$  is the maximum achievable longitudinal deceleration which is given as the product of the coefficient of friction between tire and road and the acceleration due to gravity. The time to collision at which braking must be initiated is

$$ttc_b = \frac{v_x}{2d_{\max}} \quad (2)$$

#### 3.2 Steering Avoidance

For the generation of the desired lane change paths, five functions are considered in this subsection. These functions are circular arcs, polynomial, ramp sinusoidal, sigmoid, and trapezoidal acceleration profiles. The coefficient of friction between tire and road is taken into account by the generation of the desired paths.

##### 3.2.1 Circular Arcs

The lane change path can be constructed by using two circular arcs and the length of the lane change maneuver  $L_{xca}$  is given by Sledge and Marshek (1997) as

$$L_{xca} = \sqrt{\frac{4L_y v_x^2}{a_{\max}} - L_y^2} \quad (3)$$

where  $a_{\max}$  is the maximum achievable lateral acceleration. This acceleration is limited by the tire-road friction coefficient. The time to collision at which the lane change maneuver must be initiated is

$$ttc_{ca} = \sqrt{\frac{4L_y}{a_{\max}} - \frac{L_y^2}{v_x^2}} \quad (4)$$

### 3.2.2 Polynomial

A fifth-order polynomial has been used to represent a lane change path (Shah et al., 2015). The length of the lane change maneuver  $L_{xp}$  is given by Weber (2012) as

$$L_{xp} = v_x \sqrt{\frac{10L_y}{\sqrt{3}a_{\max}}} \quad (5)$$

The time to collision at which the steering avoidance must be initiated is

$$ttc_p = \sqrt{\frac{10L_y}{\sqrt{3}a_{\max}}} \quad (6)$$

### 3.2.3 Ramp Sinusoidal

In the previous studies (Sledge & Marshek, 1997; Kim et al., 2017), ramp sinusoidal is used to generate the desired lane change path. In the paper by Sledge and Marshek (1997), the length of the lane change maneuver  $L_{xrs}$  is written as

$$L_{xrs} = v_x \sqrt{\frac{2\pi L_y}{a_{\max}}} \quad (7)$$

The time to collision at which the steering avoidance has to be initiated is

$$ttc_{rs} = \sqrt{\frac{2\pi L_y}{a_{\max}}} \quad (8)$$

### 3.2.4 Sigmoid

Isermann et al. (2008) used a sigmoid function to represent a lane change path. The generation of the lane change path by using sigmoid takes into account the maximum lateral acceleration and maximum lateral jerk. The lateral position as given by Arbitmann et al. (2012) can be written as a function of the longitudinal position along the path as

$$y(x) = \frac{L_y}{1 + e^{-\beta(x-l)}} \quad (9)$$

where  $\beta$  is the slope of the sigmoid and  $l$  represents half of the length of the lane change maneuver  $L_{xs}$ . The length  $L_{xs}$  is given by

$$L_{xrs} = 2l \quad (10)$$

The steering avoidance has to be initiated at

$$ttc_s = \frac{2l}{v_x} \quad (11)$$

### 3.2.5 Trapezoidal Acceleration Profile

The trapezoidal acceleration profile as shown in Figure 3 is used for the lane change maneuvers (Chee & Tomizuka, 1994; Soudbakhsh et al., 2013). The parameters  $T_a$ ,  $T_b$ ,  $T_c$ ,  $T_d$ , and  $T_e$  given by Chee & Tomizuka (1994) can be written as

$$T_a = \frac{a_{\max}}{j_{\max}} \quad (12)$$

$$T_b = \frac{-T_a^2 + \sqrt{T_a^4 + 4T_a \frac{L_y}{j_{\max}}}}{2T_a} \quad (13)$$

$$T_c = 2T_a + T_b \quad (14)$$

$$T_d = T_a + 2T_b \quad (15)$$

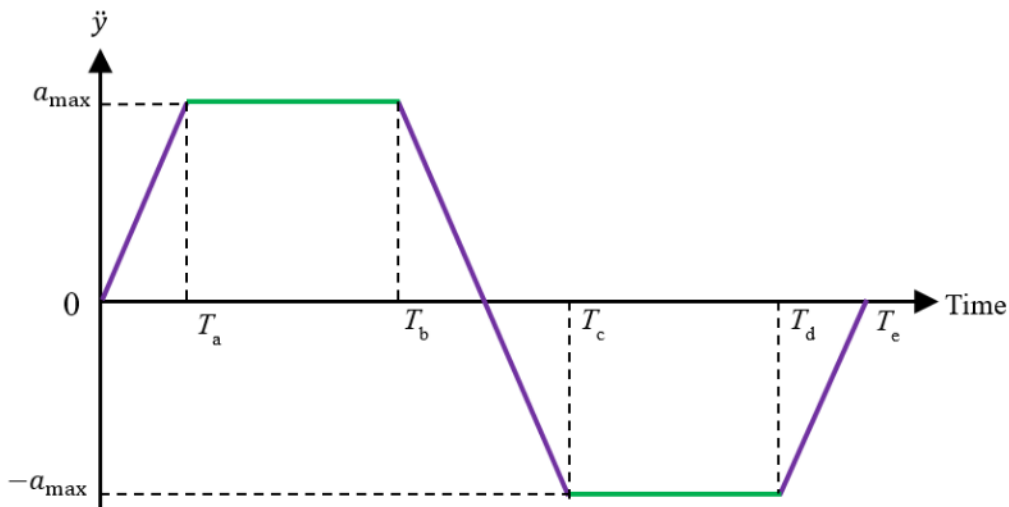
$$T_e = 2T_a + 2T_b \quad (16)$$

where  $j_{\max}$  is the maximum lateral jerk. The length of the lane change maneuver  $L_{xtap}$  is as follows:

$$L_{xtap} = v_x T_e \quad (17)$$

The time to collision for the initiation of the steering avoidance using trapezoidal acceleration profile is

$$ttc_{tap} = T_e \quad (18)$$

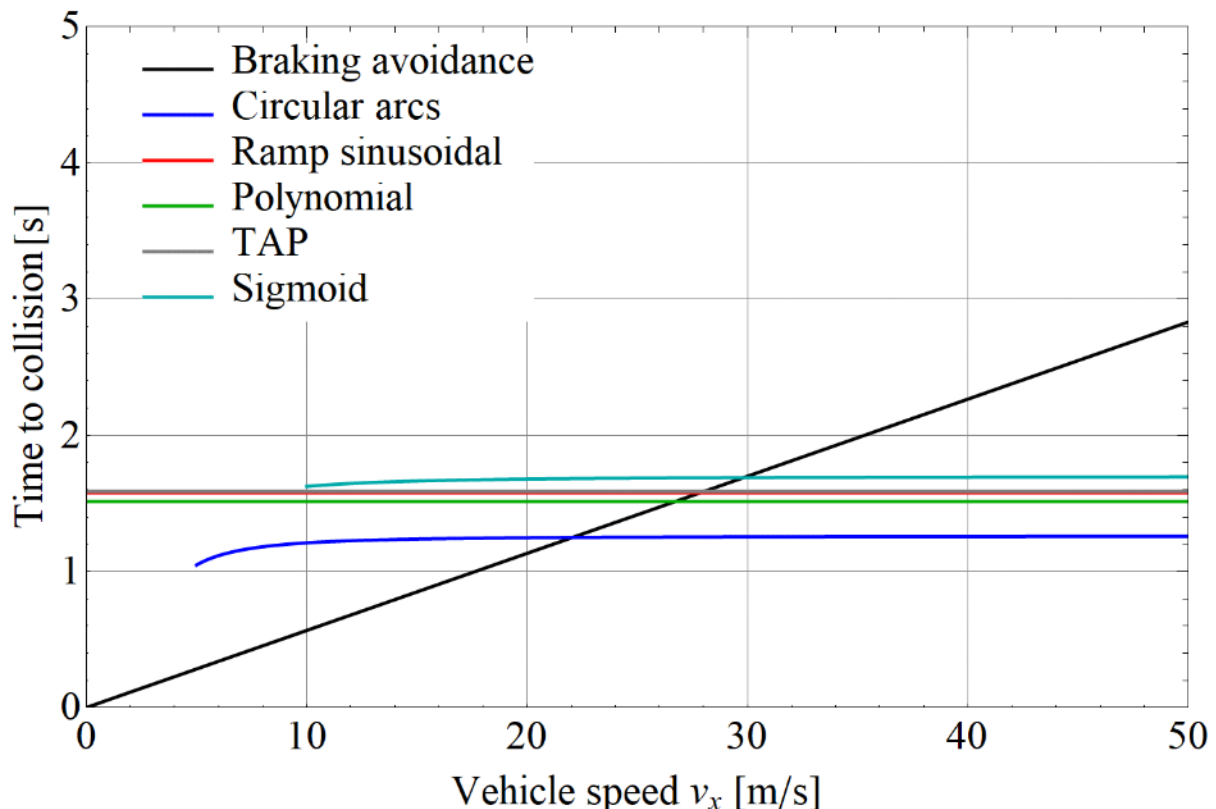


**Figure 3:** Trapezoidal acceleration profile

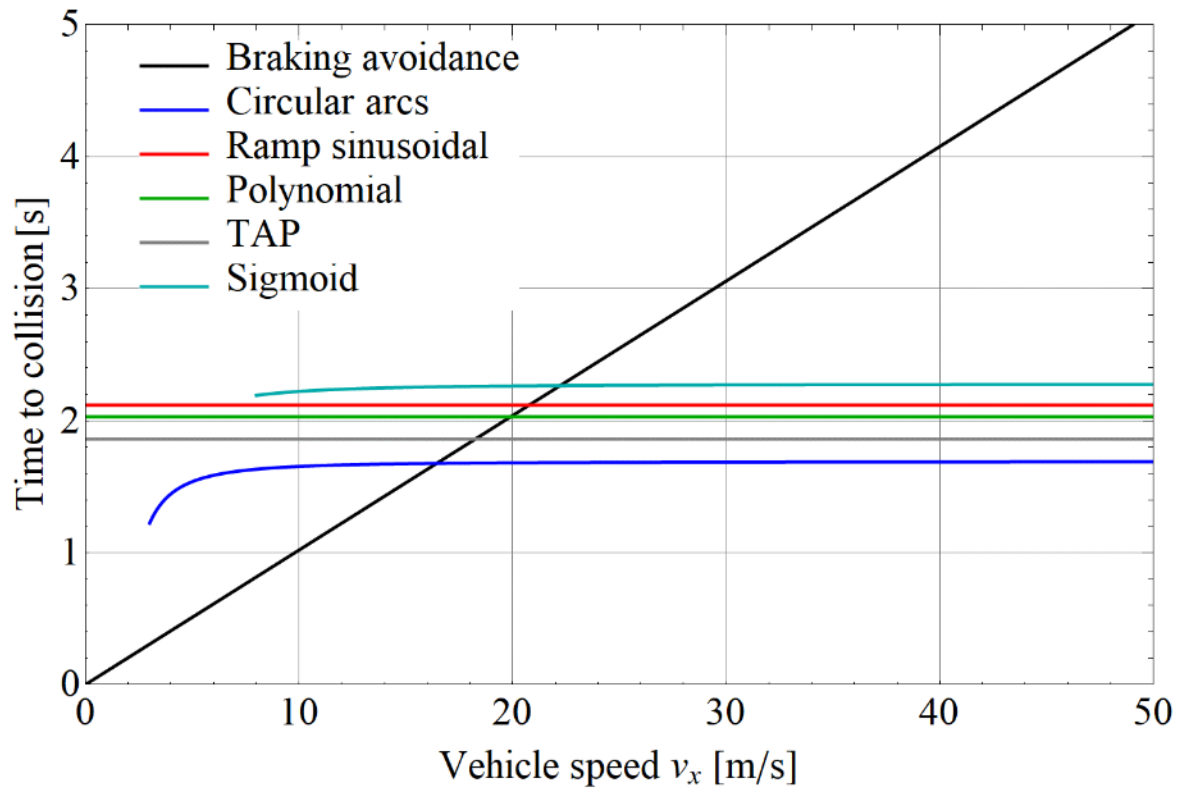
#### 4.0 NUMERICAL EXAMPLES

This section computes and compares the times to collision for the five functions which are used to generate the emergency lane change paths. The width of the lane change maneuver is assumed to be 3.5 m in all cases. For the trapezoidal acceleration profile and sigmoid function, a maximum lateral jerk of 30 m/s<sup>3</sup> is assumed. Figures 4 through 6 show the time to collision as a function of vehicle speed for coefficients of friction of 0.9, 0.5, and 0.2, respectively. The black line indicates braking avoidance, and the blue, red, green, gray, and cyan lines indicate steering avoidance using circular arcs, ramp sinusoidal, polynomial, trapezoidal acceleration profile, and sigmoid functions, respectively. The intersection points in these figures are where the times to collision for the braking avoidance and steering avoidance are the same.

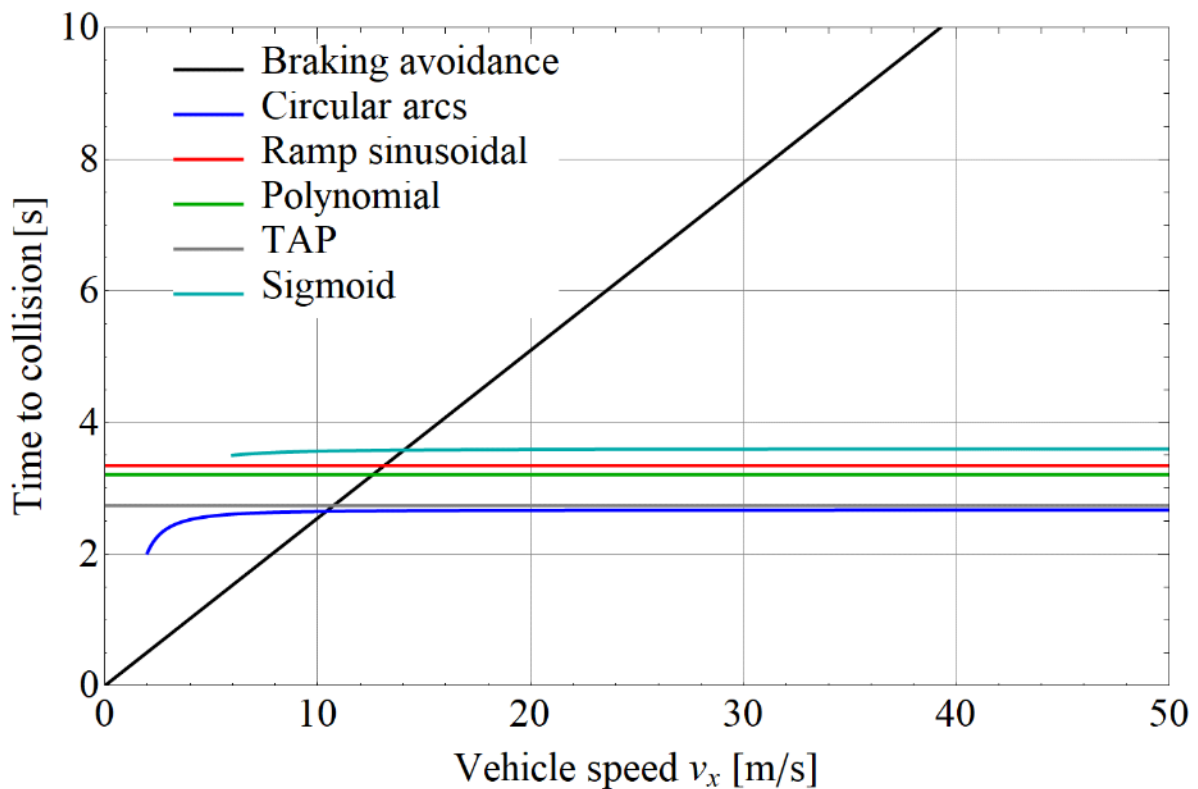
In Figure 4, it can be seen that the functions for steering avoidance can be arranged from the shortest to longest time to collision in the following order: circular arcs, polynomial, ramp sinusoidal, trapezoidal acceleration profile, and sigmoid. The times to collision for ramp sinusoidal and trapezoidal acceleration profiles are similar. For Figure 5, the functions for steering avoidance from the shortest to longest time to collision are circular arcs, trapezoidal acceleration profile, polynomial, ramp sinusoidal, and sigmoid. As shown in Figure 6, the functions for steering avoidance arranged according to the time to collision from shortest to longest, in order, are circular arcs, trapezoidal acceleration profile, polynomial, ramp sinusoidal, and sigmoid. For friction coefficients of 0.9, 0.5, and 0.2, the shortest and longest times to collision for steering avoidance are given by circular arcs and sigmoid, respectively.



**Figure 4:** Time to collision for a lane change maneuver width of 3.5 m and a friction coefficient of 0.9



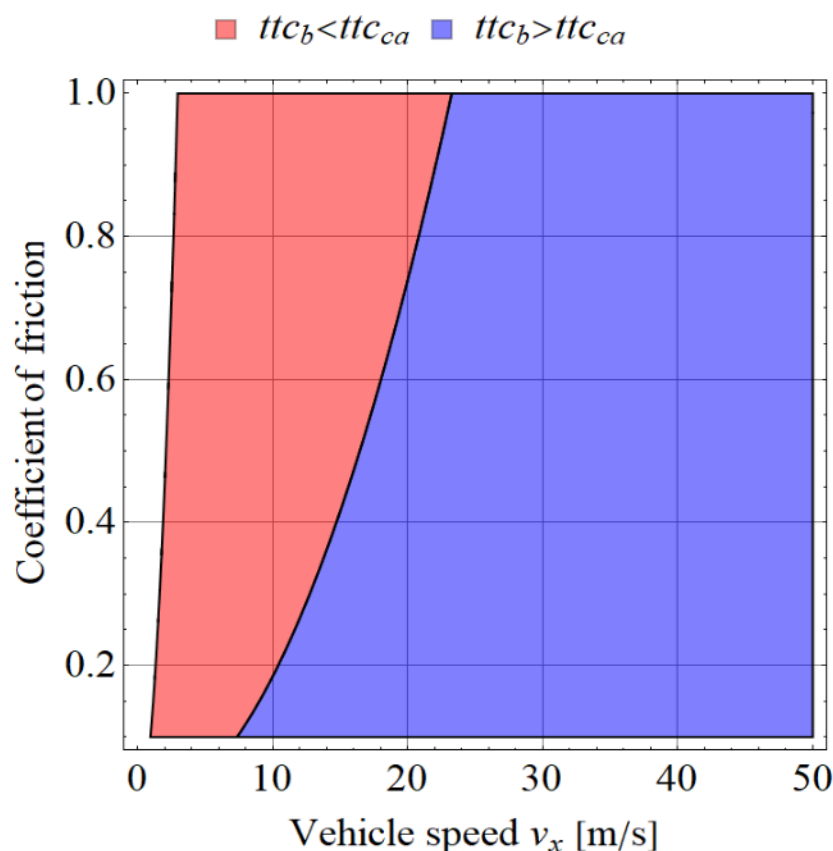
**Figure 5:** Time to collision for a lane change maneuver width of 3.5 m and a friction coefficient of 0.5



**Figure 6:** Time to collision for a lane change maneuver width of 3.5 m and a friction coefficient of 0.2

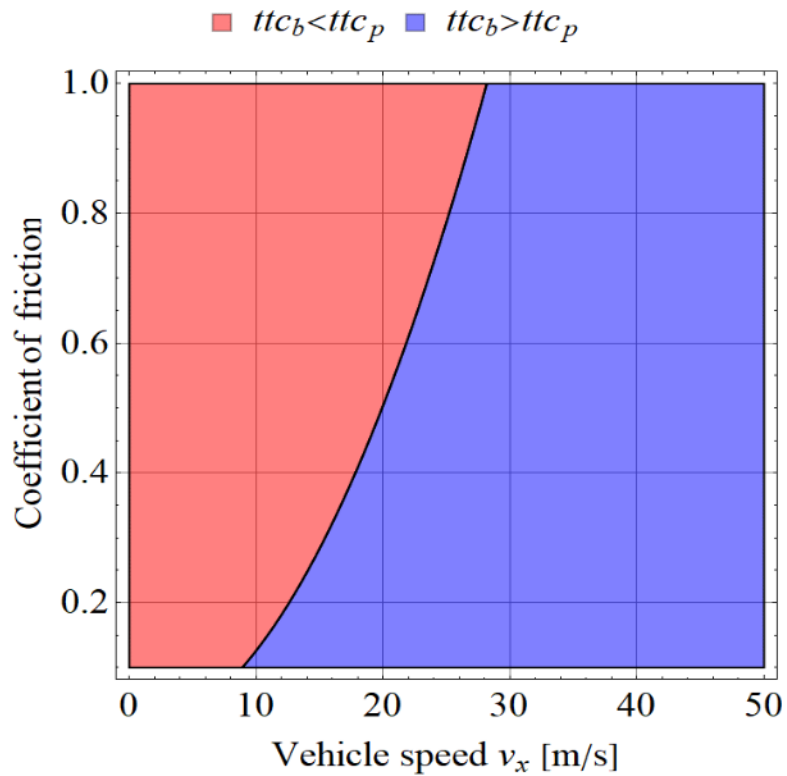
Figures 7 through 11 show the regions in which the time to collision for braking avoidance is either shorter or longer compared to that of steering avoidance by using circular arcs, polynomial, ramp sinusoidal, sigmoid, and trapezoidal acceleration profile, respectively. In these figures, the red region indicates that the time to collision for braking avoidance is less than that of steering avoidance and the blue region indicates that the time to collision for braking avoidance is greater than that of steering avoidance. The avoidance maneuver which gives the shortest time to collision is considered to be the most effective. It can be seen that among the five functions, the circular arc function gives the largest region in which steering avoidance is more effective compared to braking avoidance.

Besides the steering avoidance and braking avoidance considered in this paper, a collision with the obstacle may be avoided by using combined steering and braking. Singh and Nishihara (2018) derived the optimal steering and braking control for emergency obstacle avoidance and it was demonstrated that steering and braking avoidance are more effective compared to steering avoidance. Dimensionless indices for collision avoidance, more specifically, the dimensionless longitudinal distance to the obstacle and dimensionless time to collision were proposed by Singh and Nishihara (2016). The benefit of the dimensionless collision avoidance indices is that there is a unique switching point between the avoidance maneuvers. Besides, the effects of the vehicle speed, coefficient of friction, and lane change maneuver width on the time to collision can be studied on a single two-dimensional decision-making diagram. For more details, see Singh and Nishihara (2016).

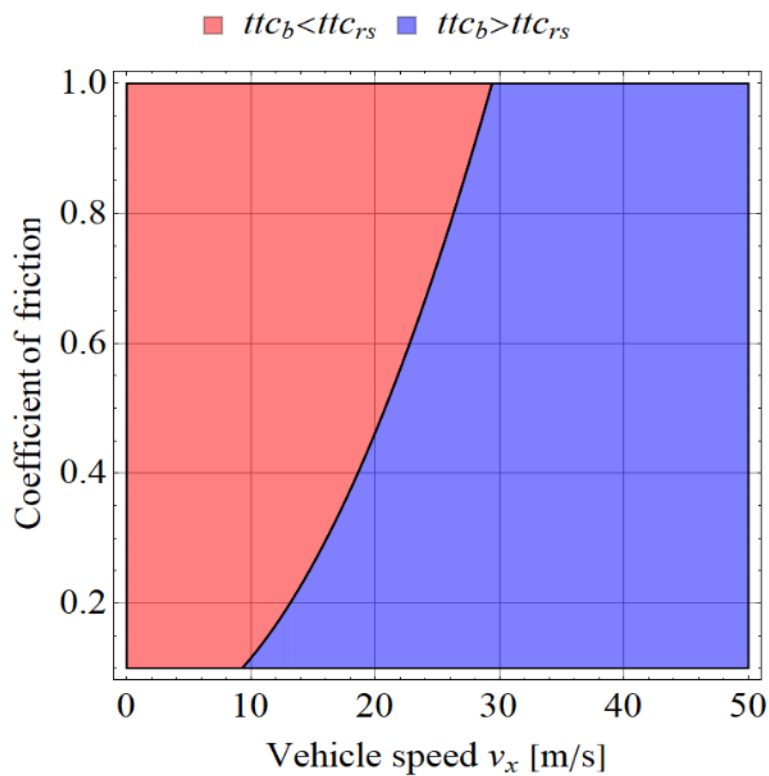


**Figure 7:** Regions in which either braking or steering avoidance gives the shortest time to collision at which the avoidance maneuver must be initiated to avoid collision for circular arcs

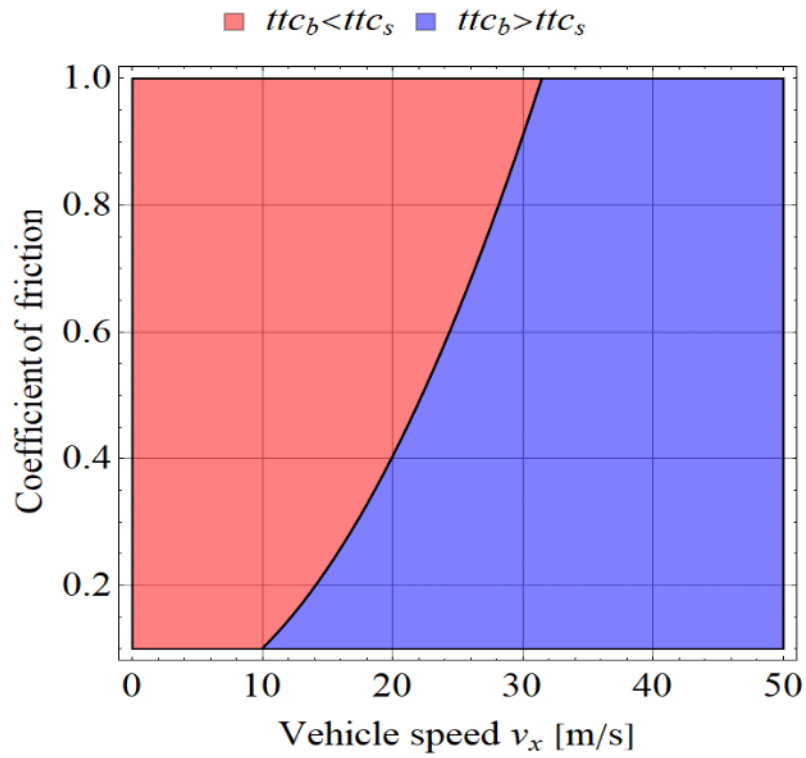




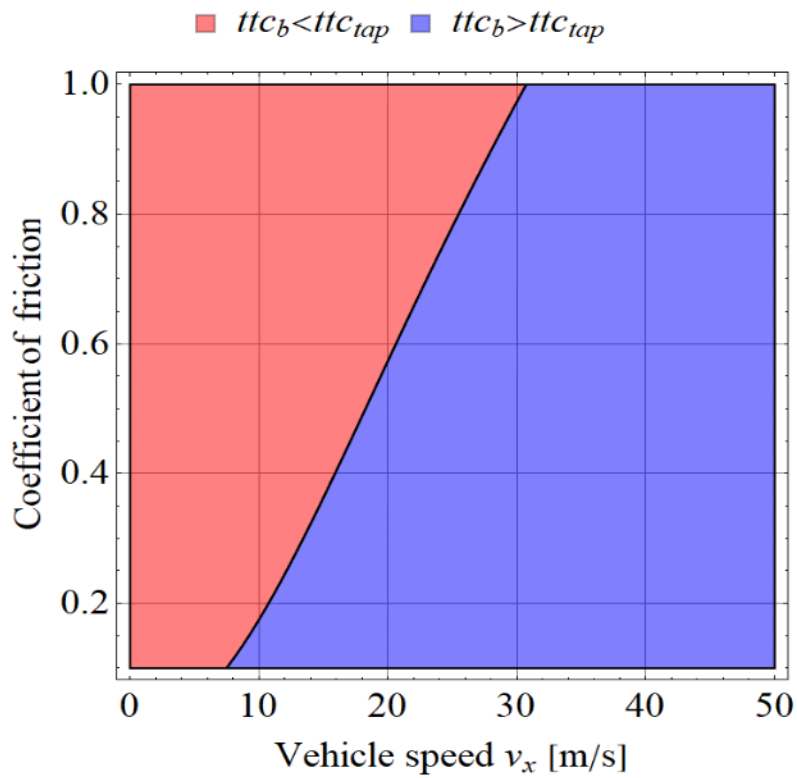
**Figure 8:** Regions in which either braking or steering avoidance gives the shortest time to collision at which the avoidance maneuver must be initiated to avoid collision for polynomial



**Figure 9:** Regions in which either braking or steering avoidance gives the shortest time to collision at which the avoidance maneuver must be initiated to avoid collision for ramp sinusoidal



**Figure 10:** Regions in which either braking or steering avoidance gives the shortest time to collision at which the avoidance maneuver must be initiated to avoid collision for sigmoid



**Figure 11:** Regions in which either braking or steering avoidance gives the shortest time to collision at which the avoidance maneuver must be initiated to avoid collision for trapezoidal acceleration profile

## 5.0 CONCLUSION

In the present paper, five functions namely circular arcs, polynomial, ramp sinusoidal, sigmoid, and trapezoidal acceleration profiles are considered for the generation of emergency lane change paths and the times to collision given by these functions are compared. For steering avoidance, the shortest time to collision is obtained by using circular arcs, and the longest is obtained by using sigmoid. The regions in which either steering avoidance or braking avoidance is more effective are identified.

## ACKNOWLEDGEMENTS

The authors would like to acknowledge the support from the New Car Assessment Program for Southeast Asian Countries (ASEAN NCAP) through the ASEAN NCAP Collaborative Holistic Research – Phase III (ANCHOR III) research grant (No. A3-C391). This grant is registered at UTeM with the grant number ANTARABANGSA-ANCHOR/2020/FKM-CARE/A00029. The authors would also like to acknowledge the support from Universiti Teknikal Malaysia Melaka (UTeM) for providing the research facilities.

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