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Axisymmetric Hybrid Nanofluid Flow Due to a Convectively Heated Stretching/Shrinking Disk

Najiyah Safwa Khashi'ie[1,*], Iskandar Waini[1], Ioan Pop[2], Nurul Amira Zainal[1], Abdul Rahman Mohd Kasim[3]

[1] Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
[2] Department of Mathematics, Babeş-Bolyai University, R-400084 Cluj-Napoca, Romania
[3] Centre for Mathematical Sciences, College of Computing & Applied Sciences, Universiti Malaysia Pahang, Lebuhraya Tun Razak, Gambang 26300, Pahang, Malaysia

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ABSTRACT

This significant study is designed to analyze the axisymmetric hybrid nanofluid flow with heat transfer on a convectively heated stretching/shrinking disk. The combination of metal (Cu) and metal oxide (Al_2O_3) nanoparticles with water (H_2O) as the base fluid is used for the analysis. Similarity transformation is adopted to reduce the complexity of the PDEs into a system of ODEs. The utilization of suction in maintaining the steady flow solution for the shrinking disk case discloses the presence of dual solutions. Besides, an upsurge of Biot number and suction's strength enhances the heat transfer operation. The application of Cu-Al_2O_3/water nanofluid can extend the range of solutions' existence and consequently, decelerate the separation of laminar flow.

1. Introduction

Hybrid nanofluids are widely used in the experimental and numerical investigations of fluid dynamics due to its significance in the thermal and energy performances. The composite/hybrid nanoparticles are developed from two types of nanoparticles: metal oxides (i.e., Al_2O_3/alumina, CuO/cupric oxide, Fe_3O_4/magnetite, Fe_2O_3/hematite), metals (i.e., Ag/silver, Cu/copper), carbon materials (i.e., CNT/carbon nanotube, graphite, MWCNT/multi-walled carbon nanotubes) or metal carbide. Idris *et al.,* [1] examined the heat transfer performance of alumina-silica/water nanofluid in cooling plate of PEMFC (Proton Electrolyte Membrane fuel cells) which is applicable for the automotive industry. The brief discussion on the preparation, stability, recent applications, and thermal conductivity of the hybrid and single nanofluids can be found from the review papers by Halim and Sidik [2,3], Sidik *et al.,* [4], Babu *et al.,* [5] and Sajid and Ali [6]. Suresh *et al.,* [7] highlighted the advantages of the Cu-Al_2O_3 hybrid nanoparticle which then led to the proposal of thermophysical

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[*] *Corresponding author.*
E-mail address: najiyah@utem.edu.my

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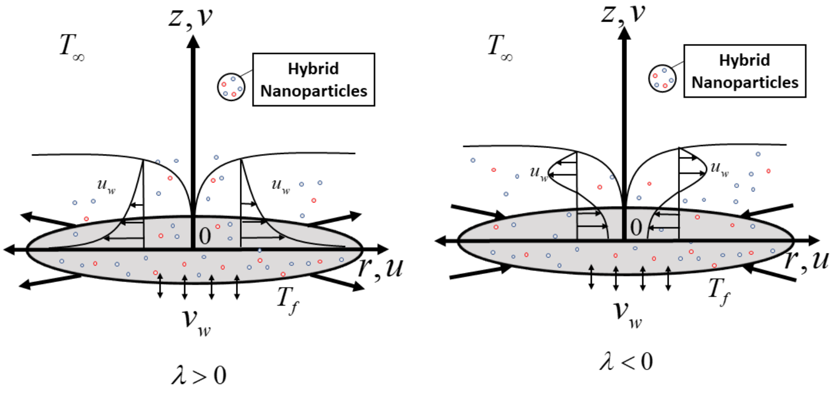
properties' correlations for hybrid nanofluids by Devi and Devi [8]. They used these correlations to analyze a steady boundary layer flow problem due to a stretching sheet.

Another correlations of hybrid nanofluids which were proposed by Takabi and Salehi [9] have been widely used in the numerical simulation of fluid motion. There is only a slight of difference between the numerical values using both correlations as discussed by Xu [10] and Khashi'ie *et al.,* [11]. A detail comparison of the published papers with different types of correlations was initiated and discussed by Khashi'ie *et al.,* [12]. The numerical exploration of dual solutions in hybrid nanofluid flow is not only limited to a flat plate surface, but also have been actively investigated for a thin needle, circular cylinder, disk, Riga plate and curved surface [11-31].

Inspired by the capability of hybrid nanofluids in enhancing the heat transfer operation, this work is devoted to (i) discover the control parameters which can delay the separation of fluid motion and enhance the heat transfer operation, and (ii) to observe the non-unique solutions and the solutions' stability for a hybrid nanofluid flow on a convectively heated stretching/shrinking disk. The hybrid Cu-Al_2O_3/water nanofluid with the correlations by Takabi and Salehi [9] are used in this work. We are confident that this study is novel and new for a disk geometry which significantly can help other researchers in examining the hybrid nanofluid motion with heat transfer. However, the future findings are only conclusive to the pair of this hybrid nanoparticle.

2. Problem Formulation

Consider an axisymmetric flow of hybrid nanofluid over a permeable stretching/shrinking disk with convective boundary condition (CBC). The hybrid nanoparticles from metal (Cu) and metal oxide (Al_2O_3) groups are modeled with the water (H_2O) base fluid using the applicable correlations of hybrid nanofluid proposed by Takabi and Salehi [9]. For the geometry of the problem as exhibited in Figure 1, cylindrical coordinate (r,α,z) is used with the fluid motion on the plane $z=0$. The stretching/shrinking disk is moved with a linear velocity $u_w(r)=ar$ where a is a constant. The bottom of the disk is convectively heated from the hybrid nanofluid with a fixed temperature T_f and a heat transfer coefficient h_f. Meanwhile the constant ambient temperature is represented by T_∞.



(a) Stretching disk (b) Shrinking disk

Fig. 1. The physical model

With the consideration of the boundary layer approximations, the mathematical model in the form of partial differential equations is [11,29]

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rv)}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial z^2}, \tag{3}$$

subject to the boundary conditions [16,29]

$$\begin{aligned} &u=\lambda u_w(r),\quad v=v_w,\quad -k_{hnf}\frac{\partial T}{\partial z}=h_f\left(T_f-T\right),\\ &u\to 0,\quad T\to T_\infty,\qquad \text{as}\qquad z\to\infty. \end{aligned} \tag{4}$$

where $v_w=-S\sqrt{a\nu_f}$ is the mass flow with $v_w<0$ stands for suction and $v_w>0$ denotes the injection process. The hybrid nanofluid velocities are represented by u ($r-$direction) and v ($z-$direction), while T is the hybrid nanofluid temperature. The thermophysical properties of hybrid nanofluid used in this work are presented in Table 1. It is worth to mention that these correlations are introduced by Takabi and Salehi [9] which can be reduced to a case of nanofluid if $\phi_1=0$ or $\phi_2=0$. Nonetheless, these correlations are applicable to a pure water (viscous fluid) model if $\phi_1,\phi_2\approx 0$. It is worth to point out that the total concentration of the hybrid nanoparticles is denoted as $\phi_{hnf}=\phi_1+\phi_2$.

Table 1
The thermophysical properties between hybrid and traditional nanofluid [9]

Properties	Hybrid nanofluid's correlations
Density	$\rho_{hnf}=\left(1-\phi_{hnf}\right)\rho_f+\phi_1\rho_{s1}+\phi_2\rho_{s2}$
Heat Capacity	$(\rho C_p)_{hnf}=\left(1-\phi_{hnf}\right)(\rho C_p)_f+\phi_1(\rho C_p)_{s1}+\phi_2(\rho C_p)_{s2}$
Dynamic Viscosity	$\dfrac{\mu_{hnf}}{\mu_f}=\dfrac{1}{\left(1-\phi_{hnf}\right)^{2.5}}$
Thermal Conductivity	$\dfrac{k_{hnf}}{k_f}=\left[\dfrac{\left(\dfrac{\phi_1k_1+\phi_2k_2}{\phi_{hnf}}\right)+2k_f+2\left(\phi_1k_1+\phi_2k_2\right)-2\phi_{hnf}k_f}{\left(\dfrac{\phi_1k_1+\phi_2k_2}{\phi_{hnf}}\right)+2k_f-\left(\phi_1k_1+\phi_2k_2\right)+\phi_{hnf}k_f}\right]$

From Table 1, ϕ_1 and ϕ_2 denote the volumetric concentration of the Al_2O_3 and Cu nanoparticles, respectively, the subscripts $s1$, $s2$, hnf, nf and f represent the Al_2O_3 nanoparticle, Cu nanoparticle, hybrid nanofluid, nanofluid and base fluid, accordingly. For the numerical exploration, the thermophysical properties of the Al_2O_3 nanoparticle, Cu nanoparticle and pure water are used and listed in Table 2.

Table 2
Thermophysical properties of the Al_2O_3, Cu and H_2O [32]

Thermophysical properties	$\rho\left(\mathrm{kg/m^3}\right)$	$C_p\left(\mathrm{J/kgK}\right)$	$k\left(\mathrm{W/mK}\right)$
Al_2O_3	3970	765	40
Cu	8933	385	400
H_2O	997.1	4179	0.6130

The complexity of Eq. (2)-(4) is reduced by applying the similarity transformation in Eq. (5). This transformation can be adopted if and only if it fulfils Eq. (1) and for this problem [16,29],

$$u = arf'(\eta), \quad v = -2\sqrt{a\nu_f}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = z\sqrt{\frac{a}{\nu_f}}. \tag{5}$$

The reduced similarity equations are then, given by

$$\left(\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\right) f''' + 2ff'' - f'^2 = 0, \tag{6}$$

$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\theta'' + 2f\theta' = 0, \tag{7}$$

inclusive of the boundary conditions [16]

$$\begin{aligned} & f(0) = S, \quad f'(0) = \lambda, \quad \frac{-k_{hnf}}{k_f}\theta'(0) = Bi\left[1 - \theta(0)\right], \\ & f'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{8}$$

where $\Pr = \frac{\mu_f \left(C_p\right)_f}{k_f}$ is the Prandtl number where $\Pr = 6.2$ represents water, $Bi = \frac{h_f}{k_f}\sqrt{\frac{\nu_f}{a}}$ is the Biot number and S is the suction/injection parameter.

The skin friction coefficient with the local Nusselt number, are mathematically expressed by

$$C_f = \frac{2\tau_w}{\rho_f u_w^2}, \qquad Nu_r = \frac{rq_w}{k_f\left(T_f - T_\infty\right)}, \tag{9}$$

where τ_w and q_w are the shear stress and heat flux of the surface/heated disk, respectively

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial z}\right)_{z=0}, \quad q_w = -k_{hnf}\left(\frac{\partial T}{\partial z}\right)_{z=0}. \tag{10}$$

With the substitution of (5) and (10) into (9), the reduced skin friction coefficient and heat transfer rate are respectively given by

$$0.5\mathrm{Re}_r^{1/2}C_f=\frac{\mu_{hnf}}{\mu_f}f''(0),\ \ \mathrm{Re}_r^{-1/2}Nu_r=-\frac{k_{hnf}}{k_f}\theta'(0), \tag{11}$$

where $\mathrm{Re}_r=\dfrac{ru_w}{\nu_f}$.

3. Stability Analysis

There are a few of steps in the implementation of stability analysis which are,

i. consider the unsteady form of Eq. (2) and Eq. (3) as suggested by Merkin [33]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial z^2}, \tag{12}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial z^2}, \tag{13}$$

with a new set of similarity transformations including the dimensionless time variable τ

$$u=ar\frac{\partial f(\eta,\tau)}{\partial\eta},\quad v=-2\sqrt{a\nu_f}\,f(\eta,\tau),\quad \theta(\eta,\tau)=\frac{T-T_\infty}{T_f-T_\infty},\quad \eta=z\sqrt{\frac{a}{\nu_f}},\quad \tau=at. \tag{14}$$

By substituting Eq. (14) into Eq. (12) and Eq. (13), the transformed equations are

$$\left(\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\right)\frac{\partial^3 f}{\partial\eta^3}+2f\frac{\partial^2 f}{\partial\eta^2}-\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\partial^2 f}{\partial\eta\partial\tau}=0, \tag{15}$$

$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}\frac{\partial^2\theta}{\partial\eta^2}+2f\frac{\partial\theta}{\partial\eta}-\frac{\partial\theta}{\partial\tau}=0, \tag{16}$$

with the transformed conditions

$$\begin{gathered}f(0,\tau)=S,\quad \frac{\partial f(0,\tau)}{\partial\eta}=\lambda,\quad \frac{-k_{hnf}}{k_f}\frac{\partial\theta(0,\tau)}{\partial\eta}=Bi\left[1-\theta(0,\tau)\right],\\ \frac{\partial f(\eta,\tau)}{\partial\eta}\to 0,\quad \theta(\eta,\tau)\to 0,\quad \text{as}\quad \eta\to\infty.\end{gathered} \tag{17}$$

ii. use the perturbation equations to test the solutions' stability. The perturbation equations are given by [34]

$$\left.\begin{aligned}f(\eta,\tau)&=f_0(\eta)+e^{-\gamma\tau}F(\eta)\\ \theta(\eta,\tau)&=\theta_0(\eta)+e^{-\gamma\tau}H(\eta)\end{aligned}\right\}, \tag{18}$$

where $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$, γ is an unknown eigenvalue whereas $F(\eta)$ and $H(\eta)$ are a small relative to $f_0(\eta)$ and $\theta_0(\eta)$, respectively. Hence, by inserting Eq. (18) into Eq. (15)-(17), the linearized eigenvalue problem is

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} F''' + 2 f_0 F'' - \left(2 f_0' - \gamma\right) F' + 2 F f_0'' = 0, \tag{19}$$

$$\frac{1}{\mathrm{Pr}} \frac{k_{hnf}/k_f}{\left(\rho C_p\right)_{hnf} / \left(\rho C_p\right)_f} H'' + 2F\theta_0' + 2 f_0 H' + \gamma H = 0, \tag{20}$$

with

$$\begin{aligned} &F(0)=0, \quad F'(0)=0, \quad F''(0)=1, \quad \frac{k_{hnf}}{k_f} H'(0) = BiH(0), \\ &F'(\eta)\to 0 \text{ (relaxed)}, \quad H(\eta)\to 0, \quad \text{as} \quad \eta\to\infty, \end{aligned} \tag{21}$$

after considering the relaxation of boundary condition as implemented by Harris *et al.,* [35]. The sign of the smallest eigenvalue γ_1 will determine the stability of solutions.

4. Results and Discussions

In this section, the numerical solutions are obtained by solving Eq. (6)-(8) using the bvp4c application in the Matlab software. The impact of the respective parameters namely suction S, Biot number Bi and the volumetric concentrations of the nanoparticles (ϕ_1, ϕ_2) on the reduced skin friction coefficient and local Nusselt number/heat transfer rate are briefly discussed including the analysis of solutions' stability. For this purpose, the values are chosen based on the main references and from the trial-and-error basis due to the existence of two solutions. As stated in Section 2, the value of Prandtl number is set as $\mathrm{Pr} = 6.2$ which denotes the water base fluid at $25°C$. Meanwhile the values for other parameters are within these ranges $3 \le S \le 3.1$, $0.1 \le Bi \le 0.102$ and $0 \le \phi_1, \phi_2 \le 0.02$. Using the correlations of hybrid nanofluid by Takabi and Salehi [9], the total volumetric concentration of the Cu-Al_2O_3 hybrid nanoparticles is $\phi_{hnf} = \phi_1 + \phi_2$. The verification of model is conducted by comparing the values of $0.5\mathrm{Re}_r^{1/2} C_f$ and $\mathrm{Re}_r^{-1/2} Nu_r$ with Khashi'ie *et al.,* [11] when $Bi \to \infty$ (constant temperature condition), $\mathrm{Pr} = 6.2$ and $\lambda = 1$ as shown in Table 3. The present numerical values are in accordance with the previous data, hence, justify the accuracy of the present bvp4c codes.

The stability of the solutions is obtained by solving Eq. (19)-(21) using the continuous code of Eq. (6)-(8) in the bvp4c solver. Table 4 presents the smallest eigenvalue γ_1 of the solutions towards λ. The positive sign of γ_1 for the first solution implies the reliability and stability of the solution while opposite result is obtained for the second solution.

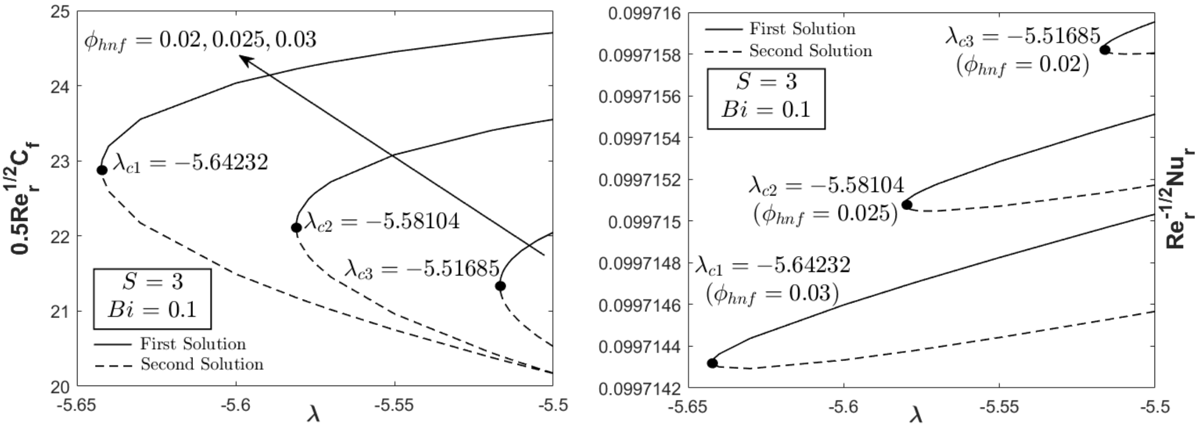
Table 3
Comparison values of $0.5\mathrm{Re}_r^{1/2}C_f$ and $\mathrm{Re}_r^{-1/2}Nu_r$ when $Bi\to\infty$, $\mathrm{Pr}=6.2$ and $\lambda=1$

S	ϕ_1	ϕ_2	Present		Khashi'ie *et al.,* [11]	
			$0.5\mathrm{Re}_r^{1/2}C_f$	$\mathrm{Re}_r^{-1/2}Nu_r$	$0.5\mathrm{Re}_r^{1/2}C_f$	$\mathrm{Re}_r^{-1/2}Nu_r$
0	0	0	-1.17372	2.54642	-1.17372	2.54642
	0	0.005	-1.20437	2.55839	-1.20437	2.55839
	0.005	0	-1.18987	2.56044	-1.18987	2.56044
	0.01	0.01	-1.26788	2.59961	-1.26788	2.59961
2.1	0	0	-4.52863	26.43654	-4.52863	26.43654
	0	0.005	-4.70055	26.41731	-4.70055	26.41731
	0.005	0	-4.59544	26.40581	-4.59544	26.40581
	0.01	0.01	-5.00649	26.33703	-5.00649	26.33703

Table 4
Smallest eigenvalue γ_1 when $S=\varepsilon=\delta=0$ and $\lambda=1$

λ	γ_1 (First solution)	γ_1 (Second solution)
-5.5168	0.0222	-0.0221
-5.5164	0.0782	-0.0774
-5.516	0.1085	-0.107
-5.51	0.3143	-0.3017
-5.5	0.4981	-0.4671

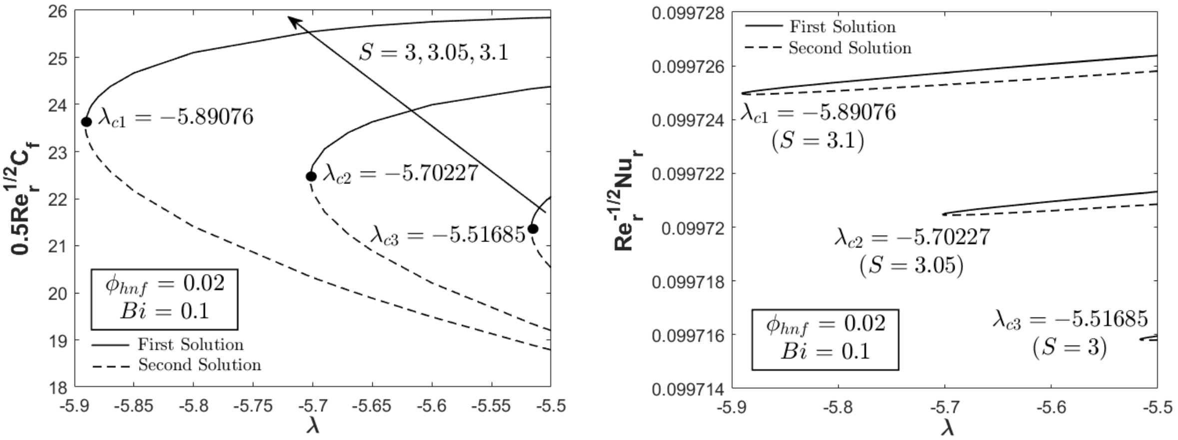
The values of $0.5\mathrm{Re}_r^{1/2}C_f$ and $\mathrm{Re}_r^{-1/2}Nu_r$ towards λ when $\phi_{hnf}=0.02,\ 0.025$ and 0.03 are presented in Figure 2(a) and Figure 2(b), respectively. The total of volumetric concentrations for hybrid Cu-Al$_2$O$_3$/water nanofluid is allocated from equal concentrations of alumina and copper nanoparticles where $\phi_{hnf}=0.02\ (\phi_1,\phi_2=0.01)$, $\phi_{hnf}=0.025\ (\phi_1,\phi_2=0.0125)$ and $\phi_{hnf}=0.03\ (\phi_1,\phi_2=0.015)$. From Figure 2, the dual solutions exist for given ϕ_{hnf} when $S=3$ and $Bi=0.1$. An increase of ϕ_{hnf} extends $|\lambda_c|$ where λ_c is the approximated separation value of the laminar flow. The first and second solutions meet at λ_c and beyond this value, no solution exists. These findings also show that the high concentration of ϕ_{hnf} can slightly reduce the heat transfer rate of the working fluid. The performance of hybrid nanofluid in enhancing the heat transfer rate was affected using high suction strength $(S>2)$ [11,29]. Moreover, as $\lambda\to\lambda_c$, both skin friction coefficient and heat transfer rate of the hybrid Cu-Al$_2$O$_3$/water nanofluid marginally deteriorate, and these results are seen for all values of ϕ_{hnf}.



(a) (b)

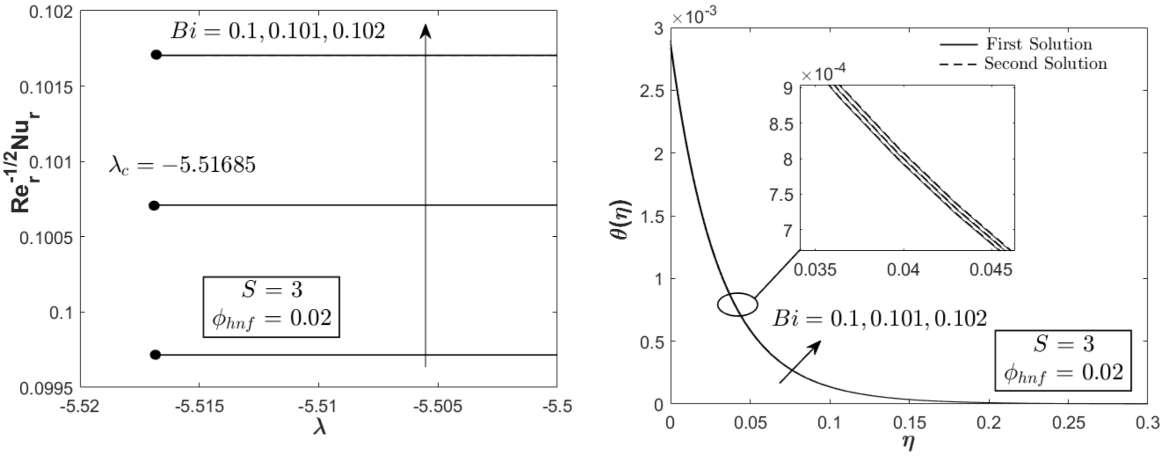
Fig. 2. The distribution of (a) $0.5\mathrm{Re}_r^{1/2}C_f$, and (b) $\mathrm{Re}_r^{-1/2}Nu_r$ towards λ for various ϕ_{hnf}

The impact of suction on the skin friction coefficient and the heat transfer rate is portrayed in Figure 3(a) and Figure 3(b), respectively. An increase of suction's strength by 5% $(S = 3, 3.05, 3.1)$ enhances both $0.5\mathrm{Re}_r^{1/2}C_f$ and $\mathrm{Re}_r^{-1/2}Nu_r$. The suction parameter also delays the separation of boundary layer flow and this is seen from the extension of the separation point where $\lambda_c = -5.51685\ (S = 3)$, $\lambda_c = -5.70277\ (S = 3.05)$ and $\lambda_c = -5.89076\ (S = 3.1)$. Meanwhile, Figure 4(a) demonstrate the effect of Biot number which represents the convective heat transfer process on the performance of the Cu-Al$_2$O$_3$/water nanofluid. As the Biot number slightly augments, the enhancement of heat transfer rate is seen. From Eq. (8), the Biot number is comparable with $-\theta'(0)$ which implies the operation of convective heating process in regulating and augmenting the wall temperature as clearly exhibited in Figure 4(b).



(a) (b)

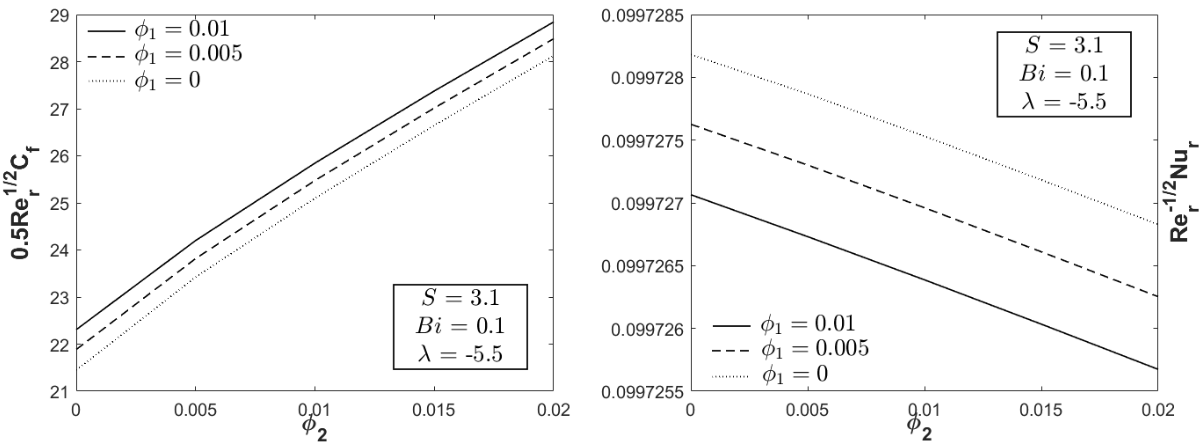
Fig. 3. The distribution of (a) $0.5\mathrm{Re}_r^{1/2}C_f$, and (b) $\mathrm{Re}_r^{-1/2}Nu_r$ towards λ for various S



(a) (b)

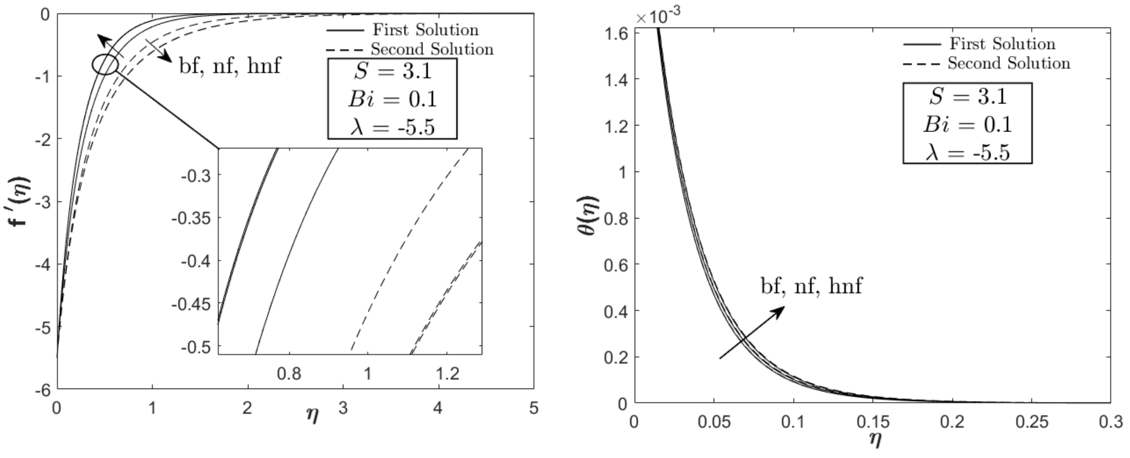
Fig. 4. The distribution of (a) $\mathrm{Re}_r^{-1/2} Nu_r$ towards λ, and (b) temperature profile with various Bi

Figure 5(a) and Figure 5(b) present the relationship between ϕ_1 and ϕ_2 on the distribution of $0.5\mathrm{Re}_r^{1/2} C_f$ and $\mathrm{Re}_r^{-1/2} Nu_r$. From Figure 5(a), the pure water $(\phi_1, \phi_2 = 0)$ has the lowest skin friction coefficient while the Cu-Al$_2$O$_3$/water nanofluid with $\phi_1 = 0.01$ and $\phi_2 = 0.02$ has the highest $0.5\mathrm{Re}_r^{1/2} C_f$. As the volumetric concentrations for Al$_2$O$_3$ (ϕ_1) and Cu (ϕ_2) increase, the skin friction coefficient also increases. Surprisingly, the use of Cu-Al$_2$O$_3$/water nanofluid in this work does not contribute to the development of the heat transfer rate as it is evident from Figure 5(b) that the pure water has the highest $\mathrm{Re}_r^{-1/2} Nu_r$. Further observation shows that the addition of ϕ_1 and ϕ_2 reduce the heat transfer performance of the working fluid. However, it is worth to mention that this discussion is only valid for the case of shrinking sheet $(\lambda = -5.5)$. Figure 6(a) and Figure 6(b) display the profiles for velocity and temperature, respectively with different fluids where bf, nf and hnf stand for pure water $(\phi_1, \phi_2 = 0)$, Cu-water nanofluid $(\phi_1 = 0, \phi_2 = 0.01)$ and Cu-Al$_2$O$_3$/water nanofluid $(\phi_1, \phi_2 = 0.01)$, accordingly. Theoretically, the boost of ϕ_1 and ϕ_2 may generate energies within the fluid particles and then, potential to increase the velocity and temperature profile.



(a) (b)

Fig. 5. The distribution of (a) $0.5\mathrm{Re}_r^{1/2} C_f$, and (b) $\mathrm{Re}_r^{-1/2} Nu_r$ towards ϕ_2 for various ϕ_1



(a) (b)

Fig. 6. (a) Velocity, and (b) temperature profiles for different fluids when $S = 3.1$, $Bi = 0.1$ and $\lambda = -5.5$

5. Conclusions

The analysis of hybrid Cu-Al_2O_3/water nanofluid flow on a stretching/shrinking and permeable disk with convective boundary condition is conducted using the bvp4c solver. The thermophysical properties of hybrid nanofluid are evaluated numerically using the existing correlation and the results are evaluated within a certain range of the control parameters namely suction S, Biot number Bi and the volumetric concentrations of the nanoparticles (ϕ_1, ϕ_2). The conclusions are

i. The separation of laminar flow can be delayed by enhancing the suction's strength and the hybrid nanoparticles volumetric concentration. This observation is based on the location of critical/separation value.
ii. The augmentation of heat transfer rate is seen for the addition of suction and Biot number.
iii. The stability analysis verifies the stability of the first solution.
iv. An increase of Biot number slightly increases the temperature profile of the Cu-Al_2O_3/water nanofluid.
v. The Cu-Al_2O_3/water nanofluid has higher distribution of the velocity and temperature profiles than the Cu-water nanofluid and pure water.

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