



Faculty of Mechanical Engineering

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**INSTABILITY AND SENSITIVITY TO IMPERFECTION OF
CONICAL SHELL SUBJECTED TO AXIAL COMPRESSION**

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UNIVERSITI TEKNIKAL MALAYSIA MELAKA

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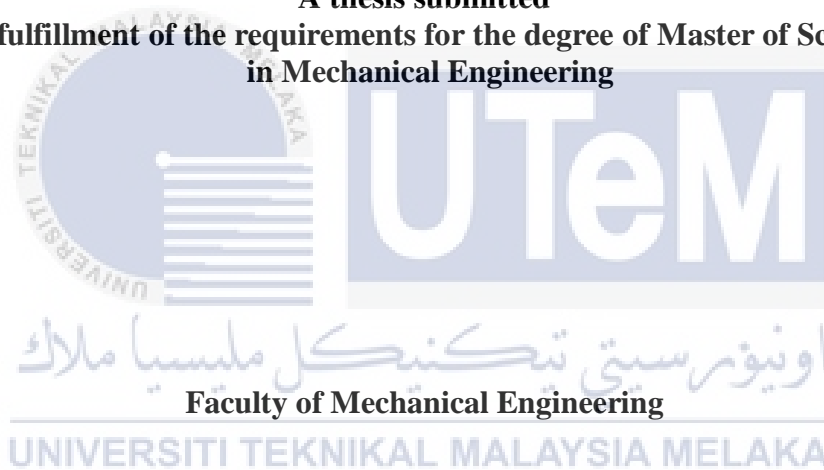
Master of Science in Mechanical Engineering

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**INSTABILITY AND SENSITIVITY TO IMPERFECTION OF CONICAL
SHELL SUBJECTED TO AXIAL COMPRESSION**

FAIRUZ MARDHIAH BINTI MAHIDAN

**A thesis submitted
in fulfillment of the requirements for the degree of Master of Science
in Mechanical Engineering**




UNIVERSITI TEKNIKAL MALAYSIA MELAKA

2021

DECLARATION

I declare that this thesis entitled “Instability and Sensitivity to Imperfection of Conical Shell Subjected to Axial Compression” is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.



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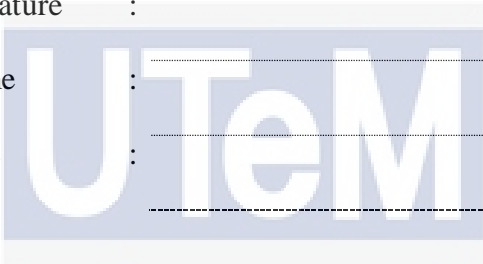
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I hereby declare that I have read this thesis and in my opinion, this thesis is sufficient in terms of scope and quality for the award of the degree of Master of Science in Mechanical Engineering.

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DEDICATION

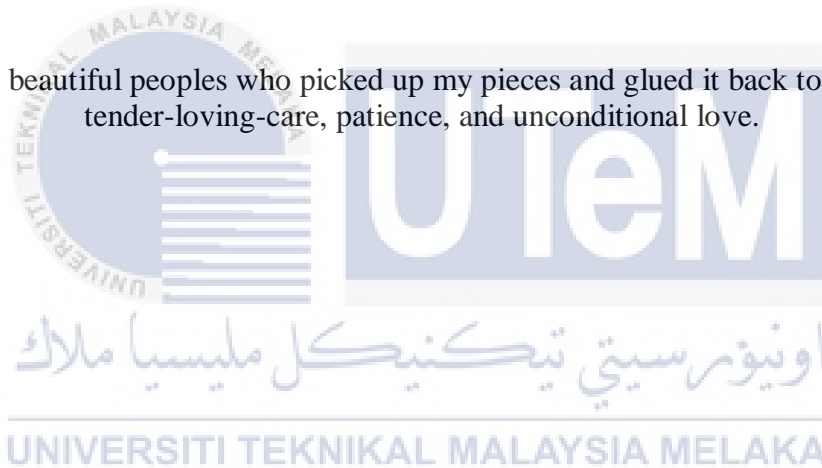
My all: Allah SWT

My very own Godsend blessings: Ayah and Ibu

My crutches: Iti, Abang, Umi

and

To all the beautiful peoples who picked up my pieces and glued it back together with tender-loving-care, patience, and unconditional love.



ABSTRACT

Shell structures have been widely used in engineering applications such as pipelines, aerospace, marine structures, and cooling towers. Occurring suddenly and generally inadvertent due to its nature, buckling is one of the main failure considerations in the design of these structures. The presence of defects, such as geometric imperfection, uneven loading, the boundary condition of the shell, material discontinuity/crack imperfection, and so on in shell structures may severely compromise their buckling behavior and jeopardize the structural integrity. In this study, experimental and numerical investigations on the buckling behavior of axially compressed conical shell with uneven axial length imperfection were carried out. The effect of imperfection amplitude, wave number, and wave type were investigated. Initial geometric imperfection in the form of (i) sinusoid waves, (ii) triangle waves, and (iii) square waves having different wave number are explored. This thesis contains experimental data verification and further Finite Element (FE) prediction. Excellent repeatability between experimental results with only 0% to 7% of error was revealed. Abaqus FE was used to simulate the numerical modelling. The imperfection amplitude and shape highly influenced the load-carrying capacity of conical shells. Triangular waves yields the lowest imperfection sensitivity in comparison to other wave shape. Furthermore, the influence of wave number was also studied for each wave shapes. It was found that the wave number has insignificant influence on the buckling load of the axially compressed cones. In the next step, a comparison between different imperfection approach, namely (i) Eigenmode imperfection, (ii) Single and Multiple Load Indentation (SLI and MLI), (iii) crack imperfection, and (iv) uneven axial length imperfection was carried out to determine the worst knockdown factor (KDF) for axially compressed steel conical shell. As predicted, imperfection severely affected the buckling strength of conical shells, and the decrease in buckling strength is heavily reliant on the imperfection approach. It is apparent that for axially compressed cones with radius-to-thickness ratio, $r_1/t = 25$, uneven axial length imperfection was seen to produce the lowest buckling load, followed by eigenmode imperfection, crack imperfection, and load indentation for imperfection amplitude $0 < A < 1.68$. Increasing the imperfection amplitude, A , beyond this level ($A \geq 1.68$), the highest reduction in buckling load was found to be eigenmode imperfection, followed by uneven axial length, crack and load indentation. Furthermore, based on ECCS 2008 recommendation for imperfection tolerance, the lower bound curve which can be used for design recommendation purposes has been proposed for the worst imperfection approach case (uneven axial length and eigenmode imperfection) for different conical shell geometry configurations. Finally, the proposed lower bound curve was compared with the plot of NASA SP-8019 recommended imperfection correlation factor for axially compressed cone. Results showed that the proposed lower bound curve for axially compressed conical shells with uneven axial length imperfection is notably higher than the NASA SP-8019 KDF by 7%. However, axially compressed conical shells with eigenmode imperfection were seen to underestimate NASA's KDF by 55%, particularly for elastic buckling.

KETAKSTABILAN DAN KESENSITIVITIAN TERHADAP KETAKSEMPURNAAN OLEH KELOMPANG BERKON DI BAWAH MAMPATAN PAKSI

ABSTRAK

Struktur kelompang banyak digunakan dalam aplikasi kejuruteraan seperti saluran paip, aeroangkasa, laut, dan menara penyejuk. Berlaku tiba-tiba dan tidak sengaja, lengkokan adalah salah satu pertimbangan kegagalan utama dalam struktur ini. Ketaksempurnaan geometri, beban yang tidak rata, keadaan sempadan kelompang, ketakselajaran bahan/ketaksempurnaan retak dalam kelompang menjejaskan tingkah laku lengkokan dan integriti struktur. Dalam kajian ini, penyelidikan ujikaji dan berangka mengenai tingkah laku lengkokan kelompang berkon termampat paksi dengan ketaksempurnaan panjang paksi dilakukan. Kesan ketaksempurnaan amplitud, bilangan gelombang dan jenis gelombang disiasat. Tiga jenis ketaksempurnaan geometri dengan bilangan gelombang berbeza dianalisis, iaitu (i) gelombang bentuk sinus, (ii) gelombang segitiga, dan (iii) gelombang segiempat sama. Pengesahan data ujikaji dan lanjutan analisis unsur terhingga disediakan. Kebolehhulangan data ujikaji yang baik dinyatakan melalui keputusan ujian dengan hanya 0% hingga 7% ralat. Unsur terhingga Abaqus digunakan untuk menyelakutkan pemodelan berangka. Ketaksempurnaan bentuk dan amplitud mempengaruhi beban kon. Gelombang berbentuk segitiga menghasilkan kepekaan terhadap ketaksempurnaan yang terendah berbanding gelombang yang lain. Seterusnya, kesan bilangan gelombang juga dikaji untuk setiap bentuk gelombang. Ianya didapati bahawa bilangan gelombang adalah takbererti kepada keupayaan menanggung beban kon. Perbandingan antara pendekatan ketaksempurnaan yang berbeza iaitu (i) mod Eigen, (ii) Lekukan Beban Tunggal dan Berganda (LBT and LBB), (iii) keretakan and (iv) ketaksempurnaan panjang paksi direalisasikan untuk menentukan faktor kejatuhan terburuk bagi kelompang berkon keluli. Seperti yang diramalkan, ketaksempurnaan sangat mempengaruhi kekuatan lengkokan kelompang berkon. Kon yang dimampatkan secara paksi dengan nisbah jejari-ke-ketebalan, $r_1/t=25$, ketaksempurnaan panjang paksi dilihat menghasilkan beban lengkokan terendah, diikuti oleh mod eigen, ketaksempurnaan retak dan lekukan beban untuk amplitud ketaksempurnaan $0 < A < 1.68$. Apabila amplitud, A , ditingkatkan, melebihi tahap ini ($A \geq 1.68$), pengurangan tertinggi dalam beban lengkokan didapati adalah mod eigen, diikuti oleh panjang paksi yang tidak rata, ketaksempurnaan retak dan lekukan beban. Berdasarkan saranan ECCS 2008 untuk had terima ketaksempurnaan, lengkung batasan bawah telah dicadangkan untuk kes ketaksempurnaan terburuk (panjang paksi yang tidak rata) untuk tatarajah geometri kelompang berkon yang berbeza. Akhirnya, lengkung batasan bawah yang dicadangkan dibandingkan dengan plot faktor sekaitan ketaksempurnaan yang disarankan oleh NASA SP 8019 untuk kon termampat paksi. Hasil kajian menunjukkan bahawa lengkung batasan bawah yang dicadangkan untuk kelompang berkon termampat paksi dengan ketaksempurnaan panjang paksi lebih tinggi daripada NASA SP-8019 KDF sebanyak 7%. Walau bagaimanapun, kelompang berkon yang dimampatkan secara paksi dengan mod eigen dilihat lebih rendah dari KDF NASA sebanyak 55%, terutamanya untuk lengkokan elastik.

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UNIVERSITI TEKNIKAL MALAYSIA MELAKA

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LIST OF SYMBOLS AND ABBREVIATIONS

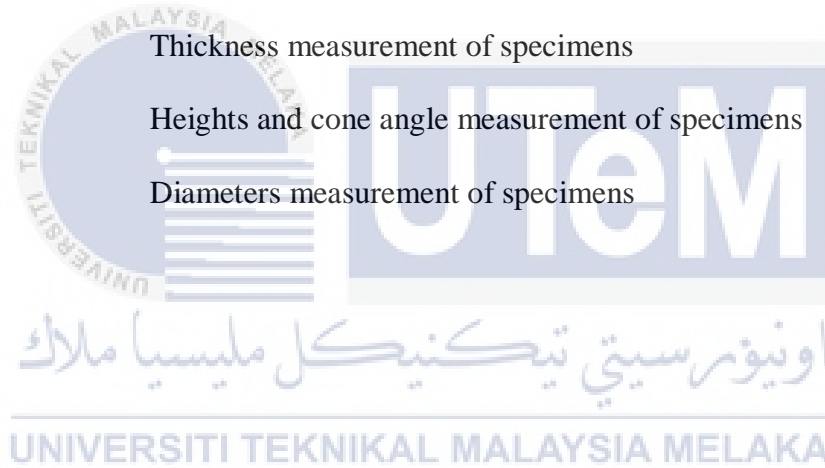
A	-	Imperfection amplitude
E	-	Young's modulus
F	-	Axial force
F_{coll}	-	Axial collapse force
FEA	-	Finite Element Analysis
GPa	-	Giga Pascal
I	-	Number of load indentation
kN	-	Kilo Newton
L	-	Axial length of cone
LBA	-	Linear Bifurcation Analysis
l_{gx}	-	Gauges of length according to ECCS 2008
L_{slant}	-	Slant length of cone
mm	-	Millimeter
MPa	-	Mega Pascal
n	-	Eigenmode number
N	-	Number of waves
r_1	-	Cone's top radius

- r_2 - Cone's bottom radius
- R - Cylinder's radius
- t - Wall thickness
- U_{0x} - Load indentation tolerance parameter according to ECCS 2008
- β - Cone semi-vertex angle
- θ - Crack angle
- ν - Poisson's ratio
- σ_{yp} - Yield strength of material



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LIST OF PUBLICATIONS

Journal with Impact Factor

Ifayefunmi, O., and Mahidan, F.M., 2020. Buckling of Cracked Cones subjected to Axial Compression. *Latin American Journal of Solids and Structures*, 17(9), pp. 1-13. <https://doi.org/10.1590/1679-78256241> (ISI indexed, Q2, IF = 1.289 (2018))

Ifayefunmi, O., and Mahidan, F.M., 2021. Collapse of Conical Shells having Single Dimple Imperfection under Axial Compression. *Journal of Pressure Vessel and Technology, Transactions of the ASME*, 143(1), pp. 011301-011308. <https://doi.org/10.1115/1.4047681> (ISI indexed, Q2, IF = 1.142 (2019))

Mahidan, F.M., and Ifayefunmi, O., 2020. Buckling of Axially Compressed Cones with Imperfect Axial Length. *Latin American Journal of Solids and Structures*, 17(7), pp. 1-20. <https://doi.org/10.1590/1679-78256189> (ISI indexed, Q2, IF = 1.289 (2018))

Mahidan, F.M., and Ifayefunmi, O., 2020. The Imperfection Sensitivity of Axially Compressed Steel Conical Shells – Lower Bound Curve. *Thin-Walled Structures*, 159, pp. 107323-107335. (ISI indexed, Q1, IF = 4.108 (2019))

Indexed Journal

Ifayefunmi, O., Mahidan, F.M., and Wang, S.H., 2019. Buckling of Cones with Imperfect Length subjected to Axial Compression. *International Journal of Mechanical and*

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<https://doi.org/10.24247/ijmperdaug201923>

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<https://doi.org/10.24247/ijmperdapr20206>

Non-indexed Journal

Ifayefunmi, O., Mahidan, F.M., and Abu Bakar, N., 2020. Buckling of Axially Compressed Imperfect Steel Cones with Local Dents. *International Journal of Mechanical and Production Engineering Research and Development*, 10(3), pp. 10807-10816. <https://doi.org/10.24247/ijmperdjun20201036>

Ifayefunmi, O., Mahidan, F.M., and Maslan, M.H., 2020. Instability of Conical Shells with Multiple Dimples under Axial Compression. *International Journal of Recent Technology and Engineering*, 8(5), pp. 1022–1027.
<https://doi.org/10.35940/ijrte.e6102.018520>

Conference Proceedings

Mahidan, F.M., Ifayefunmi, O., and Fadzullah, S.H.S.M., 2020. The Influence of Contact Interaction on the Buckling Behaviour of Axially Compressed Conical Shell with Imperfect Axial Length. In: *International Conference on Design and Concurrent Engineering*, Malacca, Malaysia, 21-22 September 2020.

CHAPTER 1

INTRODUCTION

1.1 Background

Conical shell structures found its application in various industries such as offshore, marine, mechanical, civil, and aeronautical. Different industries used different thicknesses of conical shell as this will determine the failure mode of the structures. For thin-walled conical shells, the failure is usually governed by elastic buckling, while for thicker shells, the failure is often at the plastic region. For instance, in the offshore industry, relatively thick conical structures are often being used as pressure vessels, pipelines, legs for oil drilling platform and connectors between two cylinders that have different diameters (Błachut, 2016; Ifayefunmi, 2017). Whilst, thin-walled conical shells were applied to most aeronautical, aerospace, and civil industries (Khakimova et al., 2014). In aerospace applications, thin conical structures are used as parts of launcher transport systems and adapters between cylindrical shells of different diameters, as stated by Khakimova et al. (2016b) and Wagner et al. (2018), respectively. This statement is also supported in the work presented by Hao et al. (2016). Furthermore, Chahardoli and Alavi Nia (2017) expressed that thin-walled conical structures are used in rail and car industry as energy absorber, as seen also in the work of Jafarian and Rezvani (2019).

When in use, conical shells are often subjected to various types of loading such as axial compression, external pressure, internal pressure, or a combination of loads, which can lead to instability. This technical challenge has led to extensive research in the area of axially compressed conical shells. A collection of experimental data on isotropic conical

shells under axial compression has been presented in Seide et al. (1960) with different top radius-to-thickness ratios, r_1/t . Following this, Seide (1961) derived a simple formula to calculate the critical elastic buckling load, F_{crit} of a perfect isotropic axially compressed conical shell, see Equation (1.1):

$$F_{crit} = \frac{2\pi Et^2 \cos^2 \beta}{\sqrt{3(1 - \nu^2)}} \quad (1.1)$$

where

E = Young's modulus

t = wall thickness

β = cone semi-vertex angle

ν = Poisson's ratio

Nevertheless, Equation (1.1) is restricted to the failure of truncated cones in the elastic region. To account for the plastic mechanism of conical shells, Chryssanthopoulos and Poggi (2001) proposed a formula for the collapse strength, F_{coll} , of unstiffened conical shell subjected to axial compression as expressed in Equation (1.2).

$$F_{coll} = 2\pi r_1 t \sigma_{yp} \cos \beta \quad (1.2)$$

where

r_1 = the top radius of the cone

t = wall thickness

σ_{yp} = yield strength of material

β = cone semi-vertex angle

However, early research on the subject matter reports large discrepancies between