



Faculty of Mechanical Engineering

**WIDEBAND NONLINEAR DYNAMIC VIBRATION ABSORBER
USING PIECEWISE LINEAR STIFFNESS FOR EFFECTIVE
STRUCTURAL VIBRATION SUPPRESSION**

اونيورسيتي تيكنيكل مليسيا ملاك
UNIVERSITI TEKNIKAL MALAYSIA MELAKA

Muhammad Harith bin Mustaffer

Master of Science in Mechanical Engineering

2020

DECLARATION

I declare that this thesis entitled “Wideband Nonlinear Dynamic Vibration Absorber Using Piecewise Linear Stiffness for Effective Structural Vibration Suppression” is the result of my own research except as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in the candidature of any other degree.



Signature :

[Handwritten Signature]

Name :

MUHAMMAD HARTH BIN MUSTAFFER

Date :

30/11/2021

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

APPROVAL

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of Master of Science in Mechanical Engineering.

Signature :

Supervisor Name :

Date :



[Handwritten Signature]
Rosmida Ramla
1/12/2021

اونيورسيتي تيكنيكل مليسيا ملاك

UNIVERSITI TEKNIKAL MALAYSIA MELAKA

DEDICATION

To my beloved mother and father



ABSTRACT

Vibration has become a major concern nowadays due to its tendency to produce undesirable noise and to potentially result in a harmful response. Generally, there are two ways to control the level of vibration in machines or structures. One of the techniques is by isolating the receiver of the vibration from the source. Isolation is a feasible solution if the level of vibration of the source cannot be altered. In some cases, the source of the vibration must be suppressed, hence isolation may not be a feasible solution. For these cases, the dynamic vibration absorber (DVA) is normally used. This is done by attaching another single-degree-of-freedom (SDOF) oscillating system onto the vibrating primary structure. The DVA is designed to have a natural frequency similar to the troublesome frequency of the primary structure. Many of the currently available passive dynamic vibration absorbers are not fully efficient in suppressing the vibration of the primary structure due to narrow operating frequency bandwidth. The performance of the DVA deteriorates even more in the application where the structure's troublesome frequency varies over time and it requires constant retuning of its natural frequency. Its low tolerance towards frequency mistuning may increase the level of vibration. Thus, it is necessary to design a DVA with efficient tuning capability and less sensitive towards mistune. In this study, the nonlinear dynamic vibration absorber (NDVA) with a tuneable piecewise linear stiffness mechanism which behaves similar to hardening stiffness mechanism was designed. The hardening stiffness is proven to perform better due to the larger suppression bandwidth. However, unlike the hardening stiffness mechanism, the proposed piecewise linear stiffness mechanism offers better tuning capability. The mechanism is composed of a cantilever beam constrained by two limit blocks which are adjustable in both horizontal and vertical directions. Firstly, the analytical study was performed before developing the NDVA to study its static and dynamic characteristics. The characterization study of the NDVA includes different limit block configurations (horizontal position and vertical gap), input amplitude, mass, and stiffness. Once the NDVA was fabricated, the analytical results were then validated experimentally by conducting quasi-static and dynamic measurements. The quasi-static measurement was done by exciting the base of the NDVA at low frequency to measure for force-deflection relationship. As for dynamic measurement, the base of the NDVA was once again excited on the electrodynamic shaker using sweep-up and sweep down of the excitation frequency between 10 Hz to 40 Hz. Finally, the performance of the NDVA in suppressing the vibration of the primary structure was measured and compared with its equivalent linear DVA. This was done by attaching the NDVA on the structure connected to the shaker and was excited using a similar range of sweep-up and sweep down excitation frequency. The results show a promising performance of the NDVA with an increase in suppression frequency bandwidth compared to its equivalent linear DVA.

**PENYERAP GETARAN DINAMIK JALUR LEBAR TAK LINEAR MENGGUNAKAN
KEKAKUAN LINEAR SESECEBIS UNTUK PENYERAPAN GETARAN STRUKTUR
YANG BERKESAN**

ABSTRAK

Getaran telah menjadi kebimbangan utama pada masa kini disebabkan oleh kecenderungannya untuk menghasilkan getaran yang tidak diinginkan dan menghasilkan tindak balas yang berpotensi berbahaya. Secara umumnya, terdapat dua cara untuk mengawal kadar getaran pada sesebuah mesin atau struktur. Salah satu cara adalah dengan mengasingkan penerima getaran daripada puncanya. Pengasingan adalah cara yang sesuai jika tahap getaran pada puncanya tidak berubah. Dalam sesetengah keadaan, punca getaran mesti dikurangkan, oleh itu pengasingan bukan cara yang sesuai. Untuk keadaan ini, penyerap getaran dinamik (DVA) biasanya digunakan. Ia dilaksanakan dengan menghubungkan sistem getaran satu darjah kebebasan (SDOF) pada struktur utama yang bergetar. DVA ini direka supaya mempunyai frekuensi tabii yang sama dengan frekuensi bermasalah pada struktur utama. Kebanyakan penyerap getaran dinamik pasif sedia ada tidak berkesan sepenuhnya dalam mengurangkan getaran struktur utama kerana ia mempunyai lebar jalur frekuensi yang kecil. Prestasi DVA juga menjadi lebih merosot dalam aplikasi di mana frekuensi bermasalah pada struktur berubah-ubah mengikut masa dan sering kali memerlukan penyesuaian semula frekuensi tabiinya. Daya tahannya yang rendah terhadap salah suaian frekuensi akan meningkatkan lagi tahap getaran. Oleh itu, adalah perlu untuk mencipta sebuah DVA yang mempunyai kemampuan penyesuaian yang cekap dan kurang sensitif terhadap salah suaian frekuensi. Dalam kajian ini penyerap getaran dinamik tak linear (NDVA) dengan mekanisma kekakuan linear sesecebis boleh ubah yang mempunyai sifat yang sama seperti mekanisma pengerasan kekakuan telah direka. Pengerasan kekakuan ini telah terbukti lebih berkesan kerana mempunyai lebar jalur penyerapan yang lebih besar. Walau bagaimanapun, tidak seperti mekanisma pengerasan kekakuan, mekanisma linear sesecebis ini mempunyai kemampuan penyesuaian yang lebih baik. Mekanisma ini terdiri daripada jalur rasuk yang dikekang oleh dua blok penghalang, di mana jarak melintang dan jarak menegaknya boleh dilaraskan. Pertamanya, kajian analitikal pada NDVA dijalankan sebelum mereka bentuk NDVA ini untuk mengkaji ciri-ciri statik dan dinamik. Kajian pencirian ini dijalankan dengan menggunakan beberapa parameter seperti konfigurasi blok penghalang (jarak melintang dan menegak), tahap ketinggian masukan, jisim dan kekakuan. Setelah NDVA direka bentuk, hasil ujikaji analitikal kemudiannya disahkan secara eksperimen dengan menggunakan cara pengukuran seakan-statik dan dinamik. Pengukuran seakan-statik dilakukan dengan mengenakan getaran pada tapak NDVA menggunakan penggongcang pada frekuensi rendah untuk mengukur daya dan pemesongan. Untuk pengukuran dinamik tapak NDVA digetarkan menggunakan frekuensi menaik dan menurun antara 10 Hz ke 40 Hz. Akhirnya prestasi NDVA dalam menyerap getaran pada struktur utama diukur dan dibandingkan dengan DVA linear. Hasilnya menunjukkan prestasi NDVA yang baik dengan peningkatan lebar jalur frekuensi berbanding dengan DVA linear.

ACKNOWLEDGEMENTS

First and foremost, I would like to express my sincerest gratitude to those who are involved in the completion of this thesis, especially to my supervisor Associate Professor Dr. Roszaidi bin Ramlan and co-supervisor Associate Professor Dr. Azma Putra from the Faculty of Mechanical Engineering Universiti Teknikal Malaysia Melaka (UTeM) who have directly gave guidance as well as teaching me the value of being highly independent. Additionally, I would also keen on extending my deepest gratitude to those involved in the technical aspects of the research, in particular towards Mr. Johardi bin Abdul Jabar, the Assistant Engineer from the Vibroacoustics Laboratory of Faculty of Mechanical Engineering Universiti Teknikal Malaysia Melaka for his assistance, time spent and effort in all the lab works that have been done.

Special thanks are dedicated to my loving parents and siblings for continuous support morally. Lastly, special thanks to my colleagues either directly or indirectly for their support throughout the period of master study together.

TABLE OF CONTENTS

	PAGE
DECLARATION	
APPROVAL	
DEDICATION	
ABSTRACT	i
ABSTRAK	ii
ACKNOWLEDGEMENTS	iii
TABLE OF CONTENTS	iv
LIST OF TABLES	vii
LIST OF FIGURES	ix
LIST OF APPENDICES	xx
LIST OF SYMBOLS	xxi
LIST OF PUBLICATIONS	xxiv
CHAPTER	
1. INTRODUCTION	1
1.1 Research background	1
1.2 Problem statement	3
1.3 Objectives of research	4
1.4 Research scopes	4
1.5 Thesis outline	6
2. LITERATURE REVIEW	8
2.1 Introduction	8
2.2 Fundamental of vibration	8
2.3 Vibration isolation	11
2.3.1 Linear vibration isolation	12
2.3.2 Passive control vibration isolator	15
2.3.3 Active control vibration isolator	18
2.3.4 Non-linear vibration isolator	23
2.4 Vibration absorption	26
2.4.1 Linear dynamic vibration absorption	28
2.4.2 Passive control dynamic vibration absorber	31
2.4.3 Active control dynamic vibration absorber	38
2.4.4 Non-linear dynamic vibration absorber	41
2.4.5 Piecewise-linear stiffness dynamic vibration absorber	48
2.5 Summary	50

3.	METHODOLOGY	51
3.1	Introduction	51
3.2	Theoretical analysis	53
3.2.1	Theoretical prediction of the primary structure natural frequency	53
3.2.2	Static analysis of NDVA	54
3.2.3	Equivalent stiffness method	56
3.2.4	Dynamic analysis	57
3.3	Device development	58
3.4	Experimental analysis	61
3.4.1	Natural frequency of the primary structure	61
3.4.2	Quasi-static measurement	62
3.4.3	Dynamic measurement	65
3.5	Performance of NDVA	68
3.6	Summary	71
4.	THEORETICAL RESULT AND DISCUSSION	73
4.1	Introduction	73
4.2	Static analysis of NDVA	73
4.2.1	Static beam deflection using double integration method	73
4.2.2	Equivalent stiffness method	80
4.3	Dynamic analysis of NDVA	83
4.4	Summary	88
5.	EXPERIMENTAL RESULT AND DISCUSSION	90
5.1	Introduction	90
5.2	Characterization of the static properties	90
5.3	Characterization of the dynamic properties	93
5.3.1	Effect of the limit block configuration (horizontal position, x_i and vertical gap, y_i)	93
5.3.2	Effect of the amplitude of input displacement	102
5.3.3	Effect of the different tip mass	105
5.3.4	Effect of the beam length (stiffness)	107
5.4	Performance of the linear DVA and the NDVA	109
5.4.1	Performance of the linear DVA	110
5.4.2	Performance of the NDVA	115
5.4.3	Effect of the mistuned frequency ratio, f_a/f_s	116
5.4.4	Effect of the limit block configuration with $f_a/f_s = 1$	121
5.4.5	Effect of the input displacement amplitude with $f_a/f_s = 1$	124
5.4.6	Effect of the mass ratio (μ) with $f_a/f_s = 1$	126
5.5	Summary	127
6.	CONCLUSION AND RECOMMENDATIONS	129
6.1	Conclusion	129

6.1.1	Characterization of static and dynamic properties of the NDVA	129
6.1.2	Performance of the attached NDVA on the primary structure	131
6.2	Recommendations	132
REFERENCES		133
APPENDICES		142



LIST OF TABLES

TABLE	TITLE	PAGE
3.1	Specification of the cantilever beam structure	53
3.2	Mechanical properties and dimensions of the steel beam and brass tip mass	61
3.3	Variable configuration parameters for static analysis	63
3.4	Configuration used for the dynamic analysis	66
5.1	Configuration of the device based on the different horizontal positions, x_i at the vertical gap, $y_i = 1$ mm	98
5.2	Configuration of the device based on the different vertical gaps, y_i at the horizontal position, $x_i = 45$ mm	101
5.3	Comparison of response of the primary structure for linear DVA	113
5.4	Comparison of swept-up frequency structural response of the NDVA	119
5.5	Comparison of swept-down frequency structural response of the NDVA	120
5.6	Vibration reduction bandwidth of NDVA over linear DVA on the structural response from different limit block configurations	123
5.7	Vibration reduction bandwidth with the application of the NDVA for different input amplitude	125



LIST OF FIGURES

FIGURE	TITLE	PAGE
2.1	Harmonic motion of simple pendulum oscillation	9
2.2	Two harmonic motions with a phase angle of φ (Den Hartog, 1985)	10
2.3	Schematic diagram of vibration isolation: (a) mass excitation (force, $f(t)$ is applied by equipment and transmitted force, $f_T(t)$ to the foundation); (b) base excitation (foundation motion, $y(t)$ is transmitted to equipment motion, $x(t)$)	12
2.4	Transmissibility curve with different damping ratio (Rao, 2010)	14
2.5	Common elastomeric isolators (Robert, 2007)	15
2.6	Typical metal spring isolators (Kinetic Noise Control, 2019)	16
2.7	Common types of air springs. (a) Air spring with convolutions. (b) A rolling lobe air spring. (c) Rolling diaphragm air spring. (d) Air spring having a diaphragm and an elastomeric sidewall (Harris and Piersol, 2002)	17
2.8	Schematic diagram of active vibration isolation (a) parallel type (b) series type which consists of an actuator and a control unit	19
2.9	Schematic structure of an active vibration isolator composed of electromagnetic actuator and air spring (Li et al., 2015)	20

2.10	(a) Design concept of the component-level frequency tuneable isolator; (b) Experimental setup for the frequency tuneable ability validation (Zhang et al., 2016)	21
2.11	Configuration of wind turbine blade section with active external trailing-edge flap: (a) side view; (b) bottom view; (c) layout of actuator array; double-lever mechanical linkage (Sun et al., 2017)	22
2.12	(a) A simple structure for mounting and constraining Euler Spring. (b) Force-displacement contribution from the individual spring (Winterflood et al., 2002)	24
2.13	Schematic diagram of typical quasi-zero stiffness mechanism (a) vertical coil with two buckled beams, (b) links restraint against horizontal springs, (c) buckled beam isolator and (d) Vertical and horizontal spring system (Alabuzhev et al., 1989)	25
2.14	Experimental setup for quasi-zero stiffness using magnetic spring (Robertson et al., 2013)	26
2.15	DVA installed on top of Taipei 101 (Taipei 101, 2018)	28
2.16	Model of linear dynamic vibration absorber (DVA) for (a) undamped and (b) damped (Den Hartog, 1985)	29
2.17	FRF of the structure without DVA and with DVA for $f = 1$ and $\mu = 0.05$ at different damping ratio, ζ_a (Den Hartog, 1985)	30
2.18	Design of DVA for suppressing vibration of the mass, m_s due to ground motion, y (Wong and Cheung, 2008)	32

2.19	(a) Photo of the experimental setup of main system with PVA; (b) Schematic diagram of the primary system mounted with a dual-bean PVA (Huang and Lin, 2014)	33
2.20	(a) Photo of the experimental setup and schematic diagram of the MDVA; (b) Photo of the experimental setup and schematic diagram of magnetic damper (Wang et al., 2017)	34
2.21	(a) Numerical study on the natural frequency of the Sun Deck (b) Schematic shift position of the masses at the ends of beams; (c) Position of TMD placed half the high of the girder of the Sun Deck (Pais and Boote, 2017)	35
2.22	Schematic diagram of the milling spindle with the absorber (Gafsi et al., 2017)	36
2.23	A laminate metamaterial with a spring-mass subsystem (b) A unit cell of the laminate metamaterial (He et al., 2017)	36
2.24	The TVA-LEC device for (a) upper (b) lower membrane (Yao et al., 2018)	37
2.25	Active DVA control with (a) fully active method and (b) hybrid combination of passive and active method (Trindade and Benjeddou, 2002)	39
2.26	Exploded view of the sandwich beam with ER (Lin et al., 2017)	40
2.27	Schematic of the VCM structure b) Chart of the distribution of the magnetic lines of force (Chen et al., 2005)	41

2.28	(a) Schematic diagram of nonlinear spring; (b) force-deflection relationship for softening and hardening nonlinear spring compared to linear spring (Kovacic and Brennan, 2011)	42
2.29	Response of nonlinear DVA under base excitation for hardening and softening compared to linear DVA	43
2.30	Single pendulum nonlinear energy absorber coupled with the piston assembly (Dolatabadi et al., 2017)	45
2.31	Magnetic levitation system with threaded supports to position the outer magnet and copper wire that was wound around the outer casing (Mann and Sims, 2009)	46
2.32	Photographs of energy the energy harvesting device: (a) full view and (b) the arrangement of the magnets with hardening and bi-stable mode characteristic (Ramlan et al., 2010)	47
2.33	Coupled system consists of a nonlinear system attached to an electro-dynamic shaker. (a) Photograph of the system, (b) Photograph showing the details of the nonlinear system attached to the shaker, (c) Schematic view (Gatti et al., 2010)	48
2.34	Schematic of wideband MPG (Soliman et al., 2009)	49
3.1	Flowchart of the research methodology	52
3.2	(a) Schematic of the cantilever beam system (b) NDVA hardening mechanism constrained by two limit blocks	55
3.3	Restoring force-displacement relationship for Duffing oscillator with the linearized equivalent stiffness (Wagg and Neild, 2010)	57

3.4	Exploded view of the NDVA parts	59
3.5	Isometric view of the NDVA assembly	59
3.6	Schematic diagram of the FRF measurement of the primary structure	62
3.7	Schematic diagram of the quasi-static measurement setup	64
3.8	Experimental setup for the quasi-static measurement	64
3.9	Schematic diagram for the dynamic characterization measurement setup	67
3.10	Actual setup arrangement for the dynamic characterization measurement	67
3.11	Schematic diagram for dynamic performance measurement setup	69
3.12	Actual setup arrangement for dynamic performance measurement	69
3.13	Response of the primary structure without DVA (dashed-dotted) and the suppressed response of the structure due to the effect of DVA (solid) with vibration reduction level and vibration reduction bandwidth (Hsu, 2013)	71
4.1	The cantilever beam system with horizontal position, x_i and vertical gap, y_i of the limit block	74
4.2	The elastic curve of the cantilever beam mechanism with unconstrained and constrained region	74
4.3	Free body diagram of a cantilever beam system with a limit block	75
4.4	The schematic of part 1	76
4.5	The schematic of part 2	77

4.6	Comparison of force-deflection relation of different x_i position of $x_i = 25$ mm (dotted black), $x_i = 35$ mm (dashed-dotted blue), and $x_i = 45$ mm (dashed red) with $y_i = 1$ mm	79
4.7	Comparison of force-deflection relation of different y_i gap of $y_i = 3$ mm (dotted black), $y_i = 2$ mm (dashed-dotted blue), and $y_i = 1$ mm (dashed red) with $x_i = 35$ mm	80
4.8	Illustration of the equivalent restoring force	81
4.9	Comparison of stiffness-deflection relation for different x_i ; $x_i = 25$ mm (dotted black), $x_i = 35$ mm (dashed-dotted blue), and $x_i = 45$ mm (dashed red) with $y_i = 1$ mm	82
4.10	Comparison of stiffness-deflection relation for different y_i ; $y_i = 3$ mm (dotted black), $y_i = 2$ mm (dashed-dotted blue), and $y_i = 1$ mm (dashed red) with $x_i = 35$ mm	83
4.11	Jump-up and jump-down point (red-dot circle) on low-amplitude branch and high amplitude branch, respectively, for NDVA frequency response	86
4.12	Frequency response curve of NDVA at damping factor, $c_a = 0.2067$, mass, $m_a = 40$ g plotted for different limit block configuration $x_i = 25$ mm (black), $x_i = 35$ mm (blue), $x_i = 45$ mm (red) with $y_i = 1$ mm	87
4.13	Frequency response curve of NDVA at damping factor, $c_a = 0.2067$, mass, $m_a = 40$ g, input amplitude, $X_b = 0.5$ mm plotted for different limit block configuration $y_i = 1$ mm (red), $y_i = 2$ mm (blue), and $y_i = 3$ mm (black) with $x_i = 35$ mm	87

4.14	Frequency response curve of NDVA with damping factor, $c_a = 0.2067$, mass, $m_a = 0.040$ g plotted for different input amplitude, X_b of 0.5 mm (black), 0.65 mm (blue) and 0.75 mm (red) for $x_i = 35$ mm and $y_i = 1$ mm	88
5.1	Comparison of the force-deflection relation for different horizontal configurations x_i while vertical configuration y_i is fixed at 1 mm; $x_i = 25$ mm (\times), $x_i = 35$ mm (*), and $x_i = 45$ mm (+)	91
5.2	Comparison of the force-deflection relation for different vertical configurations y_i while the horizontal configuration is fixed at 45 mm; $y_i = 1$ mm (+), $y_i = 2$ mm (*), and $y_i = 3$ mm (\times)	91
5.3	Measured frequency response of real (left) and amplitude (right) curves of the NDVA system of configuration $x_i = 25$ mm and $y_i = 1$ mm [sweep up (—grey), sweep down (--black)] with $X_b = 0.5$ mm	95
5.4	Measured frequency response of real (left) and amplitude (right) curves of the NDVA system of configuration $x_i = 35$ mm and $y_i = 1$ mm [sweep up (—grey), sweep down (--black)] with $X_b = 0.5$ mm	95
5.5	Measured frequency response of real (left) and amplitude (right) curves of the NDVA system of configuration $x_i = 45$ mm and $y_i = 1$ mm [sweep up (—grey), sweep down (--black)] with $X_b = 0.5$ mm	96
5.6	Comparison of the frequency response curve for different x_i of 25 mm (o), 35 mm (*) and 45 mm (\square) [sweep up (—grey), sweep down (--black)] at $y_i = 1$ mm and $X_b = 0.5$ mm	97

5.7	Comparison of the phase angle for different x_i of 25 mm (o), 35 mm (*) and 45 mm (□) [sweep up (—grey), sweep down (--black)] at $y_i = 1$ mm and $X_b = 0.5$ mm	97
5.8	Measured frequency response of real (left) and amplitude (right) curves of the NDVA system of configuration $x_i = 45$ mm and $y_i = 2$ mm [sweep up (—grey), sweep down (--black)] with $X_b = 0.5$ mm	99
5.9	Measured frequency response of real (left) and amplitude (right) curves of the NDVA system of configuration $x_i = 45$ mm and $y_i = 3$ mm [sweep up (—grey), sweep down (--black)] with $X_b = 0.5$ mm	99
5.10	Comparison of the frequency response curve for different y_i of 3 mm (o), 2 mm (*) and 1 mm (□) [sweep up (—grey), sweep down (--black)] at $x_i = 45$ mm and $X_b = 0.5$ mm	100
5.11	Comparison of the phase angle for different y_i of 3 mm (o), 2 mm (*) and 1 mm (□) [sweep up (—grey), sweep down (--black)] at $x_i = 45$ mm and $X_b = 0.5$ mm	100
5.12	Comparison of the frequency response curve for different input amplitudes, X_b of 0.125 mm (o), 0.25 mm (*), and 0.5 mm (□) [sweep up (—grey), sweep down (--black)] for $x_i = 45$ mm and $y_i = 0.5$ mm	104
5.13	Comparison of the frequency response curve for different input amplitudes, X_b of 0.5 mm (Δ), 0.65 mm (o), and 0.75 mm (□) [sweep up (—grey), sweep down (--black)] for $x_i = 35$ mm and $y_i = 0.5$ mm	104

5.14	Comparison of the frequency response curve for different mass (m) of 20 g (\square), 40 g (x), 60 g (Δ) and 80 g (o) [sweep up (—grey), sweep down (--black)] of different natural frequency for $x_i = 35$ mm, $y_i = 1$ mm and $X_b = 0.5$ mm	106
5.15	Comparison of the frequency response curve in term of frequency ratio, f_b/f_a for different mass, m of 20 g (\square), 40 g (x), 60 g (Δ) and 80 g (o) [sweep up (—grey), sweep down (--black)] of different natural frequency for $x_i = 35$ mm, $y_i = 1$ mm and $X_b = 0.5$ mm	106
5.16	Comparison of the frequency response curve for different mass, m of 20 g (o), 40 g (Δ), and 60 g (\square) [sweep up (—grey), sweep down (--black)] with fixed natural frequency of 19.5 Hz for $x_i = 35$ mm, $y_i = 1$ mm and $X_b = 0.5$ mm	107
5.17	Comparison of the frequency response curve for different beam length, L of 90 mm (Δ), 100 mm (x) 110 mm (\square), 120 mm (*) and 130 mm (o) [sweep up (—grey), sweep down (--grey)] for $x_i = 35$ mm and $y_i = 1$ mm	108
5.18	Comparison the frequency response curve in terms of frequency ratio, f_b/f_a for different beam length, L of 90 mm (Δ), 100 mm (x) 110 mm (\square), 120 mm (*) and 130 mm (o) [sweep up (—grey), sweep down (--black)] for $x_i = 35$ mm and $y_i = 1$ mm	108
5.19	Measured (a) frequency response and (b) phase angle of the structure without DVA for $X_b = 0.25$ mm	110

5.20	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of the linear DVA for $f_a/f_s = 1.0$ and $X_b = 0.25$ mm	111
5.21	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of the linear DVA for $f_a/f_s = 0.85$ and $X_b = 0.25$ mm	112
5.22	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of the linear DVA for $f_a/f_s = 0.75$ and $X_b = 0.25$ mm	113
5.23	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of NDVA for $f_a/f_s = 1.0$ [sweep up (—), sweep down (--)] with $X_b = 0.25$ mm	117
5.24	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of NDVA for $f_a/f_s = 0.85$ [sweep up (—), sweep down (--)] with $X_b = 0.25$ mm	118
5.25	Measured (a) receptance plot of the primary structure, (b) phase angle and (c) absolute transmissibility of NDVA for $f_a/f_s = 0.75$ [sweep up (—), sweep down (--)] with $X_b = 0.25$ mm	119
5.26	Measured (a) response and (b) phase angle of the structure for the linear DVA (solid) and the NDVA with different limit block position, $x_i = 25$ mm (dotted), $x_i = 35$ mm (dashed-dotted), and $x_i = 45$ mm (dashed) at $y_i = 1$ mm and $X_b = 0.25$ mm	122

- 5.27 Measured (a) response and (b) phase angle of the structure for the linear DVA (solid) and the NDVA with different limit block position, $y_i = 1$ mm (dashed), $x_i = 2$ mm (dashed-dotted), and $x_i = 3$ mm (dotted) at $x_i = 45$ mm and $X_b = 0.25$ mm 123
- 5.28 Measured response of the primary structure for different input amplitude, $X_b = 0.1$ mm (solid), $X_b = 0.15$ mm (dotted), and $X_b = 0.2$ mm (dashed-dotted), and $X_b = 0.25$ mm (dashed) 125
- 5.29 Measured response of the primary structure for different mass ratio. $\mu = 0.04$ (solid), $\mu = 0.05$ (dotted), and $\mu = 0.06$ (dashed) with $X_b = 0.25$ mm 127



LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Assembly drawing of the NDVA	142
B1	Part drawing of the NDVA – vibrating plate	143
B2	Part drawing of the NDVA – stand block	144
B3	Part drawing of the NDVA – rack	145
B4	Part drawing of the NDVA – base	146
B5	Part drawing of the NDVA – rack block	147
B6	Part drawing of the NDVA – limit block	148
B7	Part drawing of the NDVA – horizontal slider	149
B8	Part drawing of the NDVA – pinion	150
B9	Part drawing of the NDVA – oil free bushing	151
B10	Part drawing of the NDVA – collar	152
B11	Part drawing of the NDVA – knob	153
B12	Part drawing of the NDVA – locating pin	154