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Effect of suction on the MHD flow in a doubly-stratified micropolar fluid over a shrinking sheet

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This paper investigates the influence of suction on the flow, heat and mass transfer characteristics over a permeable shrinking sheet immersed in a doubly stratified micropolar fluid. The model which consists of partial differential equations is converted into a set of nonlinear equations using similarity transformations and then solved using the bvp4c solver. Numerical results obtained are presented graphically for the distributions of velocity, angular velocity, temperature and concentration profiles within the boundary layer for various values of the magnetic parameter and wall mass suction parameter. It is visualized that the enhancement of suction parameter will increase the skin friction, heat transfer rate (local Nusselt number) and Sherwood number. It is also found that as the magnetic parameter increase, there is an increment in the skin friction while opposite results are obtained for the local Nusselt number and Sherwood number.

Keywords: *double stratification, magnetic field, micropolar fluid, mixed convection, suction effect.*

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1. Introduction

Low-cost energy consumption with optimum heat-transfer performance is one of the requirements in related industrial and technological sectors. For that purpose, the optimization of thermal rate with minimum cost is requisite for the heating/cooling industries for example chemical plants, power stations, petrochemical and air conditioning. It is dependable on those particular sectors either to maximize or minimize the heating/cooling rate. Convenient use of suction is acceptable as one of the low-cost methods for controlling the boundary layer flow of working (cooling/heating) fluid and heat transfer. The suction effect is progressive in reducing the drag of an external flow, including depreciating energy losses [1]. The advantage/disadvantage of the suction effect in controlling the boundary layer of working fluid flow can be found in these works [2–5]. Traditional working fluid like water is less effective in increasing the thermal rate of the engineering systems even though water is categorized under low-cost working fluid. Furthermore, many engineering and industrial processes nowadays use non-Newtonian and nanofluid or the combination of non-Newtonian fluid with nanoparticles. Micropolar fluid is one of the non-Newtonian fluid subclasses which was theoretically introduced by Eringen [6]. Later, many interesting boundary layer flow studies were reported for a micropolar fluid (see [7–20]).

The topic of fluid stratification is related to the difference in concentration (solutal stratification), temperature (thermal stratification), or both (double stratification). Theoretically, the phenomenon

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when a hot fluid layer is horizontally located above the cool fluid layer is called thermal stratification. The experimental work by Bouhal [21] highlighted the thermal stratification effect for a laminar viscous fluid motion. The double stratification effect is applicable in the engineering and technological applications like thermal energy storage, condensers of power plants and solar ponds. Chang & Lee [22] analyzed the behavior of the free convection flow with thermal stratification and uniform heat flux on a vertical plate in a thermally stratified micropolar fluid. The results were numerically obtained using a combination of cubic spline method with a finite difference scheme. They showed that an increase in the stratification parameter could reduce the fluid temperature, skin friction, and the wall couple stress. In 2013, Srinivasacharya & Upendar [23] considered free convection micropolar fluid flow with double stratification and magnetic field effects which were then extended by Mishra et al. [24]. Recently, Khashi'ie et al. [25] discovered the dual solutions in the mixed convection flow of hybrid nanofluid with a double stratification effect. However, the micropolar fluid model in [25] was slightly different from the presented model in [23] and [24]. Other interesting works on the impact of fluid stratification in the boundary layer flow can be referred to in these papers [26–30].

Upon the motivation by the above investigations and using the micropolar fluid model in Srinivasacharya & Upendar [23] and Mishra et al. [24], the novelty of this work is to study the MHD flow, heat and mass transfer induced by a shrinking sheet immersed in a micropolar fluid with the double stratification and suction effects which have not been reported by any researchers.

2. Mathematical formulation

Consider a steady, laminar, free convective heat and mass transfer flow of an incompressible, doubly stratified and electrically conducting micropolar fluid past a shrinking sheet. The shrinking sheet is assumed to be permeable in order to give way for possible wall fluid suction or injection (see [29]). The wall temperature ($T_w(x)$) and concentration ($C_w(x)$) are given in a variable form. Meanwhile, for the ambient medium, both temperature and concentration are in the form of linear stratification such that $T_\infty(x) = T_{\infty,0} + A_1 x$ and $C_\infty(x) = C_{\infty,0} + B_1 x$. The constants A_1, B_1 determine the intensity of fluid stratification [23]. Further, the initial ambient temperature $T_{\infty,0}$ and concentration $C_{\infty,0}$ are used and indicated at $x = 0$.

The magnetic field is applied perpendicular to the direction of flow with the exclusion of induced magnetic field. Under the Boussineq approximation, the simplified two dimensional governing equations for the MHD boundary layer flow of the doubly stratified micropolar fluid are given as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (\mu+\kappa)\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial \omega}{\partial y} + \rho\, g^*\left[\beta_T(T-T_\infty) + \beta_c(C-C_\infty)\right] - \sigma B^2 u, \tag{2}$$

$$\rho\, j\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right) = \gamma\frac{\partial^2 \omega}{\partial y^2} - \kappa\left(2\omega + \frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{5}$$

where u, v are the corresponding velocities in x, y directions, ω denotes the microrotation component, g^* refers to the gravitational acceleration, T, C are the corresponding fluid temperature and concentration, β_T, β_c are the corresponding thermal and solutal expansions' coefficient, B and σ correspond to the magnetic field strength and magnetic permeability of the fluid while μ, ν refer to the fluid's dynamic and kinematic viscosities. Meanwhile, the parameters related to the micropolar fluid are κ (vortex viscosity), j (density of micro-inertia) and γ (spin-gradient viscosity). Further, the fluid thermal diffusivity is represented by α, while D represents the fluid molecular diffusivity.

The boundary conditions are:

$$\left.\begin{array}{c} u(x,0)=-U_w(x)=-ax, \quad v(x,0)=V_w, \quad \omega(x,0)=-m\frac{\partial u}{\partial y}, \\ T(x,0)=T_w(x), \quad C(x,0)=C_w(x), \\ u(x,\infty)=0, \quad \omega(x,\infty)=0, \quad T(x,\infty)=T_\infty(x), \quad C(x,\infty)=C_\infty(x), \end{array}\right\} \tag{6}$$

where the subscripts w and ∞ indicate the conditions at wall and at the outer edge of the boundary layer, respectively. In the boundary condition (6), m is a constant where $0 \leqslant m \leqslant 1$; $m=0$ is called strong concentration by Guram and Smith [31], which indicates that $\omega=0$ is near to the surface and represents concentrated particle flows in which the microelements close to the surface are unable to rotate [32]. The case $m=\frac{1}{2}$ indicates that the vanishing of antisymmetrical part of the stress tensor and denotes weak concentration [33]. The case $n=1$, as suggested by Peddieson [34], is used for the modeling of turbulent boundary-layer flows. V_w is the mass flux through the sheet where

$$V_w=\begin{cases} V_w<0, & \text{suction,} \\ V_w>0, & \text{injection.} \end{cases} \tag{7}$$

The governing equations (1)–(5) subject to the boundary conditions (6) can be reduced into a system of ordinary differential equations using the following similarity transformation (see [8, 23])

$$\begin{gathered} \eta=\sqrt{\frac{a}{\nu}}y, \quad \psi=\sqrt{a\,\nu}x\,f(\eta), \quad \omega=a\,x\sqrt{\frac{a}{\nu}}\,g(\eta), \\ \theta(\eta)=\frac{T-T_{\infty,0}-A_1x}{\Delta T}, \quad \Delta T=T_w(x)-T_{\infty,0}=M_1x, \\ \phi(\eta)=\frac{C-C_{\infty,0}-B_1x}{\Delta C}, \quad \Delta C=C_w(x)-C_{\infty,0}=N_1x, \end{gathered} \tag{8}$$

where η is the similarity variable and ψ is the stream function such that

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x}, \tag{9}$$

which satisfies (1). Using (8),

$$u=a\,x\,f'(\eta) \quad \text{and} \quad v=-\sqrt{a\,\nu}f(\eta) \tag{10}$$

and from (10), the mass flux, V_w can be determined as

$$V_w=-\sqrt{a\,\nu}S, \tag{11}$$

where $S=f(0)$ is the wall mass suction parameter. Hence, the transformed nonlinear ordinary differential equations are:

$$f'''=(1-N)\left[(f')^2-f f''-\left(\frac{N}{1-N}\right)g'-\mathrm{Gr}(\theta+L\phi)+Mf'\right], \tag{12}$$

$$g''=\left(\frac{1}{\lambda}\right)\left[f'g-f\,g'+\left(\frac{N}{1-N}\right)J(2g+f'')\right], \tag{13}$$

$$\theta''=\mathrm{Pr}\left[f'\theta-f\theta'+\varepsilon_1 f'\right], \tag{14}$$

$$\phi''=\mathrm{Sc}\left[f'\phi-f\phi'+\varepsilon_2 f'\right], \tag{15}$$

where $N=\frac{\kappa}{\mu+\kappa}$ is the coupling number, $\mathrm{Gr}=\frac{g*}{a^2x}\beta_T\Delta T$ is the Grashof number, $L=\frac{\beta_c}{\beta_T}\frac{\Delta C}{\Delta T}$ is the buoyancy parameter, $M=\frac{\sigma B^2}{\rho\,a}$ is the magnetic field parameter, $\lambda=\frac{\gamma}{j\rho\nu}$ is the spin-gradient viscocity, $J=\frac{\nu}{j\,a}$ is the micro-inertia density, $\mathrm{Pr}=\frac{\nu}{\alpha}$ is the Prandtl number and $\mathrm{Sc}=\frac{\nu}{D}$ is the Schmidt number. The thermal and solutal stratification parameters are accordingly defined as $\varepsilon_1=\frac{x}{\Delta T}\frac{d}{dx}[T_\infty(x)]$ and $\varepsilon_2=\frac{x}{\Delta C}\frac{d}{dx}[C_\infty(x)]$.

The corresponding boundary conditions are

$$\left.\begin{array}{c} f(0)=S, \quad f'(0)=-1, \quad g(0)=-mf''(0), \quad \theta(0)=1-\delta_1, \quad \phi(0)=1-\delta_2, \\ f'(\eta)\to 0, \quad g(\eta)\to 0, \quad \theta(\eta)\to 0, \quad \phi(\eta)\to 0 \quad \text{as} \quad \eta\to\infty, \end{array}\right\} \tag{16}$$

where $\delta_1 = \frac{x}{\Delta T} A_1$ and $\delta_2 = \frac{x}{\Delta C} B_1$.

The skin friction, wall couple stress, local Nusselt and the Sherwood number are given by

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\sqrt{\frac{a}{\nu}}, \tag{17}$$

$$M_w = \frac{\lambda}{J} g'(0)\sqrt{\frac{a}{\nu}}, \tag{18}$$

$$\mathrm{Nu} = \frac{q_w}{\sqrt{\frac{a}{\nu}}k(T_w - T_\infty)}, \tag{19}$$

$$\mathrm{Sh} = \frac{q_m}{D\sqrt{\frac{a}{\nu}}(C_w - C_\infty)}, \tag{20}$$

respectively, where

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{and} \quad q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{21}$$

3. Results & discussions

The system of ordinary differential equations (12)–(15) subject to the boundary conditions (16) is solved numerically using the bvp4c solver. For numerical results, the non dimensional parameter values as $\mathrm{Gr} = L = \lambda = \mathrm{Pr} = 1$, $J = \varepsilon_1 = \delta_1 = 0.1$, $N = 0.5$ and $\mathrm{Sc} = \varepsilon_2 = \delta_2 = 0.2$ have been considered. These values are kept as constant in the entire study except the varied parameters as shown in figures and tables. The values of λ and J are chosen based on the thermodynamic restrictions (see Eringen [6]). The results obtained show the influences of the non-dimensional governing parameters, namely magnetic parameter, M and wall mass suction parameter, S on velocity, microrotation, temperature and concentration profiles.

The boundary conditions for η at ∞ are replaced by a large value of where the velocity, microrotation, temperature and concentration profile are satisfied asymptotically and, thus, supporting the validity of the numerical results obtained. In the absence of suction effect with $\mathrm{Gr} = 1$, $J = 0.1$, $L = 1$, $\lambda = 1$, $N = 0.5$, $\mathrm{Pr} = 1.0$, $\mathrm{Sc} = 0.2$, $M = 1.0$, $\varepsilon_1 = \delta_1 = 0.1$ and $\varepsilon_2 = \delta_2 = 0.2$, the numerical results obtained are in favorable agreement with results by Srinivasacharya and Upendar [23] as presented in Table 1.

Table 1. Comparison result of $f''(0)$, $-g'(0)$, $-\theta'(0)$ and $-\phi'(0)$ obtained using present method and Srinivasacharya and Upendar [23].

M	$f''(0)$		$-g'(0)$		$-\theta'(0)$		$-\phi'(0)$	
	[23]	Present	[23]	Present	[23]	Present	[23]	Present
0	0.75428	0.754277	0.02772	0.027717	0.62289	0.622886	0.28042	0.280419
1	0.63048	0.630479	0.02099	0.020992	0.55703	0.557033	0.23895	0.238948
2	0.55448	0.554476	0.01710	0.017098	0.51043	0.510425	0.21176	0.211760

Figures 1–4 present the comparison results between flow over a shrinking sheet with various suction and magnetic parameters. The figures show that the velocity, temperature and concentration profile decrease with the influence of wall mass suction parameter while opposite result obtained for angular velocity profile. This results support the fact that suction decreases the boundary layer thickness and effective in boundary layer laminarization [1]. It is also found in Fig. 1 that the velocity will decrease slowly as the magnetic parameter (M) increase. Magnetic field produces a drag known as the Lorentz force [23, 35]. This force tends to resist the flow and hence will reduce the velocity distribution of the fluid. Figs. 3 and 4 show that the temperature and concentration profile increase with the increasing value of M for both cases. This also might be due to the Lorentz force which tends to enhance

the friction between fluid layers and simultaneously, resulting an increment in fluid temperature and concentration [23, 35].

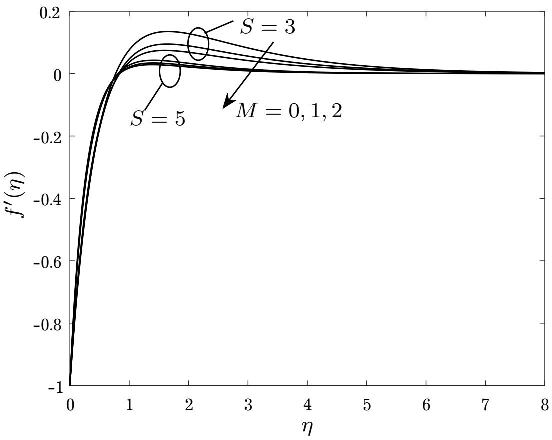

Fig. 1. Velocity profile for various values of magnetic parameter M with $S = 3$ and $S = 5$.

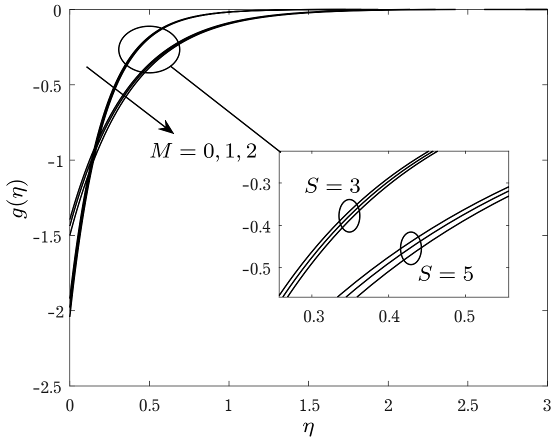

Fig. 2. Angular velocity profile for various values of magnetic parameter M with $S = 3$ and $S = 5$.

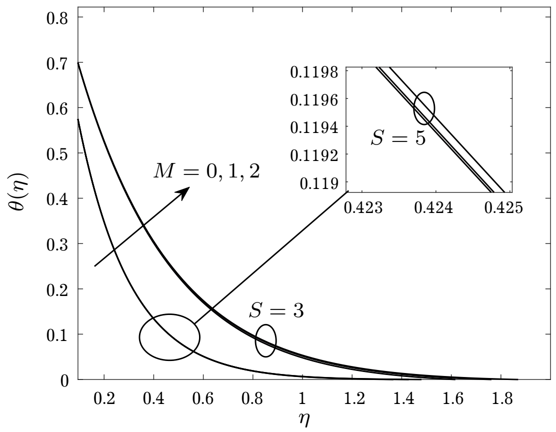

Fig. 3. Temperature profile for various values of magnetic parameter M with $S = 3$ and $S = 5$.

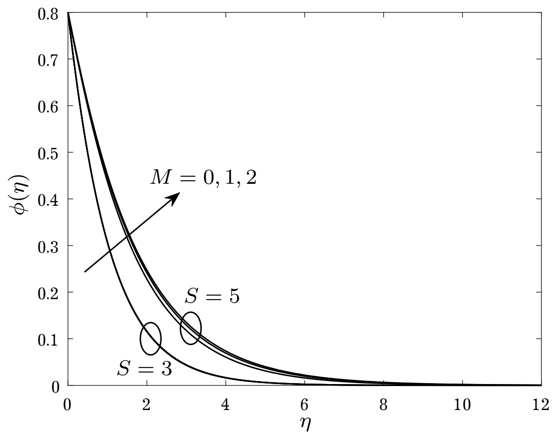

Fig. 4. Concentration for various values of magnetic parameter M with $S = 3$ and $S = 5$.

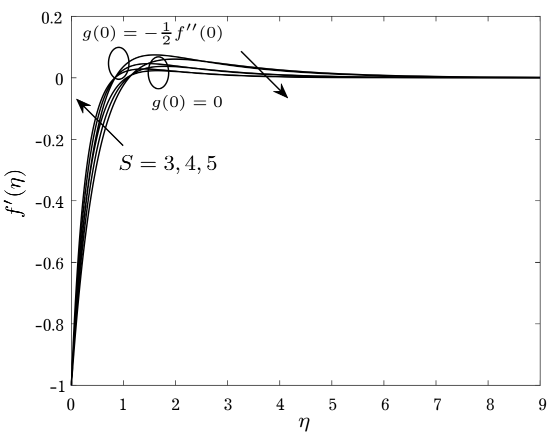

Fig. 5. Velocity profile for various values of suction parameter, S and $M = 1$.

The wall mass suction is very significant in maintaining the steady boundary layer near the sheet by delaying the separation. In practical, it is important to determine the minimum suction fluid necessary to keep the boundary layer laminar, because an excess of suction flow rate may result in such a power consumption. Hence, the effect of several values of mass suction parameter using $m = 0$ and $m = \frac{1}{2}$ (see boundary conditions (16)) on the velocity, angular velocity (microrotation), temperature and concentration profiles are demonstrated in the Figs. 5–7, respectively. There is a reduction in velocity, temperature and concentration distribution as the wall mass suction parameter increase for both cases. These findings also supported the results obtained by Daniel & Daniel [35] and Takhar et al. [36] that the imposition of wall fluid suction and increasing value of the suction parameter will reduce the hydrodynamic and thermal boundary layer.

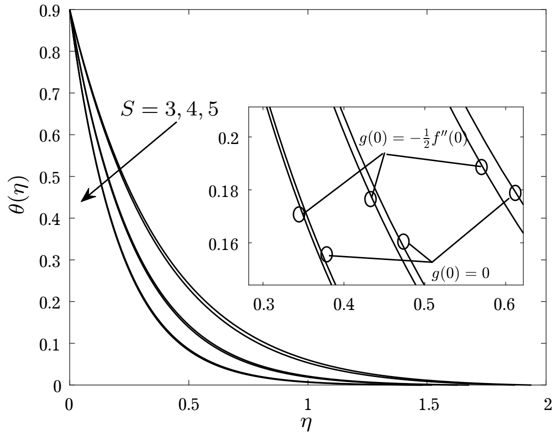

Fig. 6. Temperature profile for various values of suction parameter, S and $M = 1$.

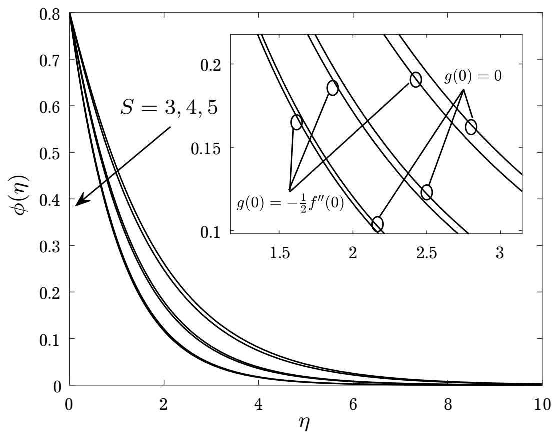

Fig. 7. Concentration profile for various values of suction parameter, S and $M = 1$.

Table 2 shows the values of skin friction ($f''(0)$), wall couple stress ($g'(0)$), Nusselt number ($-\theta'(0)$) and Sherwood number ($-\phi'(0)$) for case $g(0) = 0$ (Case 1) and $g(0) = -\frac{1}{2}f''(0)$ (Case 2) with $M = 1$. It can be seen that the skin friction, heat transfer and mass transfer rate increase as the suction parameter increases.

Table 2. The numerical data of skin friction, wall couple stress, Nusselt and Sherwood number for $M = 1$ and various values of S.

S	$f''(0)$		$g'(0)$		$-\theta'(0)$		$-\phi'(0)$	
	Case 1	Case 2	Case1	Case 2	Case1	Case 2	Case1	Case 2
3	2.20610	2.87645	-0.10678	3.83914	2.33143	2.39639	0.39005	0.42006
4	2.57778	3.40589	-0.10432	6.34012	3.29743	3.34288	0.55018	0.57362
5	2.97945	3.95804	-0.10298	9.42965	4.24526	4.27932	0.71668	0.73514

4. Conclusion

In the present paper, we have extended the problem of the flow, heat and mass transfer of the free convection on a vertical plate with double stratification effect which has been considered by Srinivasacharya & Upendar [23] by adding the effect of suction over a shrinking sheet. Using the similarity variables, the PDEs are transformed into a system of ODEs (similarity equations). The bvp4c solver in the Matlab software is used as the main tool of the numerical computation. As the conclusions:

— The values of skin friction coefficient, $f''(0)$ for shrinking surface problem with the effect of suction show a small increment as the values of the magnetic parameter, M increase while opposite results obtained for heat and mass transfer rate. However,

— The values of skin friction coefficient, local Nusselt number (heat transfer rate) and Sherwood number (mass transfer rate) show a rise as the wall mass suction parameter increases for both conditions; $g(0) = 0$ and $g(0) = -\frac{1}{2}f''(0)$.

— As expected, suction is one of the methods in order to enhance the values of the local Nusselt number. Such higher Nusselt number is very useful in an industrial process to obtain the desired quality product. However, it depends on the industrial process the minimum suction fluid needed since it is uneconomical if there is an excess of suction flow rate.

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Вплив всмоктування на магнітогідродинамічний потік у подвійній стратифікованій мікрополярній рідині на стисливому шарі

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У статті досліджується вплив всмоктування на потік, характеристики тепло- та масопереносу на проникливому стисливому шарі, який занурений у подвійну стратифіковану мікрополярну рідину. Модель, яка складається з диференціальних рівнянь у частинних похідних, перетворена у систему нелінійних рівнянь, використовуючи перетворення подібності, а потім розв'язується за допомогою bvp4c solver. Отримані чисельні результати подано графічно для розподілу швидкості, кутових швидкостей, температурних та концентраційних профілів у межах граничного шару для різних значень магнітного параметра та параметра всмоктування стінкою. Візуалізовано, що посилення параметра всмоктування збільшить поверхневе тертя, швидкість теплопередачі (локальне число Нуссельта) та число Шервуда. Також виявлено, що зі збільшенням значення магнітного параметра збільшується поверхневе тертя, тоді як для локального числа Нуссельта та числа Шервуда отримано протилежні результати.

Ключові слова: *подвійне розшарування, магнітне поле, мікрополярна рідина, змішана конвекція, ефект всмоктування.*