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Numerical Solution for A Curvilinear Crack Phenomenon in Thermoelectric Bonded Materials Subjected to Mechanical Loadings

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ABSTRACT

In this study, a curvilinear crack phenomenon laying in the upper part of thermoelectric bonded materials subject to mechanical loadings is considered. A curvilinear crack problem in thermoelectric bonded materials subjected to shear stress is formulated. The modified complex potential (MCP) function method is used to formulate this crack phenomenon into the hypersingular integral equations (HSIEs) with the help of the continuity conditions of the resultant electric force and displacement electric function. The normal and tangential traction along the crack segment serves as the right-hand side of the integral equation. The HSIEs are solved numerically for the unknown crack opening displacement (COD) function, electric current density, and energy flux load using the appropriate quadrature formulas. The numerical solution presented the behavior of the dimensionless stress intensity factors (SIFs) at the crack tips which depend on the elastic constant's ratio, the position of the crack, the electric conductivity, and the thermal expansion coefficients.

1. Introduction

Cracks in engineering structures are defects in the material used to build and construct engineering structures such as buildings, bridges, dams, and pipelines. These cracks are primarily caused by factors such as material fatigue, corrosion, excessive strain, or stresses, which can result in the structure weakening and eventually failing. A lot of researchers have dealt with and analyzed the crack problems in elastic materials [1,2], thermal materials [3-6], magnetoelastic materials [7,8], and thermoelectric materials [9-12].

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There are several methods that were used by previous works to investigate crack problems in thermoelectric materials subject to remote stress. Zhong and Meguid [9] used the complex variable approach to analyze a circular arc crack in piezoelectric materials with anti-plane shear and in plane electric fields. It has been found that the energy release rate is significantly influenced by the crack configuration. Jiang *et al.,* [10] interpreted the complex function method and the conformal mapping theory for two unequal fractures in infinite thermoelectric materials subjected to a uniform electric current and thermal flux. By using a commercial finite element code, a good agreement with the regularized simulation result was obtained. Wang *et al.,* [11] analyzed a finite element computational method for the transient and nonlinear coupling thermal stresses in thermoelectric materials. To create a first-order system of differential-equations, the method uses finite element space discretization. The time-dependent response is resolved by using a finite difference approach to address the problem. Yang *et al.,* [12] are concerned with a piezoelectric material strip's dynamic performance with a parallel crack under thermal shock and transient stress.

By utilizing the dislocation density functions, Laplace and Fourier transform, and the governing thermal and electromechanical equations, a system of singular integral equations of the Cauchy type is precisely created. The findings show that while the wave behavior is weaker and stronger, the overshooting phenomena would be more obvious with a longer relaxation time and greater fractional order. Zhang and Wang [13] influenced the fracture mechanics problem in piezoelectric materials subject to thermal, mechanical, and electric loads at infinity is theoretically explored. Maxwell stresses acting against crack sides. The thermoelastic Green's function approach and Stroh's formalism served as the foundation for this method. Liu and Ding [14] examined a thermoelectric thin film attached to an elastic substrate's thermoelectric behavior. The singular integral equation method is used to construct a computation model for thermoelectric thin films. The distribution of thermal stress in the film and at the contact between the film and substrate is discovered.

Pang *et al.,* [15] investigated the complex variable function method and the conformal mapping methodology, and the two-dimensional problem of thermoelectric materials under the action of uniform electric current and uniform total energy flux is investigated. The studies focused on crack length and radius as intensity parameters for these nondimensional fields. Liu *et al.,* [16] analyzed in thermoelectric materials, a crack runs vertically to the applied electric flux and energy flux loads. It creates a strip saturation model with an electric field reaching a saturation limit in front of the crack while putting forth the notion of electrical nonlinearity at the crack tip. When evaluating the strength of thermoelectric materials and associated devices, the thermal stress intensity factor's results are very generic and simple to apply. Zhang and Wang [17] discussed the complex variable method of Muskhelishvili and the conformal mapping methodology, two-dimensional problems of an elliptic hole or a stiff inclusion inserted in a thermoelectric material to uniform electric current density and energy flux at infinity are explored. The effect of the major-to-minor axis ratio of the elliptic geometry and the heat conductivity of inhomogeneity on thermoelectric and stress fields is demonstrated using numerical data.

However, to the best of the author's knowledge, less work in the study of crack problems in thermoelectric bonded materials using hypersingular integral equations (HSIEs) can be found. As a result, the goal of this study is to use HSIEs to develop a formulation for a curvilinear crack phenomenon that lies in the upper part of thermoelectric bonded materials subject to mechanical loadings and obtained their numerical solution for dimensionless stress intensity factors (SIFs).

2. Methodology

2.1 Mathematical Formulation

According to Bergman and Levy [18], the governing equations for a thermoelectric material in the absence of electric charges and heat sources can be presented in the form:

$$\nabla \bullet j_e = 0 \tag{1}$$

$$\nabla \bullet q + j_e \nabla V = 0 \tag{2}$$

$$j_e = -\gamma \nabla V - \gamma \varepsilon \nabla T \tag{3}$$

$$q = -\gamma \varepsilon T \nabla V - (k + \gamma \varepsilon^2 T) \nabla T \tag{4}$$

where V is electric potential, T is absolute temperature, γ is electric conductivity, k is thermal conductivity, ε is Seebeck coefficient, j_e is electric current density vector and q is heat flux vector. Since energy is transported by both electrons and heat, then the energy flux vector j_u can be derived from the electric current density and heat flux as:

$$j_u = q + j_e V \tag{5}$$

According to Zhang and Wang [17], an analytic function F is defined as $F = V + \varepsilon T$, then we have

$$j_e = -\gamma \nabla F \tag{6}$$

$$j_u = -\gamma F \nabla F - k \nabla T \tag{7}$$

Combining Eq. (1), Eq. (2), Eq. (6) and Eq. (7), the constitutive equations become:

$$\nabla^2 F = 0 \tag{8}$$

$$k \nabla^2 T + \gamma (\nabla F)^2 = 0 \tag{9}$$

For the two-dimensional thermoelectric problem considered by this study, the solutions of analytic functions F and T can be expressed as:

$$F = \mathrm{Re}\left[f(z) \right] \tag{10}$$

$$T = \mathrm{Re}\left[g(z) \right] - \frac{\gamma}{4k} f(z) \overline{f(z)} \tag{11}$$

where $z = x + iy$, $f(z)$ and $g(z)$ are unknown analytic functions and “Re” stands for real part of a complex number.

The corresponding thermal stresses $\left(\sigma_x, \sigma_y, \sigma_{xy}\right)$, resultant electric force (X, Y) and displacement electric (u, v) functions induced by the temperature field can be obtained as:

$$\sigma_{xx} + \sigma_{yy} = 2\left[\phi'(z) + \overline{\varphi'(z)}\right] + \frac{2G\alpha^* \gamma}{k(\kappa+1)} f(z)\overline{f(z)} \tag{12}$$

$$\sigma_{xx} - \sigma_{yy} + 2i\sigma_{xy} = 2\left[\bar{z}\phi''(z) + \varphi'(z)\right] + \frac{2G\alpha^* \gamma}{k(\kappa+1)} F(z)\overline{f'(z)} \tag{13}$$

$$-Y + iX = \phi(z) + z\overline{\phi'(z)} + \overline{\varphi(z)} + \frac{G\alpha^* \gamma}{k(\kappa+1)} F(z)\overline{f(z)} \tag{14}$$

$$2G\left(u_x + iv_y\right) = \kappa\phi(z) - z\overline{\phi'(z)} - \overline{\varphi(z)} - \frac{G\alpha^* \gamma}{k(\kappa+1)} F(z)\overline{f(z)} + 2Ga^* \int g(z)\, dz \tag{15}$$

where $F'(z) = f(z)$, $\phi(z)$ and $\varphi(z)$ are complex stress potential functions, G is shear modulus, and κ and α^* are defined as follows:

$$\kappa = \begin{cases} \dfrac{3-v}{1+v}, \\ 3-4v \end{cases} \quad \alpha^* = \begin{cases} \alpha & \text{plane stress} \\ (1+v)\alpha, & \text{plane strain} \end{cases}$$

v and α is Poisson's ratio and the linear thermal expansion coefficients, respectively. The derivative in a specified direction of resultant electric force (14) with respect to z yields the normal (*N*) and tangential (*T*) components as follows:

$$\frac{d}{dz}(-Y + iX) = \phi'(z) + \overline{\phi'(z)} + z\phi''(z)\frac{d}{dz} + \overline{\varphi'(z)}\frac{d\bar{z}}{dz} + \frac{G\alpha^* \gamma}{k(\kappa+1)}\left(F'(z)\overline{f(z)} + F(z)\overline{f'(z)}\frac{d\bar{z}}{dz}\right) = N + iT \tag{16}$$

According to Chen *et al.,* [19] and Song *et al.,* [20], the complex stress potential functions, and unknown analytic functions for electric and thermal field for the case of infinite material can be expressed by:

$$\phi(z) = \frac{1}{2\pi}\int_L \frac{g(t)\, dt}{t-z} \tag{17}$$

$$\varphi(z) = \frac{1}{2\pi}\int_L \frac{\overline{g(t)} dt}{t-z} + \frac{1}{2\pi}\int_L \frac{g(t)\, d\bar{t}}{t-z} - \frac{1}{2\pi}\int_L \frac{g(t)\bar{t} dt}{(t-z)^2} \tag{18}$$

$$f(z) = \frac{ij_e^{\infty}}{2\lambda}\sqrt{z^2 - a^2} \tag{19}$$

$$F(z)=\frac{ij_e^{\infty}}{4\lambda}\left(z\sqrt{z^2-a^2}-a^2\ln\left(z+\sqrt{z^2-a^2}\right)\right) \tag{20}$$

$$g(z)=\frac{ij_u^{\infty}}{2}\sqrt{z^2-a^2} \tag{21}$$

where the crack opening displacement (COD) function $g(t)$ is defined as:

$$g(t)=\frac{2G}{i(\kappa+1)}\left[\left(u(t)+iv(t)\right)^{+}-\left(u(t)+iv(t)\right)^{-}\right] \tag{22}$$

$\left(u(t)+iv(t)\right)^{+}$ and $\left(u(t)+iv(t)\right)^{-}$ denotes the displacement at a point 't' of upper and lower faces respectively.

2.2 Modified Complex Potential for Thermoelectric Bonded Materials

Consider a curvilinear crack with radius R in the upperpart of thermoelectric materials subjected to mechanical loadings as defined in Figure 1.

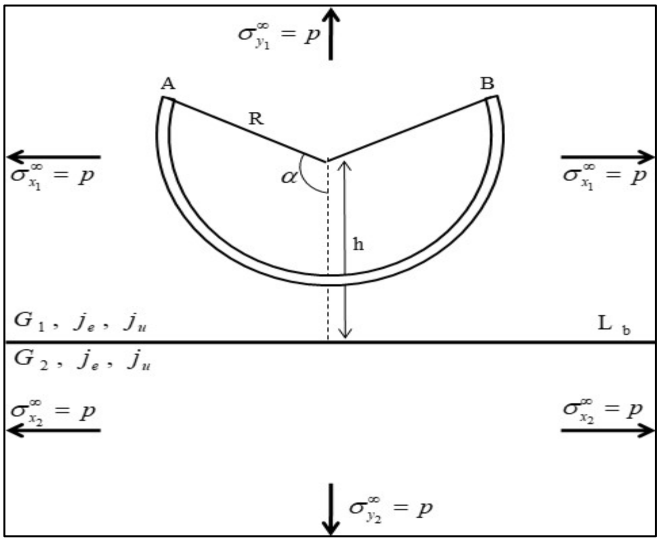


Fig. 1. A curvilinear crack with radius R in the upper part of thermoelectric materials subjected to mechanical loadings

The MCP functions for this crack phenomenon can be expressed by:

$$\phi_1(z)=\phi_{1P}(z)+\phi_{1C}(z),\quad \varphi_1(z)=\varphi_{1P}(z)+\varphi_{1C}(z) \tag{23}$$

where $\phi_{1p}(z)$ and $\varphi_{1p}(z)$ are the elementary solution for isotropic medium (infinite plane) and the principal part of the complex stress potential function, $\phi_{1c}(z)$ and $\varphi_{1c}(z)$ are the complement part of the complex stress potential function. Whereas $\phi_2(z)$ and $\varphi_2(z)$ represent the complex stress potential functions for the crack lie in the lower part of thermoelectric bonded materials. By applying continuity condition of resultant electric force (14) and displacement electric function (15), then substitute MCP functions (23), yields the following complex stress potential function as follows:

$$\phi_{1c}(z) = \Gamma_1\left(z\overline{\phi'_{1p}}(z) + \overline{\varphi_{1p}}(z)\right) + \Gamma_2\overline{F_{1p}}(z)f_{1p}(z) - \Gamma_3\int\overline{g_{1p}}(z)\,dz \tag{24}$$

$$\varphi_{1c}(z) = \Gamma_4\overline{\phi_{1p}}(z) - z\phi'_{1c}(z) + \Gamma_5\overline{F_{1p}}(z)f_{1p}(z) + \Gamma_6\overline{g_{1p}}(z)\,dz \tag{25}$$

$$\phi_2(z) = \Gamma_7\phi_{1p}(z) + \Gamma_8 F_{1p}(z)\overline{f_{1p}}(z) + \Gamma_6\int g_{1p}(z)dz \tag{26}$$

$$\varphi_2(z) = \Gamma_9\left(z\phi'_{1p}(z) + \varphi_{1p}(z)\right) - z\phi'_2(z) + \Gamma_{10}F_{1p}(z)\overline{f_{1p}}(z) - \Gamma_3\int\overline{g_{1p}}(z)\,dz \tag{27}$$

where

$$\Gamma_1 = \frac{G_2 - G_1}{G_1 + G_2\kappa_1}, \qquad \Gamma_2 = \frac{(G_2 - G_1)G_1 a_1^*\gamma_1}{k_1(G_1 + G_2 k_1)(\kappa_1 + 1)}\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$\Gamma_3 = \frac{2G_1G_2a_1^*}{(G_1 + G_2\kappa_1)}\left(\frac{k_1 - k_2}{k_1 + k_2}\right), \qquad \Gamma_4 = \frac{G_2\kappa_1 - G_1\kappa_2}{G_1\kappa_2 + G_2},$$

$$\Gamma_5 = \frac{(1+\kappa_2)G_1G_2a_2^*\gamma_2}{k_2(G_1\kappa_2 + G_2)(\kappa_2 + 1)}\left(\frac{2k_1}{k_1 + k_2}\right)^2 - \frac{G_1a_1^*\gamma_1}{k_1(\kappa_1 + 1)}$$

$$\Gamma_6 = \frac{2G_1G_2}{G_1\kappa_2 + G_2}\left(a_1^* - \frac{2k_1a_2^*}{k_1 + k_2}\right), \qquad \Gamma_7 = \frac{(\kappa_1 + 1)G_2}{G_1\kappa_2 + G_2}$$

$$\Gamma_8 = (G_1 - G_2)\frac{G_2a_2^*\gamma_2}{k_2(\kappa_2 + 1)(G_1\kappa_2 + G_2)}\left(\frac{2k_1}{k_1 + k_2}\right)^2, \qquad \Gamma_9 = \frac{G_2(1+\kappa_1)}{G_1 + G_2\kappa_1}$$

$$\Gamma_{10} = \frac{G_1G_2a_1^*\gamma_1}{k_1(G_1 + G_2\kappa_1)}\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

2.3 HSIEs for a Curvilinear Crack in Thermoelectric Bonded Materials

The new system of HSIEs for a curvilinear crack lies in the upper part of thermoelectric bonded materials can be obtained by substituted Eq. (24) and Eq. (25) into Eq. (16) and applying Eq. (17)-(21), then letting point z approaches t_0 on the crack and changing $d\bar{z}/dz$ into $d\bar{t}_0/dt_0$, yields

$$\left(N(t_0) + iT(t_0)\right)_1 \frac{1}{\pi}\int_L \frac{g(t)\,dt}{(t - t_0)^2} + \frac{1}{2\pi}\int_L M_1(t,t_0)g(t)\,dt + \frac{1}{2\pi}\int_L M_2(t,t_0)g(t)\,dt + M_3(t,t_0) \tag{28}$$

where

$$M_1(t,t_0)=\left(-\frac{1}{(t-t_0)^2}+\frac{d\bar{t}}{dt}\frac{d\overline{t_0}}{dt_0}\frac{1}{(\bar{t}-\overline{t_0})^2}\right)+\Gamma_1\left(\frac{1}{(t-\overline{t_0})^2}+\frac{2(\overline{t_0}-\bar{t})}{(\bar{t}-\overline{t_0})^3}+\left(\frac{1}{(t-\overline{t_0})^2}+\frac{1}{(\bar{t}-t_0)^2}\right)\frac{d\bar{t}}{dt}\right)$$
$$+\Gamma_1\frac{d\overline{t_0}}{dt_0}\left(\frac{2(2t_0-3\overline{t_0}+\bar{t})}{(t-\overline{t_0})^3}-\frac{6(\overline{t_0}-\bar{t})(\overline{t_0}-t_0)}{(t-\overline{t_0})^4}-\frac{1}{(t-\overline{t_0})^2}+\left(-\frac{1}{(t-\overline{t_0})^2}-\frac{2(\overline{t_0}-t_0)}{(t-\overline{t_0})^3}\right)\frac{d\bar{t}}{dt}\right)$$
$$+\Gamma_4\frac{d\overline{t_0}}{dt_0}\frac{1}{(t-\overline{t_0})^2}$$

$$M_2(t,t_0)=\left(\frac{1}{(\bar{t}-\overline{t_0})^2}\frac{d\bar{t}}{dt}+\frac{d\overline{t_0}}{dt_0}\left(\frac{1}{(\bar{t}-\overline{t_0})^2}+\frac{2(t_0-t)}{(\bar{t}-\overline{t_0})^3}\frac{d\bar{t}}{dt}\right)\right)$$
$$+\Gamma_1\left(\frac{1}{(\bar{t}-t_0)^2}+\frac{1}{(t-\overline{t_0})^2}+\left(\frac{1}{(\bar{t}-t_0)^2}+\frac{2(t_0-t)}{(\bar{t}-\overline{t_0})^3}\right)\frac{d\bar{t}}{dt}+\frac{d\overline{t_0}}{dt_0}\left(\frac{2(t_0-\overline{t_0})}{(t-\overline{t_0})^3}-\frac{1}{(t-\overline{t_0})^2}\right)\right)$$

$$M_3(t,t_0)=(\Gamma_{11}+\Gamma_{12})\frac{(j_e)^2}{4\lambda^2}\left(\sqrt{t_0^2-a^2}\sqrt{\overline{t_0}^2-a^2}+\frac{d\bar{t}_0}{dt_0}\left(t_0\sqrt{t_0^2-a^2}-a^2\ln\left(t_0+\sqrt{t_0^2-a^2}\right)\right)\frac{\overline{t_0}}{2\sqrt{\overline{t_0}^2-a^2}}\right)$$
$$+\Gamma_2\frac{(j_e)^2}{4\lambda^2}\left((t_0^2-a^2)+t\left({}_0\sqrt{t_0^2-a^2}-a^2\ln t_0+\sqrt{t_0^2-a^2}\right)\frac{t}{2\sqrt{t_0^2-a^2}}\right)$$
$$+\left(\Gamma_2-\Gamma_2\frac{d\bar{t}_0}{dt_0}+\Gamma_5\frac{d\bar{t}_0}{dt_0}\right)\frac{(j_e)^2}{4\lambda^2}\left((\overline{t_0}^2-a^2)+\left(\overline{t_0}\sqrt{\overline{t_0}^2-a^2}-a^2\ln\left(\overline{t_0}+\sqrt{\overline{t_0}^2-a^2}\right)\right)\frac{\overline{t_0}}{2\sqrt{\overline{t_0}^2-a^2}}\right)$$
$$+\Gamma_2(t_0-\overline{t_0})\frac{d\bar{t}_0}{dt_0}\frac{(j_e)^2}{4\lambda^2}\left(3\overline{t_0}+\frac{1}{2}\left(\overline{t_0}\sqrt{\overline{t_0}^2-a^2}-a^2\ln\overline{t_0}+\sqrt{\overline{t_0}^2-a^2}\right)\left(\frac{1}{\sqrt{\overline{t_0}^2-a^2}}-\frac{\overline{t_0}}{\left(\sqrt{\overline{t_0}^2-a^2}\right)^3}\right)\right)$$
$$+\left(\left(\Gamma_6\frac{d\bar{t}_0}{dt_0}+\Gamma_3\frac{d\bar{t}_0}{dt_0}-\Gamma_3\right)\sqrt{\overline{t_0}^2-a^2}+\Gamma_3(\overline{t_0}-t_0)\frac{d\bar{t}_0}{dt_0}\frac{\overline{t_0}}{\sqrt{\overline{t_0}^2-a^2}}+\Gamma_3\sqrt{t_0^2-a^2}\right)\frac{j_u}{2}$$

3. Results
3.1 Results and Discussion

The dimensionless SIFs F_{1A} and F_{1B} at crack tips A and B, respectively is defined as:

$$(K_1-iK_2)_A=\sqrt{2\pi}\lim_{t\to t_A}\sqrt{|t-t_A|}\,g'(t)=\sqrt{a\pi}F_A \quad (29)$$

$$(K_1-iK_2)_B=\sqrt{2\pi}\lim_{t\to t_B}\sqrt{|t-t_B|}\,g'(t)=\sqrt{a\pi}F_B \quad (30)$$

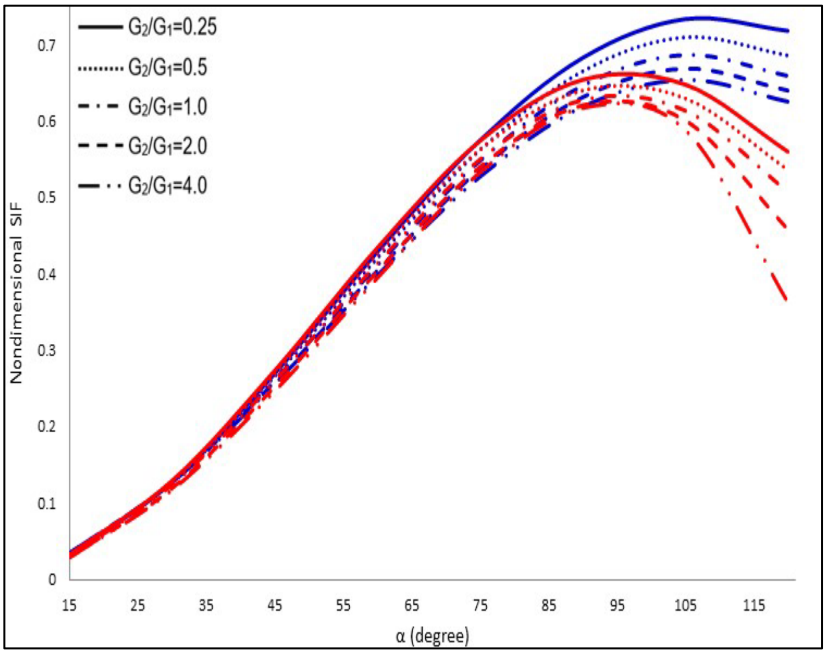
where $F_A=F_{1A}+iF_{2A}$ and $F_B=F_{1B}+iF_{2B}$

Table 1 displays the dimensionless SIFs versus α at the crack tip A for a curvilinear crack with radius *R* lies in the upperpart of thermoelectric materials subject to mix stress $\sigma x = \sigma y = p$ when $j_e = j_u = 0.0$, and is defined as in Figure 1. Our results are totally agreeing with those of Chen [21]. It is observed that the dimensionless SIFs at F_{1A} and F_{1B} are equals, and F_{2A} is equal to negative value of F_{2B}. This phenomenon is due to the equivalence of the stress acting at the tips of the cracks.

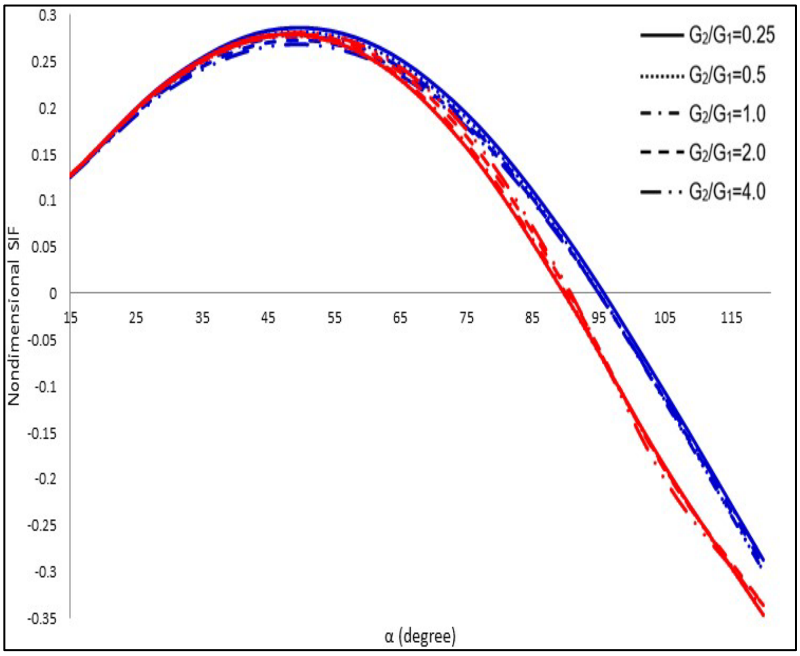
Table 1
Dimensionless SIFs F_{1A} and F_{2A} for a curvilinear crack in the upperpart of thermoelectric bonded materials

SIFs	α (degree)							
	15°	30°	45°	60°	75°	90°	105°	120°
F_{1A}	0.97503	0.90589	0.80697	0.69404	0.57988	0.47191	0.37338	0.28456
F_{1A} [18]	0.97483	0.90531	0.80584	0.69285	0.57885	0.47141	0.37357	0.28572
F_{1A} exact	0.97484	0.90528	0.80586	0.69282	0.57884	0.47140	0.37361	0.28571
F_{2A}	0.12841	0.24283	0.33442	0.40108	0.4457	0.47321	0.48879	0.49662
F_{2A} [18]	0.12835	0.24260	0.33379	0.40004	0.44417	0.47141	0.48686	0.49488
F_{2A} exact	0.12834	0.24257	0.33380	0.40000	0.44416	0.47140	0.48690	0.49487

Figure 2 shows the dimensionless SIFs versus at the crack tips *A* and *B* for a curvilinear crack with radius *R* lies in the upperpart of thermoelectric materials subject to shear stress $\sigma x = p$ when $h = 1.5R$, $j_e = 20, j_u = 0.0$ (blue), $j_e = 200, j_u = 0.0$ (red) and different values of elastic constant ratio G_2/G_1 as defined in Figure 1. It is observed that the dimensionless SIFs for Mode I (F_1) at crack tip *A* equal to the SIFs at crack tip *B*, similar to the dimensionless SIFs for Mode II (F_2). It is found that dimensionless SIFs Mode I and Mode II for electric current density vector $j_e = 20$ (blue) is higher that dimensionless SIFs for $j_e = 200$ (red). As G_2/G_1 increases the dimensionless SIFs decreases at all crack tips. Whereas, as α increases the dimensionless SIFs Mode I (F_1) increases for $\alpha < 95^o$ (Figure 2 (a)) and decreases for dimensionless SIFs Mode II (F_2) when $\alpha > 45^o$ (Figure 2 (b)) at all crack tips. This observation indicate that the strength of the materials become weaker as increases, electric current density vector decreases and elastic constant ratio G_2/G_1 decreases.



(a)



(b)

Fig. 2. (a) Dimensionless SIFs F_{1A} and F_{1B} (Mode I) and (b) Dimensionless SIFs F_{2A} and F_{2B} (Mode II), for a curvilinear crack in the upper part of thermoelectric bonded materials

4. Conclusions

The MCP function method with the unknown COD function, electric current density and energy flux load, and continuity conditions of the resultant electric force and displacement electric function were used to formulate the new system of HSIEs for a curvilinear crack phenomenon laying in the upper part of thermoelectric bonded materials subjected to mechanical loadings. From the numerical results we conclude that the dimensionless SIFs for a curvilinear crack lies in the upper part of thermoelectric bonded materials depends on the elastic constants ratio, electric current density vector, the crack geometries and the distance between the crack and the boundary. This value will have presented the strength of the materials. For the future developments, the approach used in this paper could be extended to formulate the three-dimensional cracks with and without electric problems in bonded materials with guided work from Chen and Lee [22] that focusing on an infinite material.

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References

[1] Elfakhakhre, N. R. F., NMA Nik Long, and Z. K. Eshkuvatov. "Stress intensity factor for an elastic half plane weakened by multiple curved cracks." *Applied Mathematical Modelling* 60 (2018): 540-551. https://doi.org/10.1016/j.apm.2018.03.039

[2] Kebli, B., and Z. Baka. "Annular crack in an elastic half-space." *International Journal of Engineering Science* 134 (2019): 117-147. https://doi.org/10.1016/j.ijengsci.2018.10.007

[3] Jafari, Mohammad. "Thermal stress analysis of orthotropic plate containing a rectangular hole using complex variable method." *European Journal of Mechanics-A/Solids* 73 (2019): 212-223. https://doi.org/10.1016/j.euromechsol.2018.08.001
[4] Zhang, Xue-Yang, You-Jun Xie, and Xian-Fang Li. "Transient thermoelastic response in a cracked strip of functionally graded materials via generalized fractional heat conduction." *Applied Mathematical Modelling* 70 (2019): 328-349. https://doi.org/10.1016/j.apm.2019.01.026
[5] Ngaongam, Choosak, and Rapee Ujjin. "FEM Modelling of the Heating Behaviour in Vibrothermography Based on Thermoelastic Damping on Crack Location." Journal of Advanced Research in Fluid Mechanics and Thermal Sciences 108, no. 1 (2023): 66-74. https://doi.org/10.37934/arfmts.108.1.6674
[6] Cheok, Choon Yoong, and Anusuyah Ragunathan. "Anthocyanin degradation kinetics and thermodynamic analysis of Hibiscus rosa-sinensis L. Clitoria ternatea L. and Hibiscus sabdariffa L." Progress in Energy and Environment (2022): 1-12. https://doi.org/10.37934/progee.19.1.112
[7] Lin, Chun-Bo, Shin-Cheng Chen, and Jui-Lin Lee. "Explicit solutions of magnetoelastic fields in a soft ferromagnetic solid with curvilinear cracks." *Engineering fracture mechanics* 76, no. 12 (2009): 1846-1865. https://doi.org/10.1016/j.engfracmech.2009.04.017
[8] Arhani, Afzal Akbari, and Mojtaba Ayatollahi. "Dynamic response of cracked non-homogeneous magneto-electro-elastic layer sandwiched by two dissimilar orthotropic layers." *Fatigue & Fracture of Engineering Materials & Structures* 45, no. 5 (2022): 1448-1463. https://doi.org/10.1111/ffe.13673
[9] Zhong, Z., and S. A. Meguid. "Analysis of a circular arc-crack in piezoelectric materials." *International Journal of Fracture* 84 (1997): 143-158. https://doi.org/10.1023/A:1007327931730
[10] Jiang, Dongdong, Qing-Hui Luo, Wei Liu, and Yue-Ting Zhou. "Thermoelectric field disturbed by two unequal cracks adjacent to a hole in thermoelectric materials." *Engineering Fracture Mechanics* 235 (2020): 107163. https://doi.org/10.1016/j.engfracmech.2020.107163
[11] Wang, P., B. L. Wang, and K. F. Wang. "Dynamic response of cracked thermoelectric materials." *International Journal of Mechanical Sciences* 160 (2019): 298-306. https://doi.org/10.1016/j.ijmecsci.2019.06.047
[12] Yang, Weilin, Mahsa Nourazar, Zengtao Chen, Keqiang Hu, and Xueyang Zhang. "Dynamic response of a cracked thermopiezoelectric strip under thermoelectric loading using fractional heat conduction." *Applied Mathematical Modelling* 103 (2022): 580-603. https://doi.org/10.1016/j.apm.2021.11.005
[13] Zhang, A. B., and B. L. Wang. "Crack tip field in thermoelectric media." *Theoretical and Applied Fracture Mechanics* 66 (2013): 33-36. https://doi.org/10.1016/j.tafmec.2013.11.006
[14] Liu, Qing-Nan, and Sheng-Hu Ding. "Field intensity factors of three cracks originating from a circular hole in a thermoelectric material." *Journal of Mechanics of Materials and Structures* 15, no. 5 (2020): 605-617. https://doi.org/10.2140/jomms.2020.15.605
[15] Pang, Sheng-Jie, Yue-Ting Zhou, and Feng-Jun Li. "Analytic solutions of thermoelectric materials containing a circular hole with a straight crack." *International Journal of Mechanical Sciences* 144 (2018): 731-738. https://doi.org/10.1016/j.ijmecsci.2018.06.022
[16] Liu, Y., B. L. Wang, J. E. Li, and K. F. Wang. "Thermoelectric and stress fields for a cracked thermoelectric media based on the electric field saturation model." *Mechanics Research Communications* 104 (2020): 103479. https://doi.org/10.1016/j.mechrescom.2020.103479
[17] Zhang, A. B., and B. L. Wang. "Explicit solutions of an elliptic hole or a crack problem in thermoelectric materials." *Engineering Fracture Mechanics* 151 (2016): 11-21. https://doi.org/10.1016/j.engfracmech.2015.11.013
[18] Bergman, David J., and Ohad Levy. "Thermoelectric properties of a composite medium." *Journal of Applied Physics* 70, no. 11 (1991): 6821-6833. https://doi.org/10.1063/1.349830
[19] Chen, Y. Z., X. Y. Lin, and X. Z. Wang. "Numerical solution for curved crack problem in elastic half-plane using hypersingular integral equation." *Philosophical Magazine* 89, no. 26 (2009): 2239-2253. https://doi.org/10.1080/14786430903032555
[20] Song, K., H. P. Song, P. Schiavone, and C. F. Gao. "Electric current induced thermal stress around a bi-material interface crack." *Engineering Fracture Mechanics* 208 (2019): 1-12. https://doi.org/10.1016/j.engfracmech.2019.01.004
[21] Chen, Y. Z. "Solution of integral equation in curve crack problem by using curve length coordinate." *Engineering analysis with boundary elements* 28, no. 8 (2004): 989-994. https://doi.org/10.1016/S0955-7997(03)00126-7
[22] Chen, Y. Z., and Kang Yong Lee. "Numerical solution of three-dimensional crack problem by using hypersingular integral equation." *Computer methods in applied mechanics and engineering* 190, no. 31 (2001): 4019-4026. https://doi.org/10.1016/S0045-7825(00)00374-1