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Eyring-Powell Hybrid Nanofluid with Radiative Heat Flux: Case Over a Shrinking Sheet

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ABSTRACT

This study investigates the radiative heat flux effect on Eyring-Powell hybrid nanofluid together with analysis thermal passing towards a shrinking sheet containing $Ag-MgO$ nanoparticles. The proposed mathematical model respected to the boundary condition is converted to the similarity equations by adopting the suitable transformations. In order to reduce the complexity of model, the similarity equations are then computed by applying the bvp4c function in MATLAB software. Outcomes reveal the thermal performance is upgraded in dispersing the nanoparticle to the base fluid, and it is marked under hybrid nanoparticles consideration. The presence of Eyring-Powell, radiation and shrinking parameters are the controller parameter in regulatory the sheer stress and thermal performance of the fluid.

1. Introduction

The study involving heat transport is currently at its peak due to its prominence in industries and several engineering claims. Water, oil, and glycol are typically utilised in applications although they are poor sources of heat transfer due to their truncated thermal conductivity. New fluids, such as nanofluids and nanoparticles, are being proposed to address this problem by improving thermal efficiency. A novel type of nanotechnology fluid known as a hybrid nanofluid is created by uniformly and stably suspending two or more different nanoparticles in a fluid's system. Since these particular nanofluids have better thermal properties compared to traditional fluids, they are being used in many technical and industrial applications. The design and optimization of an efficient system heavily depend on fluid flow and heat transfer [1]. As a result, by dispersing nanoparticles into fundamental fluids, scientists and engineers are able to produce sophisticated fluids so called "nanofluid". The heat conductivity of pure base fluids can be improved by this nanofluid. Choi and Eastman [2] are

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credited for being the first to add metallic nanoparticles to a base fluid in order to increase thermal conductivities and improve heat transmission in some real-world applications, such as refrigerators, air conditioners, microelectronic devices, microcomputer processors, etc. Since then, nanofluids have been employed more effectively. Hwang *et al.,* [3] presented a theoretical study of natural convection for heat transfer features in a quadrilateral permeable heated surface with water as the base fluid and nanoparticles being alumina oxide. Due to the progressive research in nanofluids, this fluid is widely employed for a variety of applications, including the cooling of nuclear reactors, the cooling of machinery and automobiles, the cooling of electronic devices, biomedicine, etc. [4]. The development is continued by further investigation numerically and experimentally from various angles. For instance, Tiwari and Das [5] proposed the mathematical models of nanofluids to examine the behaviour of the nanofluids while taking into account the impact of the solid volume fractions of nanoparticles. The finite volume method was used to numerically solve the governing equations. Numerous factors have been taken into account when studying nanofluid flow, including the flow in a rectangular enclosure, the effects of magnetic fields, chemical reactions and viscous dissipation effects, and the effects of activation energy [6-11]. Regular nanofluids have limitations in improving the thermal properties of fluid. These days, hybrid nanofluids, which are sophisticated heat transfer fluids made by two nanoparticles are proof to improve heat transmission [12,13]. Devi and Devi [14] compared the properties of heat transmission between hybrid and conventional nanofluids. They found for the presence of a magnetic field, the hybrid nanofluid has a higher heat transfer rate than the conventional nanofluid. Many researchers are keen to work toward hybrid nanofluids due to the advantages of hybrid nanofluid’s characteristics. As an illustration, Waini *et al.,* [15] described the effects of Cu-Al_2O_3 and heat transmission over a permeable moving sheet. The MHD flow across a vertical flat plate embedded with combined convective was investigated by Zainal *et al.,* [16]. For a certain case of fluid flow problem, dual solutions are developed. For example, the topic of the flow of combined convective transport filled by hybrid nanofluids moving over a vertical porous surface was examined by Waini *et al.,* [17]. It is revealed in a given set of buoyancy parameters; several dual solutions were discovered. Additionally, the boundary layer separates more slowly when hybrid nanoparticles are used. Thermal progress of a non-Newtonian hybrid nanofluid flow with temporal stability analysis was introduced by Khashi'ie *et al.,* [18]. Other reports on the dual solutions of hybrid nanofluid can be found in Waini *et al.,* [19-21].

In the theory of fluid mechanics, the flow of non-Newtonian fluids has acquired a position of highest significance and growing curiosity. Non-Newtonian fluids are defined as substances that do not follow Newton's law of viscosity. The flow of blood, food products, and dental paste are just a few examples of non-Newtonian fluids. Numerous researchers have examined boundary layer flows of non-Newtonian fluids. It is assumed that the power law model accurately captures non-Newtonian behaviour. Despite being mathematically more challenging, the Eyring-Powell model has advantages over the power-law approach. First off, the power-law model is derived from the liquid's kinematic theory rather than an empirical relationship. Second, it accurately reduces Newtonian behaviour under high-shear stress. The momentum and heat transfer of a non-Newtonian Eyring-Powell fluid over a non-isothermal stretching sheet was reported by Prasad *et al.,* [22] utilising the asymptotic boundary conditions method while the non-Newtonian Eyring-Powell fluid across a stretching sheet has been explored by Javed *et al.,* [23]. Other established reports on the Eyring-Powell fluid are documented in Hayat *et al.,* [24,25], Krishna *et al.,* [26], and Aljabali *et al.,* [27,28].

The effects of heat radiation have been extensively studied. However, there aren't many results for counterparts of hybrid nanoparticles. Hayat *et al.,* [29] and Siddiq *et al.,* [30] considered the micropolar nanofluid embedded with radiation while Mishra *et al.,* [31] investigated MHD flow in the presence of thermal radiation by considering the stretching/shrinking sheet and discovered the existence of numerous solutions. The rate of magnetic field and radiation strength could improve the thermal rate as mentioned by Babu and Sandeep [32]. Moreover, the Powell-Eyring hybrid nanofluid entropy analysis with the effects of linear thermal radiation and viscous dissipation is further explored by Aziz *et al.,* [33]. Recent publications on Powell-Eyring nanofluid can be found in Abu-Hamdeh *et al.,* [34], Ahmed *et al.,* [35], Muhammad *et al.,* [36], and Bhatti *et al.,* [37]. The investigation which offers dual solutions under the boundary layer region for different fluids and surfaces has become a hot topic among fluid mechanists and mathematicians. The challenge is to select the physical solution(s) from the range of potential options. Miklavčič and Wang [38] were among the first to explore the dual solution for the Newtonian fluid model. The developed flow field with the proper suction power was a factor in the development of dual similarity solutions. Multiple solutions (more than two) are possible in some circumstances, as discussed by Turkyilmazoglu [39], Lund *et al.,* [40], and Yahaya *et al.,* [41]. Furthermore, reported articles for analysis due to the shrinking surface under different fluid models are established in previous researches [42-48].

This study aims to contribute to the analysis of non-Newtonian hybrid Powell-Eyring nanofluids flow through a shrinking sheet with radiative heat flux. The current study, however, concentrates on the reflection of dual solutions. To the authors' knowledge, a very limited study is conducted on non-Newtonian hybrid Powell-Eyring nanofluid flow through a shrinking sheet embedded with radiative heat flux; as a result, this endeavor is noteworthy as an imminent reference for the chosen issue. The solution procedures involve the formulation of a governing equation which is deduced to a similarity equation before solving using the bvp4c function in the MATLAB software.

2. Methodology

The investigation aims to examine the features of the Eyring-Powell hybrid nanofluid flow over a shrinking sheet. The steady two-dimensional flow is considered with the surface velocity $u_w(x) = ax$ where a is a positive constant. Meanwhile, the radiative heat flux, $q_r = -(16\sigma^* / 3k^*)(\partial T / \partial y)T_\infty^{\;3}$ with the mean absorption k^* and Stefan-Boltzmann σ^* is deliberated. Figure 1 demonstrates the geometric structure of the study case. The surface temperature is taking as $T_w(x) = T_\infty + T_0 x$, for which T_0 and T_∞ are assumed to be a constant and constant ambient temperature respectively. The derivation of the Eyring-Powell hybrid nanofluid model is based on the principle of rate processes theory. This theory was proposed and explained by Powell and Eyring [49]. Meanwhile, because the hybrid nanofluid is generated as a stable compound, the size is uniform, and the agglomeration effect is excluded.

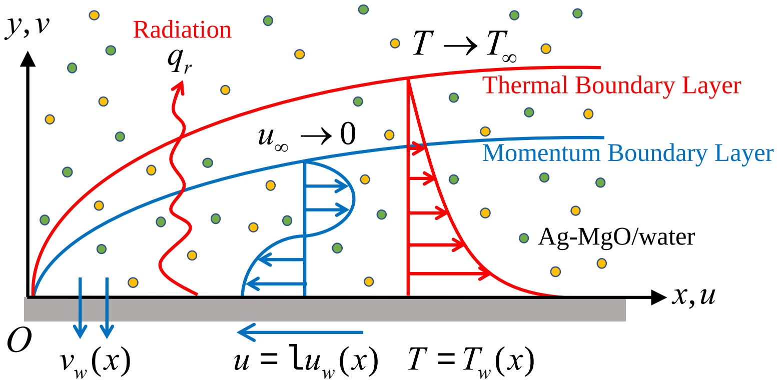

Fig. 1. Geometry of the physical problem

In Cartesian coordinate system, the governing boundary layer equations of the Eyring-Powell hybrid nanofluid can be stated as using the conventional boundary layer approximations for the continuity, momentum, and energy equations under the presumptions outlined above and depicting the fluid model can be conveyed as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}+\frac{1}{\rho_{hnf}\tilde{\beta}c^*}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{2\rho_{hnf}\tilde{\beta}c^{*3}}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$(\rho c_p)_{hnf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k_{hnf}+\frac{16\sigma^* T_\infty{}^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

with the boundary conditions,

$$\begin{aligned}&u=\lambda u_w(x),\ \ v=v_w,\ \ T=T_w(x)\ \ \text{at}\ \ y=0,\\&u\to 0,\ \ T\to T_\infty\ \ \text{as}\ \ y\to\infty.\end{aligned} \tag{4}$$

where (u,v) are the velocity components in (x,y) directions respectively, while the ρ_{hnf} is the density, μ_{hnf} defined as dynamic viscosity, c_p is the specific heat at constant pressure, $(\rho c_p)_{hnf}$ is the heat capacitance and k_{hnf} is thermal conductivity for the hybrid $Ag-MgO$/water nanofluid, Further, $\beta=\beta_0 x^{-1}$ and $\delta=\delta_0 x$ are fluid parameters of the Eyring -Powell model with constant β_0 and δ_0. Besides, the parameter λ is for the deformable plate such that $\lambda>0$ stands for a stretching plate, $\lambda<0$ indicates a shrinking plate and $\lambda=0$ represents a static plat. While $v_w(x)=-S\left(a\upsilon_f\right)^{1/2}$ denotes the constant mass velocity for the surface, and S is the suction/injection parameter such that $S>0$ corresponds to the suction effect, $S<0$ refers to the injection (fluid removal) effect. Table 1 displayed the characteristics of the water, Ag and MgO. The ϕ_1 and ϕ_2 signify Ag and MgO nanoparticles, respectively, where $\phi_{hnf}=\phi_1+\phi_2$. On the other hand, the correlation of the studied fluid is provided in Table 2.

Table 1
The thermophysical properties for the base fluid and nanoparticles [50,51]

Thermophysical Properties	Water	Ag	MgO
$\rho\left(kg/m^3\right)$	997.1	10500	3580
$c_p\left(J/kgK\right)$	4179	235	879
$k\left(W/mK\right)$	0.6130	429	30

Table 2
Correlations of hybrid nanofluid [14,15,20]

Properties	Correlations
Dynamic Viscosity	$\mu_{hnf}=\dfrac{\mu_f}{(1-\phi_{hnf})^{2.5}}$
Density	$\rho_{hnf}=(1-\phi_{hnf})\rho_f+\phi_1\rho_{s1}+\phi_2\rho_{s2}$
Heat Capacity	$(\rho c_p)_{hnf}=(1-\phi_{hnf})(\rho c_p)_f+\phi_1(\rho c_p)_{s1}+\phi_2(\rho c_p)_{s2}$
Thermal Conductivity	$\dfrac{k_{hnf}}{k_{bf}}=\left(\dfrac{\dfrac{\phi_1 k_{s1}+\phi_2 k_{s2}}{\phi_{hnf}}+2k_{bf}+2(\phi_1 k_{s1}+\phi_2 k_{s2})-2\phi_{hnf}k_{bf}}{\dfrac{\phi_1 k_{s1}+\phi_2 k_{s2}}{\phi_{hnf}}+2k_{bf}-(\phi_1 k_{s1}+\phi_2 k_{s2})+\phi_{hnf}k_{bf}}\right)$

The governing Eq. (1) to (3) are in nonlinear partial differential equations (PDEs) form and due to its complexity, a practical transformation method namely similarity transformation is introduced to convert the PDEs into a simplified set of a nonlinear ordinary differential equation (ODEs). Thus, the relevant similarity transformations which applicable to simplify on Eq. (1) to Eq. (3) inclusive with the boundary conditions (4) are,

$$\eta=\left(\frac{a}{\nu_f}\right)^{1/2}y,\qquad \psi=\left(a\nu_f\right)^{1/2}xf(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\tag{5}$$

where η defined as dimensionless similarity variable, and ψ is the stream function. The velocity component becomes,

$$u=\frac{\partial\psi}{\partial y}=axf'(\eta),\qquad v=-\frac{\partial\psi}{\partial x}=-\left(a\nu_f\right)^{1/2}f(\eta)\tag{6}$$

which satisfied the continuity equation (1). Next, the momentum (2) and energy (3) equations become,

$$\frac{\mu_{hnf}}{\mu_f}f'''+\beta_1 f'''-\beta_1\delta_1\left(f''\right)^2 f'''+\frac{\rho_{hnf}}{\rho_f}\left(ff''-f'^2\right)=0,\tag{7}$$

$$\frac{1}{\Pr}\frac{1}{(\rho c_p)_{hnf}/(\rho c_p)_f}\left(\frac{k_{hnf}}{k_f}+\frac{4}{3}R\right)\theta''+f\theta'-f'\theta=0.\tag{8}$$

with the boundary conditions,

$$\begin{aligned}&f(0)=S,\;\; f'(0)=\lambda,\;\; \theta(0)=1;\\&f'(\eta)\to 0,\;\; \theta(\eta)\to 0\;\text{ as }\eta\to\infty.\end{aligned}\tag{9}$$

where a notation prime ($'$) is derivative with respect to η, the Eyring -Powell fluid parameters β_1 and δ_1, the radiation parameter R, the Prandtl number $\Pr$, and the kinematic viscosity ν_f. These parameters can be defined as,

$$u = \frac{\partial \psi}{\partial y} = axf'(\eta), \quad v = -\frac{\partial \psi}{\partial x} = -\left(a\nu_f\right)^{1/2} f(\eta) \tag{10}$$

The quantities of physical to be highlighted are in the form of the skin friction coefficient C_f and the local Nusselt number Nu_x

$$C_f = \frac{\tau_w}{\rho_f u_w^2(x)}, \qquad Nu_x = \frac{xq_w}{k_f\left(T - T_\infty\right)}, \tag{11}$$

where τ_w and q_w are the wall shear stress and heat flux from the plate, respectively such that,

$$\tau_w = \left(\left(\mu_{hnf} + \frac{1}{\beta\delta}\right)\frac{\partial u}{\partial y} - \frac{1}{6\beta\delta}\left(\frac{\partial u}{\partial y}\right)^3\right)_{y=0}, \qquad q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} + \left(q_r\right)_{y=0} \tag{12}$$

Thus, one obtains,

$$C_f\,\mathrm{Re}_x^{1/2} = \frac{\mu_{hnf}}{\mu_f} f''(0) + Mf''(0) - \frac{B}{3}Mf''^3(0), \qquad Nu_x\,\mathrm{Re}_x^{-1/2} = -\left(\frac{k_{hnf}}{k_f} + \frac{4}{3}R\right)\theta'(0). \tag{13}$$

where $\mathrm{Re}_x = (ax^2/\nu_f)$ is the Reynolds number.

3. Results and Discussion

The formulation of the present model is started by reducing the complexity of governing equations using similarity variables. The obtained equations are then numerically solved by bvp4c function embedded in MATLAB software. This solution uses the limited difference Lobatto IIIa three-stage scheme and formula, where a set of initial guesses with the combination of suitable boundary layer thickness, η_∞ depends on the applied parameters. The correct results are obtained within the specified accuracy when the far-field boundary conditions are asymptotically satisfied and no error is generated from the MATLAB software. The procedures of this method are clearly discussed in Shampine *et al.,* [52]. The computations are carried out to study the effect of physical parameters involved likes Eyring-Powell fluid parameters β_1 and δ_1, radiation parameter R, stretching/shrinking parameter λ, and nanoparticle volume fraction parameters ϕ_1 and ϕ_2. The influences of the pertinent parameters are addressed graphically on the velocity distribution, temperature distribution, skin friction coefficient and also heat transfer rate. The validation procedures are done by comparing the values of $C_f\,\mathrm{Re}_x^{1/2}$ with the established report by Javed *et al.,* [23] for regular fluid ($\phi_1 = \phi_2 = 0$) with various values of β_1 and δ_1 as clarified in Table 3, which designate the present outputs are valid. It can be concluded the developed codes for present study are acceptable and trustworthy. From the results shown, it can be concluded the parameters β_1 and δ_1 were the controlling parameters on the rate of $C_f\,\mathrm{Re}_x^{1/2}$. However, it is noticed the presence β_1 and δ_1 give the different impact where the increasing in β_1 reducing the $C_f \operatorname{Re}_x^{1/2}$ but give contradict influence for growing in δ_1.

Table 3
Comparison values of $C_f \operatorname{Re}_x^{1/2}$ for various values of β_1 and δ_1 when $S = R = \phi_1 = \phi_2 = 0$, and $\lambda = 1$

δ_1	Javed *et al.*, [23]			Present Result		
	$\beta_1 = 0$	$\beta_1 = 0.2$	$\beta_1 = 0.4$	$\beta_1 = 0$	$\beta_1 = 0.2$	$\beta_1 = 0.4$
0.0	-1.0000	-1.0954	-1.1832	-1.000000	-1.095445	-1.183216
0.2	-1.0000	-1.0924	-1.1784	-1.000000	-1.092445	-1.178431
0.4	-1.0000	-1.0894	-1.1735	-1.000000	-1.089381	-1.173490

Figure 2 and Figure 3 captured the values of $C_f \operatorname{Re}_x^{1/2}$ and $Nu_x \operatorname{Re}_x^{-1/2}$ against ϕ_1 respectively. For these figures the sheet is taking as shrinking ($\lambda = -1$) and it can be seen for the fixed value of ϕ_2, the value of $C_f \operatorname{Re}_x^{1/2}$ are decreasing for larger value of ϕ_1 but shows contradict behavior in $Nu_x \operatorname{Re}_x^{-1/2}$ Further, it is remarked the presence of strong ϕ_2 lessening the value of $C_f \operatorname{Re}_x^{1/2}$ but improved the $Nu_x \operatorname{Re}_x^{-1/2}$. On the other hand, the fluid without nanoparticle ($\phi_1 = \phi_2 = 0$) was observed to offer a higher value of $C_f \operatorname{Re}_x^{1/2}$ and smaller value of $Nu_x \operatorname{Re}_x^{-1/2}$ compare with the inclusive of nanoparticles. The conclusion we can draw from those output is the presence of nanoparticle Ag and MgO contributed significant impact to the fluid characteristics. Physically, the drag force drops by intensifying of nanoparticles concentration in the base fluid. Hence, the nanoparticle concentration compromises some resistance towards the drag force hence decreasing the skin friction. The increasing amount of nanoparticle concentration in the base fluid contributed in advanced conductivity of thermal. Additionally, the collision within the particles in the fluid flow dissipates the energy's rate and upsurges the inclusive temperature. It is also predicted the heat transfer rate of hybrid nanofluid escalate faster compared to conventional nanofluids (single particle).

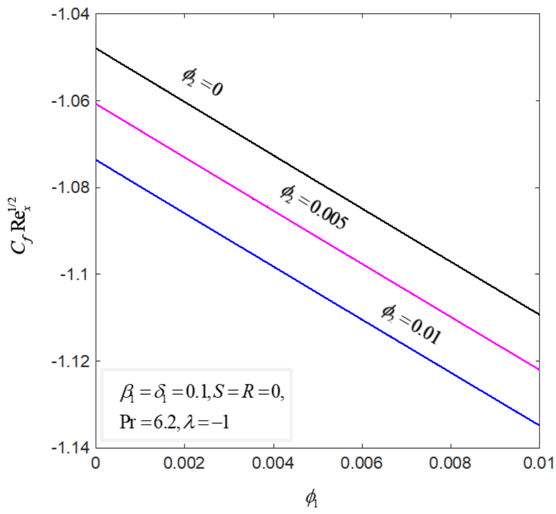

Fig. 2. Variation of $C_f \operatorname{Re}_x^{1/2}$ against ϕ_1 and ϕ_2

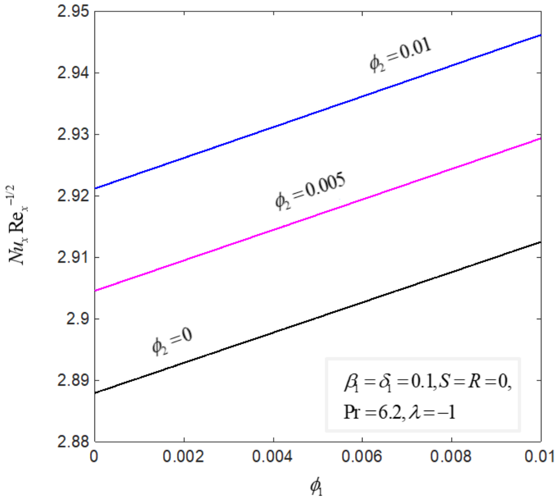

Fig. 3. Variation of $Nu_x \operatorname{Re}_x^{-1/2}$ against ϕ_1 and ϕ_2

Variation of skin friction and Nusselt number against fluid parameter β_1 in shrinking environment can be perceived in Figure 4 and Figure 5. It can be seen that the rise of β_1 contributes smaller values of $C_f \,\mathrm{Re}_x^{1/2}$ and greater quantity of $Nu_x \,\mathrm{Re}_x^{-1/2}$. Physically, the larger values of β_1 generate hurdles to the shear-thinning rate which lessen the connection between fluid and surfaces. Hence it caused the minor drag forces. On the other hand, the raising in λ enhanced the values of $C_f \,\mathrm{Re}_x^{1/2}$ significantly. However, for $\beta_1 = 0$ it is noticed the value of $C_f \,\mathrm{Re}_x^{1/2}$ only increasing on the certain value of λ and later shows decreasing trend. Besides, it is detected the dual solutions for $C_f \,\mathrm{Re}_x^{1/2}$ and $Nu_x \,\mathrm{Re}_x^{-1/2}$ which attained up to some critical values of λ The critical value reached up to $\lambda_{c1} = -1.2890$, $\lambda_{c2} = -1.1792$ and $\lambda_{c3} = -1.0856$ for $\beta_1 = 0, 0.1$ and 0.2 respectively. No other solutions were achieved beyond those mentioned critical value.

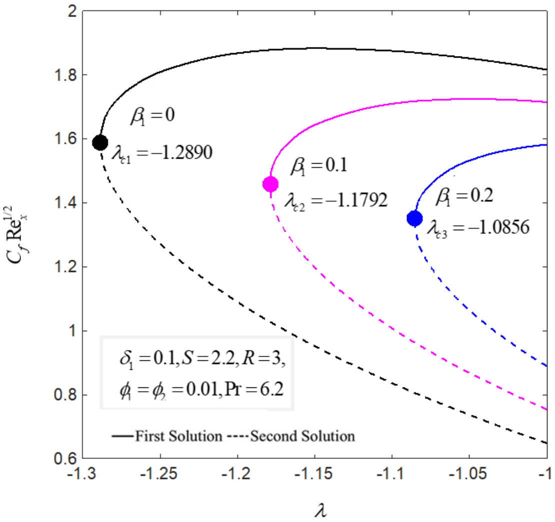

Fig. 4. Variation of $C_f \,\mathrm{Re}_x^{1/2}$ against λ and β_1

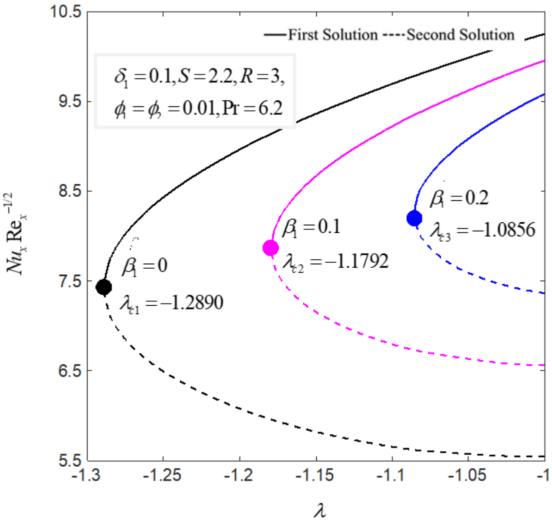

Fig. 5. Variation of $Nu_x \,\mathrm{Re}_x^{-1/2}$ against λ and β_1

Graph in Figure 6 illustrated the behavior of $Nu_x \,\mathrm{Re}_x^{-1/2}$ against the thermal radiation R and λ. At a fixed value of λ, it is apparently showing the quantity of $Nu_x \,\mathrm{Re}_x^{-1/2}$ are reducing for strong rate of radiation. In this case, we can say the presence of radiation prevent the transfer of heat. According to the graph, this behavior happened on shrinking environment which is one of the factors contributing to this fallout. Besides that, the increasing in λ boost the quantity of $Nu_x \,\mathrm{Re}_x^{-1/2}$. It is because, the shrinking environment of the sheet release the energy which contributed in heat transference of fluid. The dual solutions are acquired at $\lambda_c = -1.1792$. The existing dual solutions are supported by the temperature distributions as in Figure 7. It is clearly shows both solution (first and second) developed the temperature profile. The dual solutions are supported by the velocity and the temperature distribution as captured in Figure 8 and Figure 9. In the increasing pattern of the β_1, the first solution of the velocity show deceleration while the second solution increasing. Meanwhile, the temperature distribution shown a contradicts behavior.

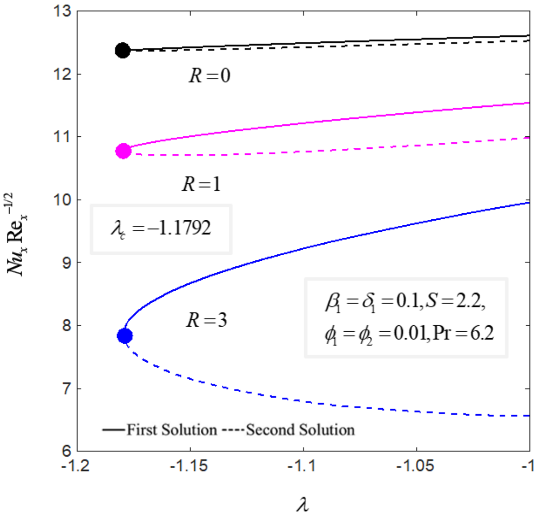

Fig. 6. Variation of $Nu_x \mathrm{Re}_x^{-1/2}$ against λ and R

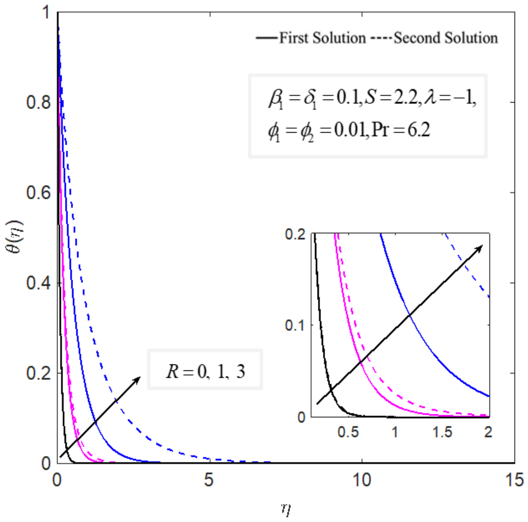

Fig. 7. Temperature profiles $\theta(\eta)$ against R

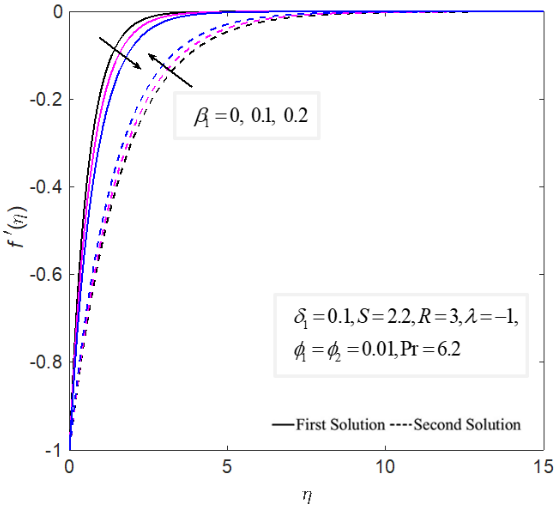

Fig. 8. Velocity profiles $f'(\eta)$ against β_1

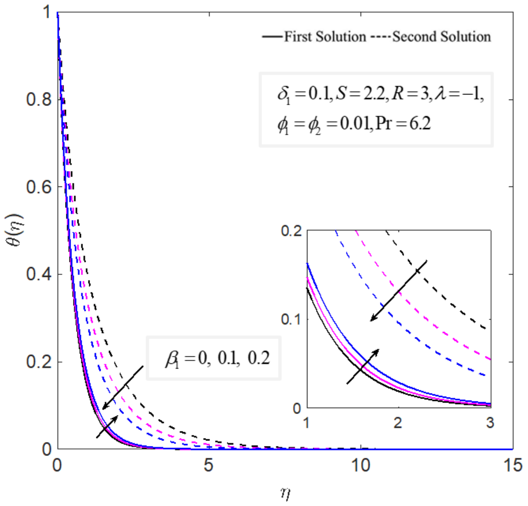

Fig. 9. Temperature profiles $\theta(\eta)$ against β_1

4. Conclusions

The present investigation focusses on compute numerical results for Eyring-Powell fluid filled with hybrid nanoparticle. The flow is taking moving over a shrinking sheet. The mathematical model was incorporated with the effect of thermal radiation and also shrinking environment. The limiting research is exposed for analysis due to Eyring-Powell hybrid nanofluid flow with radiative heat flux. The Physical variations pertinent parameters $\beta_1, \delta_1, R, \lambda, \phi_1$ and ϕ_2 are investigated and graphically illustrated for velocity, $f'(\eta)$, temperature $\theta(\eta)$, skin friction, $C_f \mathrm{Re}_x^{1/2}$ and Nusselt number, $Nu_x \mathrm{Re}_x^{-1/2}$. The conclusions from this study are:

(i) The Eyring-Powell fluid with filled with $Ag-MgO$/water is detected to offer enhanced thermal conductivity.
(ii) In higher concentration of nanoparticles, the rate of heat transfer is developed.
(iii) The augmentation of the nanoparticle volume fraction ϕ_2 (Ag) decreases the coefficient of skin friction but boost the heat transmission.

(iv) For shrinking environment, non-unique (dual) solutions are discovered with different critical values for different values of β_1.

(v) Velocity profile $f'(\eta)$ lowers with the increasing values of β_1 (first solutions) and upsurge in second solution.

(vi) Temperature distributions enhances with the increasing range of thermal radiation R and β_1.

The present formulation can be implemented in the industrial application where it can be applied in improving the heat transfer frequency and discover the best material of certain processes.

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