



COMPUTATIONAL HEMORHEOLOGY OF BLOOD FLOW WITH TiO₂ NANOPARTICLES OVER A NON-LINEARLY STRETCHING SHEET UNDER MAGNETOHYDRODYNAMICS

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Abstract

This study presents a computational analysis of the hemorheology of blood flow containing TiO₂ nanoparticles over a non-linearly stretching sheet under the influence of magnetohydrodynamics (MHD). The Sisko blood flow model is employed to capture the non-

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Newtonian characteristics of the blood. The governing equations are derived and transformed into a set of non-linear ordinary differential equations (ODEs) using appropriate similarity variables. These equations are then solved numerically using the 4th-5th order Runge-Kutta-Fehlberg (RKF45) method. The results demonstrated that an increase in Sisko material parameter enhances the velocity of nanofluid. Increase in non-linear stretching parameter leads to a thinner boundary layer, thereby enhancing heat transfer at the surface. The findings underscore the potential of TiO_2 nanoparticles in significantly improving heat transfer efficiency in biomedical and industrial applications.

1. Introduction

The ability to engineer materials at the nanoscale has opened up new possibilities for innovation, transforming industries and improving quality of life through enhanced drug delivery systems, more efficient energy storage, and advanced diagnostic techniques [1]. Within the realm of nanotechnology, nanofluids have emerged as a significant area of study and application. The evolution of nanofluids has been driven by extensive research into the types of nanoparticles used, their concentrations, and the methods of synthesis and stabilization. Fuskele and Sarviya [2] discussed nanoparticles' synthesis, preparation and stability.

The study of blood flow, or hemorheology, involves understanding of the behavior of blood as it moves through the circulatory system [3]. In recent years, the study of blood flow with the addition of nanoparticles has garnered significant attention in biomedical engineering and fluid mechanics [4]. This burgeoning area of research, known as computational hemorheology, seeks to understand the complex interactions between blood - a non-Newtonian fluid - and various nanoparticles, such as gold (Au) [5], silver (Ag) [6], titanium dioxide (TiO_2) [7], and iron oxide (Fe_3O_4) [8], which can modify its rheological properties. The inclusion of these nanoparticles in blood can enhance its thermal conductivity and viscosity, influencing the flow characteristics and heat transfer processes.

Studies on fluid flow over non-linearly stretching sheets with the addition of nanoparticles under the influence of a magnetic field have gained significant attention due to their relevance in various engineering and biomedical applications [9]. In an annulus, the impact of entropy due to ciliary motion has been discussed by Riaz et al. [10]. Chahregh and Dinarvand [11] discussed the impact of TiO₂-Ag/blood hybrid nanofluid flow through an artery for targeted drug delivery. Zaman et al. [12] investigated the impact of different nanoparticles in an overlapped stenosed channel. Ramaprasad et al. [13] explored the mass and heat transfer in unsteady MHD convective flow with heat absorption. Pati et al. [14] studied the impact of Cu nanoparticles over stretching cylinder with viscous dissipation effect.

The addition of TiO₂ nanoparticles offers several advantages, including contribution to a stable flow by reducing the viscosity of the blood. Furthermore, the magnetic properties of TiO₂ allow for better control and manipulation of the nanoparticles using external magnetic fields, enabling precise targeting and minimization of side effects. This integration of TiO₂ nanoparticles under MHD conditions thus provides a novel and effective approach for enhancing the performance and control of biomedical applications, making it a promising area for further research and development [15].

The motivation for this study stems from the need to enhance our understanding of nanoparticle behavior in biomedical applications, particularly, in targeted drug delivery and hyperthermia treatment, where precise control over thermal and flow characteristics is crucial. The novelty of this work lies in the comprehensive computational analysis of blood flow integrated with TiO₂ nanoparticles over a non-linearly stretching sheet under the influence of MHD. Moreover, this work uniquely addresses the combined effects of non-linear stretching and MHD on the rheological properties of the blood. The exploration of this uncharted territory provides new insights and contributes valuable knowledge to the fields of computational hemorheology and nanomedicine.

2. Formulation of the Problem

The current problem is posed by taking two-dimensional boundary layer flow of Sisko nanofluid at boundary layer. Heat transfer characteristics of nanofluid are taken into account for the region $y = 0^+$. The formulation of the problem is described by Sisko model as follows [6]:

$$u_x + v_y = 0, \quad (1)$$

$$uu_x + vu_y = \frac{a}{\rho_{nf}} u_{yy} - \frac{b}{\rho_{nf}} \partial_y (u_y)^n - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}}, \quad (2)$$

$$uT_x + vT_y = \alpha_{nf} T_{yy} - \frac{1}{(\rho c_p)_{nf}} \partial_y (q_r), \quad (3)$$

$$\left. \begin{aligned} u = u_w = cx^m, v = 0, T = T_w \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\}, \quad (4)$$

where the non-linear stretching surface drives the flow along x -direction with velocity $u = u_w = cx^m$, where c is a constant, m is a non-linear stretching parameter and B_0 is magnetic field strength. Further, velocity of nanofluid is $(u, v, 0)$, a , b and n are the constants of material for Sisko bio-nanofluid, T is the nanofluid's temperature, and ρ_{nf} is the density of nanofluid. Nanofluid's thermal diffusivity is given by $\alpha_{nf} = k_{nf}/(\rho c_p)_{nf}$, where c_p is the specific heat at constant pressure and k_{nf} is the nanofluid's thermal conductivity which are described by the following relations [10]:

$$\left. \begin{aligned} \rho_{nf} &= (1 - \varphi)\rho_b + \varphi\rho_t \\ (\rho c_p)_{nf} &= (1 - \varphi)(\rho c_p)_b + \varphi(\rho c_p)_t \\ \frac{k_{nf}}{k_b} &= \frac{(k_t + 2k_b) - 2\varphi(k_b - k_t)}{(k_t + 2k_b) + 2\varphi(k_b - k_t)} \end{aligned} \right\}, \quad (5)$$

where φ represents the volume fraction of nanoparticles, ρ_b and ρ_t are the densities of the blood and TiO_2 , respectively, $(\rho c_p)_b$ and $(\rho c_p)_t$ are heat

capacitances of blood and TiO₂, respectively, and k_b and k_t are thermal conductivities of blood and TiO₂, respectively. The radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3k^*} T_y^4, \tag{6}$$

where σ^* and k^* are Stefan-Boltzmann constant and coefficient of mean absorption, respectively. Expanding T^4 in powers of T using Taylor series and neglecting higher terms, following approximation is obtained:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Introduce the following non-dimensionalizing functions:

$$\left. \begin{aligned} \eta &= \frac{y}{x} Re_b^{\frac{1}{n+1}}, \quad u = u_w f'(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \\ v &= -u_w Re_b^{-\frac{1}{n+1}} \{ [m(2n-1) + 1] f(\eta) + [m(2-n) - 1] \eta f'(\eta) \} \end{aligned} \right\}. \tag{8}$$

The transformed equations take the form

$$A f''' + n(f'')^{n-1} f''' + \left[(1 - \phi) + \phi \frac{\rho_t}{\rho_b} \right] \left[\frac{m(2n-1) + 1}{n+1} f f'' - m f'^2 \right] - M f' = 0, \tag{9}$$

$$\frac{1}{Pr} \left(1 + \frac{4}{3N_R} \right) \left(\frac{k_{nf}}{k_b} \right) \theta'' + \frac{m(2n-1) + 1}{n+1} \left[(1 - \phi) + \phi \frac{(\rho c_p)_t}{(\rho c_p)_b} \right] f \theta' = 0, \tag{10}$$

and the boundary conditions become

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \tag{11}$$

where

$$A = \frac{Re_b^{\frac{2}{n+1}}}{Re_a}, \quad Pr = \frac{x u_w}{\alpha_b} Re_b^{-\frac{2}{n+1}}, \quad M = \frac{\sigma_{nf} B_0^2 x}{\rho_{nf} u_w}, \tag{12}$$

$$Re_a = \frac{\rho_b x u_w}{a}, \quad Re_b = \frac{\rho_b x^n u_w^{2-n}}{b}, \quad \alpha_b = \frac{k_b}{(\rho c_p)_b}. \quad (13)$$

Here A represents parameter for Sisko nanofluid material, Pr the Prandtl number, M the magnetic parameter, Re_a and Re_b local Reynolds numbers and α_b thermal diffusivity of blood. The engineered local Nusselt number and the skin friction can be expressed as:

$$Re_x^{1/2} Nu_x = -\theta'(0), \quad Re_x^{-1/2} C_f = -f''(0). \quad (14)$$

3. Method of Solution

The coupled non-linear differential equations obtained are first reduced into a collection of 1st order ODEs before application of the 4th-5th order Runge-Kutta-Fehlberg’s method (RKF45) integration with shooting technique. The 4th-5th order Runge-Kutta-Fehlberg integration technique solves the non-linear coupled ODEs system with initial conditions in computational software MATLAB. The values of thermophysical properties required to carry out numerical investigation are illustrated in Table 1.

Table 1. Thermophysical characteristics of blood and TiO₂ nanoparticles [7]

Property	Blood	TiO ₂
ρ (kg/m ³)	1063	4250
k (W/mK)	0.492	8.9538
C_p (J/KgK)	3594	686.2

4. Results and Discussion

The simultaneous impact of volume fraction of TiO₂ nanoparticles and non-linear stretching parameter on velocity of nanofluid is observed in Figure 1. As the non-linear stretching parameter m increases, the velocity of TiO₂-blood nanofluid decreases. The decrease in velocity of the TiO₂-blood nanofluid with increase in non-linear stretching parameter is due to the intensified stretching effect creating a higher shear force that opposes

the flow. Simultaneously, an increase in the volume fraction of TiO_2 nanoparticles raises the nanofluid's viscosity and density, which enhances internal friction and inertia, further impeding the fluid motion and resulting in a decreased velocity. Thus, the combined effect of higher shear forces from non-linear stretching and increased resistance from the augmented viscosity and density due to the nanoparticles leads to a notable reduction in the nanofluid velocity.

The increase in velocity of the TiO_2 -blood nanofluid with rising Sisko material parameter A is explored in Figure 2. This occurs because higher values of A characterize a shear-thinning behavior, reducing the effective viscosity of the fluid under shear stress, thereby facilitating easier flow and higher velocity. Conversely, increasing the volume fraction of TiO_2 nanoparticles raises the nanofluid's effective viscosity and density, resulting in greater internal friction and inertia that impede fluid motion, thus decreasing the velocity. Therefore, while the shear-thinning property (higher A) enhances the flow, the augmented viscosity and density from more nanoparticles counteract this by reducing the nanofluid's velocity.

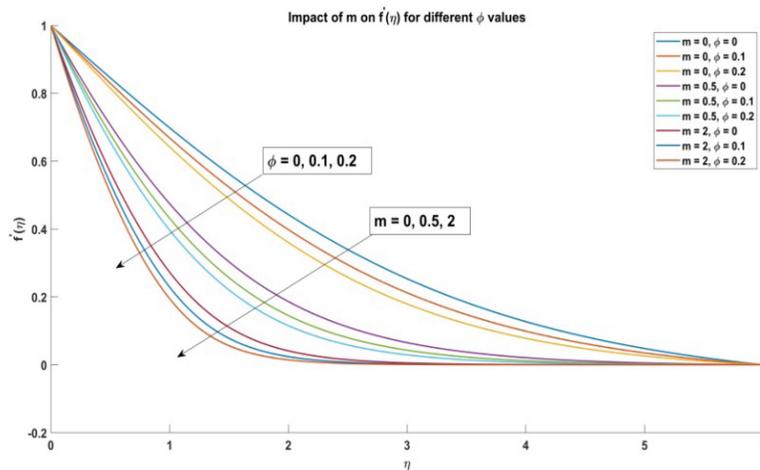


Figure 1. Impact of m on velocity profile.

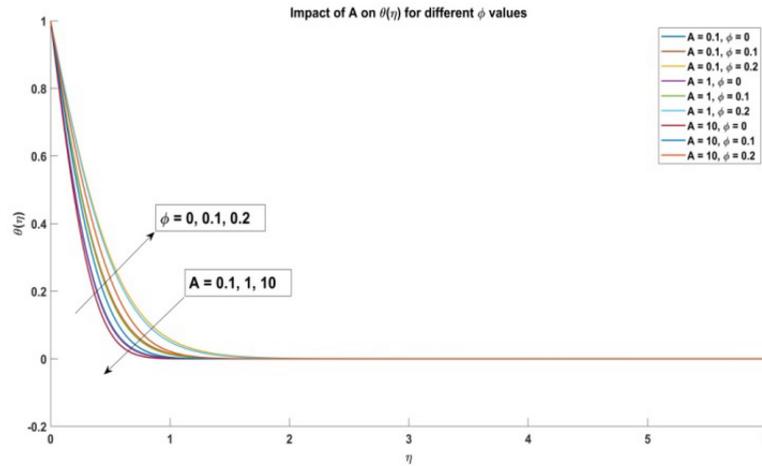


Figure 2. Impact of A on temperature profile.

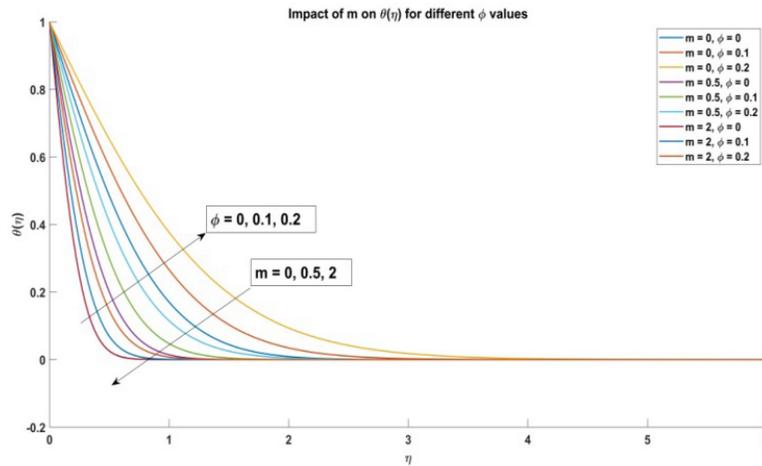


Figure 3. Impact of m on temperature profile.

The variation in values of temperature of TiO_2 -blood nanofluid due to non-linear stretching parameter m and volume fraction of nanoparticles ϕ is detailed in Figure 3. The decrease in temperature of the TiO_2 -blood nanofluid with increasing non-linear stretching parameter is observed which is due to the enhanced stretching rate, causing a thinner thermal boundary layer and more efficient heat transfer away from the surface, leading to a lower fluid temperature. Simultaneously, increase in the volume fraction of

TiO₂ nanoparticles improves the thermal conductivity of the nanofluid, promoting more efficient heat conduction, which increases the temperature.

5. Conclusion

The study reveals several key findings: The velocity of TiO₂-blood nanofluid decreases with an increase in non-linear stretching parameter and a higher volume fraction of TiO₂ nanoparticles, which raises the fluid's viscosity and density. Conversely, an increase in the Sisko material parameter leads to a rise in the fluid's velocity. Additionally, the fluid's temperature decreases with a higher nonlinear stretching parameter due to an enhanced stretching rate, resulting in a thinner thermal boundary layer and more efficient heat transfer.

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