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Anti-disturbance observer-based finite-time reliable control design for fuzzy switched systems

R. Sakthivel [a,*], R. Abinandhitha [a], S. Harshavarthini [b], A. Mohammadzadeh [c], S. Saat [d,e]

[a] *Department of Applied Mathematics, Bharathiar University, Coimbatore - 641046, India*
[b] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Chennai, India*
[c] *Department of Electrical Engineering, University of Bonab, Bonab, Iran*
[d] *School of Computing and Informatics, Albukhary International University, Alor Setar, Kedah, Malaysia*
[e] *Centre for Telecommunication Research and Innovation, Universiti Teknikal Malaysia Melaka, Melaka, Malaysia*

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Abstract

This study proposes a finite-time anti-disturbance observer-based reliable $(Q, S, R) - \theta$ dissipative feedback control technique for a class of Takagi-Sugeno fuzzy switched systems in conjunction with time-varying state/input delays and multiple disturbances. In contrast to previous findings, the external disturbances in this study are considered to be multiple disturbances, which are classified into matched and mismatched disturbances since the underlying system can be affected by various forms of disturbances [21], [25]. In order to deal with the matched disturbances that are brought about by exogenous systems, an anti-disturbance observer has been established. As well, $(Q, S, R) - \theta$ dissipative performance handles the mismatched part, which includes the concepts of H_∞, passivity, mixed H_∞ and passivity performance in a unified structure. In addition, unlike previous works in which the failure of the actuator is assumed to be deterministic [37], [38], the failure probability of the actuator in this work is determined using a distinct random variable with a probabilistic distribution in the range of [0, 1]. Furthermore, in accordance with parallel distributed compensation method and reliable control design with stochastic theory, the desired fuzzy-rule based control protocol is developed. Moreover, distinct from the asymptotic and exponential stability that are defined over an infinite interval of time [4], [14], the finite-time notion is considered in this work, where it prevents the states from exceeding a specific range within a fixed interval. A set of adequate requirements are derived utilising the Lyapunov stability theory and average dwell time technique to ensure the finite-time boundedness of the system under consideration. In the end, effectiveness of the theoretical findings is validated by means of simulation results.

Keywords: Switched systems; TS fuzzy approach; Anti-disturbance observer-based estimator; Reliable control; Finite-time boundedness

* Corresponding author.
E-mail address: krsakthivel@yahoo.com (R. Sakthivel).

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1. Introduction

Switched systems that are made up of a finite number of subsystems, have extensive interests in both theoretical and practical view point due to their substantial applications in multiple fields like power systems, vehicle industry, flight control systems, manipulator robots and so on [1–3]. More specifically, the subsystems are employed to capture the dominant dynamics of the system in different operation modes, which are activated by the respective switching signal. The switching rule regulates the coordination between those subsystems and precisely defines a specific subsystem being activated during a certain interval of time. Very interestingly, the authors in [1] proposed a fault-tolerant control strategy for switching time-delay systems by utilising average dwell-time approach and Lyapunov stability theory. On flip side, it should be noted that most industrial problems are nonlinear in nature and it induces more complexities in the system analysis and design in contrast with linear systems. Therefore, the majority of researchers have recently focused on the study of nonlinear switched systems. As a result, some interesting studies have been documented on nonlinear switched systems in the existing literatures [4–6]. Among the various methods proposed to handle nonlinear systems, Takagi-Sugeno (TS) fuzzy model approach has received attention among many researchers [7–11]. In the fuzzy model approach, the nonlinear systems can be equivalently re-drafted as a number of linear systems integrated with different weights. In particular, a robust stabilization problem of TS fuzzy systems in the presence of time-varying delays has been investigated in [7]. However, only limited number of results on the stability analysis of TS fuzzy switched systems (TSFSSs) have been reported in the existing literatures [12–17]. Specifically, in [16], the passivity analysis and feedback passification for TS fuzzy switched systems is studied via sampled-data implementation. However, we are aware of very few literatures concerning TS fuzzy switched systems, which has piqued our interest in this area. In view of this, it is necessary to further strengthen the theoretical analysis on the stabilization problems of TSFSSs.

On another point of note, the external disturbances may normally exist in practice, which results in inadequate system performance or even instability. In this connection, an abundance effort is devoted by the researchers in the development of disturbance rejection and attenuation for various kind of dynamical systems [18–20]. Moreover, estimating the disturbance or its influence from measurable variables is termed disturbance estimation and attenuating the disturbance's effect on the regulated output is known as disturbance attenuation. Particularly, disturbance observer-based control (DOBC) protocol served as an efficient tool for disturbance rejection in virtue of its robustness, efficiency, and practicability, wherein the external disturbance signals are estimated with respect to the known information. For example, in [19], the robust control problem for uncertain systems with external disturbances that is generated by an exogenous system is studied. Besides, it is worth pinpointing that in complex engineering systems, multiple disturbances with heterogeneous characteristics exist which are inevitable. Regrettably, in existence of multiple disturbances, DOBC fails to attain disturbance estimation with high precision. To overcome this, recently composite anti-disturbance control protocol has been proposed for various dynamical systems [21–25], which attenuate the outer loop disturbances (mismatched disturbances) and at the same time, reject the inner loop ones (matched disturbances). To be precise, matched disturbances enter the system via the control input path while mismatched disturbances enter through separate channels from the control input path. In the narration, one of the disturbances present in the input channel along with perturbations is generated by an exogenous system whose information is partially known is estimated via DOBC (anti-disturbance observer). To be explicit, anti-disturbance observers evaluate external disturbances based on known plant information. Another one is norm bounded with unknown information, in which the impact of disturbances are reduced in the reference output using some elegant control schemes, namely, reliable control [1], H_∞ control [12], adaptive control [5] and so on. Specifically, the composite anti-disturbance control scheme attains both disturbance rejection and disturbance attenuation by combining DOBC schemes with conventional control schemes. Remarkably, in [21], the authors developed composite anti-disturbance control for stochastic nonlinear systems. Moreover, a novel anti-disturbance control strategy was developed for singular Markovian jump systems with infinitely unobservable states and multiple disturbances in [23].

Dissipative theory serves as a prominent role in the analysis and control design of dynamical systems, because many real-world problems need to be dissipative for accomplishing satisfactory noise attenuation. Precisely, the energy attenuation level of the system is characterized by the rate of dissipation of stored energy. Moreover, the theory of dissipative concept includes H_∞, passivity and L_2 performances as special cases based on energy-related considerations. In view of this, it is considered to be a more general criterion and also witnessed wide range of applications in science and engineering domains [26–28]. For instance, the authors in [27] examined the stabilization problem of TS fuzzy systems by developing fuzzy-dependent dynamic output feedback-based dissipative control law. Additionally, the study on time delay systems have received more attention during recent years, since the existence of time delays in state and control vector are inherent feature in real process such as in chemical processes, pneumatic and hydraulic systems and industrial processes [29–34]. Specifically, in [31], the problem of stability and stabilization for TS fuzzy systems with time-varying delays has been studied. Based on state feedback and parallel distributed compensation strategy, an adaptive control scheme is developed for active suspension systems with input time delay and unknown nonlinear dynamics in [34]. Hence, in this study, both state and input delays are taken into account while studying the stabilization problem of TSFSSs, enhancing the system's suitability for real-world situations.

At the same time, as a consequence of increasing complexities in control systems, the faults in actuators and sensors are a common occurrence and lead to unsuccessful transmission of control signals. Notably, in many practical circumstances, the actuator faults are brought on by malfunction of hardware or software components, material ageing and data deformation. By virtue of these flaws, the characteristics of actuator faults may fluctuate over time. However, neglecting such faults eventually dispense unsatisfactory system performance. In order to counteract this effect, a controller has to be designed which particularly has the ability to conserve the systems stability even in the presence of actuator faults. Recently, fault-tolerant control for dynamical control systems has received great attention among researchers [35–38]. Unfortunately, most of the literatures addressed the deterministic actuator faults, however in reality, the actuator faults may occur randomly on account of abrupt changes in the environment. In light of both theory and practical viewpoint, the stochastic actuator faults are more general than the deterministic one. As a result, a slew of works have been reported on this issue [39], [40]. Therefore, it is more meaningful and important to consider the actuator faults with stochastic nature for TSFSSs. It should be mentioned that the asymptotic stability [4], [14], [27] which is hardly defined over infinite interval of time, did not confine any restrictions to system parameters. Nonetheless, in many practical systems like telecommunication networks and molecular reaction systems, the operating time needs to be short. On such grounds, the fast cognizance of system states are essential, that is the assurance of the stability in finite-time span [41–44]. For example, the work [41] studied the switched systems with external disturbances such that it guarantees H_∞ finite-time stability. By using coupling memory sampled-data control approach, finite-time stability and finite-time boundedness of TS fuzzy semi-Markov jump systems was studied in [43].

Based on the observations mentioned above and impelled by the advantages of composite anti-disturbance reliable control protocol, this study intends to establish $(Q, S, R) - \theta$ dissipativity-based conditions for TSFSSs with state/input delay and multiple disturbances. As a consequence, stochastic finite-time boundedness corresponding to the addressed TSFSSs are accomplished. More precisely, primary contributions of this article are given as:

1) A unified disturbance rejection and stabilization problem of TSFSSs with time-varying state delay, input delay, stochastic actuator faults and multi-source disturbances is formulated. Precisely, the well-known fuzzy-model based approach [7–11] is utilized to grip the nonlinear switched systems.
2) The multi-source disturbances split up into matched and mismatched disturbances. For the purpose of estimating the matched disturbance, a novel switched disturbance observer is fabricated. At the same time, the mismatched disturbance which is norm bounded is attenuated by $(Q, S, R) - \theta$ dissipative performance.
3) Moreover, a generalised actuator fault representation is provided in which the faults are considered to happen at random and stochastic variables satisfying the Bernoulli distribution are defined to describe the fault rates.
4) Motivated by the works in [21–25], by combining a reliable fuzzy-rule based state feedback controller with the disturbance observer's output, a composite anti-disturbance controller is developed to balance the effects of both types of disturbances and achieve the system's desired results.
5) In narration, the controller is designed without sharing the same premise variables as the system model, which reduces the design complexity and enhances the design flexibility.

2. Problem formulation and preliminaries

2.1. System description

By exploiting TS fuzzy modeling approach, the dynamics of the switched nonlinear plant with state delay, input delay and multiple disturbances is described by TSFSSs with f rules in the following format:
Plant rule $\Re^i_{\psi(t)}$: **IF** $\xi_1(t)$ is $\Im^i_{\psi(t)1}$, $\xi_2(t)$ is $\Im^i_{\psi(t)2}$, ..., and $\xi_f(t)$ is $\Im^i_{\psi(t)f}$ **THEN**

$$\begin{cases} \dot{x}(t) = & \mathcal{A}_{\psi(t)i}x(t) + \mathcal{A}_{d\psi(t)i}x(t-\tau(t)) + \mathcal{B}_{\psi(t)i}[u^f(t-h) + d(t)] + \mathcal{B}_{w\psi(t)i}w(t), \\ z(t) = & \mathcal{C}_{\psi(t)i}x(t) + \mathcal{C}_{w\psi(t)i}w(t), \\ x(t_0) = & \phi(t_0),\ t_0 \in [-\max(\epsilon, h), 0], \end{cases} \tag{1}$$

where $\xi_g(t)$ and $\Im^i_{\psi(t)g}$, $g = \{1, \dots, f\}$, are the premise variables and its corresponding fuzzy rule $\Re^i_{\psi(t)} = \{1, \dots, p\}$, $i = \{1, 2, \dots, N_{\psi(t)}\}$, here the function $\psi(t) : \mathbb{R}^+ \to \mathbb{M} = \{1, 2, \dots, m\}$ signifies the switching law which is a piecewise constant function of time and takes its values in a finite set and $m > 1$ denotes the number of operating modes, p indicates the number of IF-THEN rules, and $N_{\psi(t)}$ is the number of inference rules in the ψ-th switched system; $x(t) \in \mathbb{R}^n$ represents the system's state; $z(t) \in \mathbb{R}^p$ is the system's output; $u^f(t) \in \mathbb{R}^m$ describes the control vector; h is a positive constant that denotes the time delay occurred in the control input vector; $\tau(t)$ describes the time-varying delay in state dynamics with known upper bound $\epsilon > \tau(t) > 0$ and satisfies $\dot{\tau}(t) \leq \hat{\tau} < 1$; $d(t) \in \mathbb{R}^v$ denotes the disturbance signal occurred in control input; $w(t) \in \mathbb{R}^q$ indicates the norm-bounded external disturbance; $\mathcal{A}_{\psi(t)i}$, $\mathcal{A}_{d\psi(t)i}$, $\mathcal{B}_{\psi(t)i}$, $\mathcal{B}_{w\psi(t)i}$, $\mathcal{C}_{\psi(t)i}$ and $\mathcal{C}_{w\psi(t)i}$ denote the system matrices with suitable dimensions.

For notational convenience, in the rest of this paper, we denote $\psi(t) = \sigma$, $\bar{\mathcal{A}}_{\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{A}_{\sigma i}$, $\bar{\mathcal{A}}_{d\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{A}_{d\sigma i}$, $\bar{\mathcal{B}}_{\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{B}_{\sigma i}$, $\bar{\mathcal{B}}_{w\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{B}_{w\sigma i}$, $\bar{\mathcal{C}}_{\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{C}_{\sigma i}$ and $\bar{\mathcal{C}}_{w\sigma i} = \sum_{i=1}^{N_\sigma} \hbar_{\sigma i}(\xi(t))\mathcal{C}_{w\sigma i}$.

Now, the compact form of the system (1) under consideration based on fuzzy blending is given in the following form:

$$\begin{cases} \dot{x}(t) & = \bar{\mathcal{A}}_{\sigma i}x(t) + \bar{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \bar{\mathcal{B}}_{\sigma i}[u^f(t-h) + d(t)] + \bar{\mathcal{B}}_{w\sigma i}w(t), \\ z(t) & = \bar{\mathcal{C}}_{\sigma i}x(t) + \bar{\mathcal{C}}_{w\sigma i}w(t), \end{cases} \tag{2}$$

where $\hbar_{\sigma i}(\xi(t)) = \frac{\rho_{\sigma i}(\xi(t))}{\sum_{i=1}^{N} \rho_{\sigma i}(\xi(t))}$ is the normalized grade of membership and $\rho_{\sigma i}(\xi(t)) = \prod_{g=1}^{f} \Im_{ig}(\xi_g(t))$ is the membership function corresponding to the fuzzy term $\Im^i_{\sigma g}$. It also obeys the conditions $\rho_{\sigma i}(\xi(t)) \geq 0$ and $\sum_{i=1}^{N_\sigma} \rho_{\sigma i}(\xi(t)) = 1$, for all $t > 0$.

Notably, it is considered that the unknown disturbance $d(t)$ that appeared via controller path $u(t)$ was produced by dint of exogenous system given by

$$\begin{cases} \dot{\chi}(t) = & \mathcal{W}_\sigma \chi(t) + \mathcal{H}_\sigma \eta(t), \\ d(t) = & \mathcal{V}_\sigma \chi(t), \end{cases} \tag{3}$$

where $\chi(t) \in \mathbb{R}^l$ represents exogenous system's state vector; $\eta(t) \in L_2[0, \infty)$ signifies the modeling error and perturbations in the exogenous system, which confines the model accuracy; $\mathcal{W}_\sigma$, $\mathcal{H}_\sigma$ and $\mathcal{V}_\sigma$ are known matrices.

In line with the systems (2) and (3), prompted by the works in [19]-[20], the disturbance observer (DO) to estimate the exogenous system (3) is designed as follows:

$$\begin{cases} \dot{\varphi}(t) = & [\mathcal{W}_\sigma + \mathcal{L}_\sigma \bar{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma]\hat{\chi}(t) + \mathcal{L}_\sigma[\bar{\mathcal{A}}_{\sigma i}x(t) + \bar{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \bar{\mathcal{B}}_{\sigma i}u^f(t-h)], \\ \hat{\chi}(t) = & \varphi(t) - \mathcal{L}_\sigma x(t), \\ \hat{d}(t) = & \mathcal{V}_\sigma \hat{\chi}(t), \end{cases} \tag{4}$$

where $\varphi(t)$ denotes the state vector of DO; $\mathcal{L}_\sigma$ indicates suitable dimensioned DO gain matrix, which will be determined later; $\hat{\chi}(t)$ and $\hat{d}(t)$ specifies the estimations of signals $\chi(t)$ and $d(t)$. As a follow up, the disturbance estimation error is defined by subtracting the exogenous system in (3) with its observer system (4), that is $e_\chi(t) = \chi(t) - \hat{\chi}(t)$. Then, we obtain the following format of error dynamics;

$$\dot{e}_\chi(t) = [\mathcal{W}_\sigma + \mathcal{L}_\sigma \bar{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma]e_\chi(t) + \mathcal{L}_\sigma \bar{\mathcal{B}}_{\sigma i}w(t) + \mathcal{H}_\sigma \eta(t).$$

2.2. Control design

In the follow-up, a fuzzy rule controller $\Re^j_{\psi(t)}$ based on parallel distributed compensation strategy of the sequel structure is established for achieving the desired system performances. In this way, the composite anti-disturbance

controller is designed as follows:
Control rule $\Re^{j}_{\psi(t)}$: **IF** $\xi_1(t)$ is $\Im^{j}_{\psi(t)1}$, $\xi_2(t)$ is $\Im^{j}_{\psi(t)2}$, ..., and $\xi_f(t)$ is $\Im^{j}_{\psi(t)f}$ **THEN**

$$\begin{aligned} u^f(t-h) =& \mathcal{G}\sum_{j=1}^{N_\sigma} \hbar_{\sigma j}(\xi(t))\mathcal{K}_{\sigma j}x(t-h) - \hat{d}(t) \\ =& \sum_{\ell=1}^{k} \beta_\ell \Delta_\ell \sum_{j=1}^{N_\sigma} \hbar_{\sigma j}(\xi(t))\mathcal{K}_{\sigma j}x(t-h) - \hat{d}(t), \end{aligned} \tag{5}$$

where $\mathcal{K}_{\sigma j}$ indicates the state feedback control gain matrix to be determined, $\mathcal{G} = diag\{\beta_1, \ldots, \beta_k\}$ is the actuator fault matrix with $\beta_\ell \in [0, 1]$, $\ell = 1, 2, \ldots, k$ are k unrelated random variables and $\Delta_\ell = diag\{\underbrace{0, \ldots, 0}_{l-1}, 1, \underbrace{0, \ldots, 0}_{m-l}\}$.

Based on the techniques in [39], [40] and also to graft the stochastic process onto the actuator faults, we assume that β_ℓ has the probability density function $p_\ell(\beta_\ell)$ over the interval [0, 1] and the expectation and variance of fault β_ℓ are respectively denoted as ϑ_ℓ and δ^2_ℓ. As a consequence, the expectation of the fault matrix $\mathcal{G}$ is given by $\bar{\mathcal{G}} = \mathbb{E}\{\mathcal{G}\}$ and from (5), we obtain

$$\begin{cases} \mathbb{E}\{\mathcal{G} - \bar{\mathcal{G}}\} = 0, \\ \mathbb{E}\{\mathcal{G} - \bar{\mathcal{G}}\}^T \mathrm{H}\{\mathcal{G} - \bar{\mathcal{G}}\} = \sum_{\ell=1}^{k} \delta^2_\ell \Delta^T_\ell \mathrm{H} \Delta_\ell, \ \text{for H} > 0. \end{cases} \tag{6}$$

In the sort of control law defined in (5) and denoting $\bar{\omega}(t) = \begin{bmatrix} w^T(t) & \eta^T(t) \end{bmatrix}^T$, $E_1 = \begin{bmatrix} \bar{\mathcal{B}}_{w\sigma i} & 0 \end{bmatrix}$, $E_2 = \begin{bmatrix} \mathcal{L}_\sigma \bar{\mathcal{B}}_{w\sigma i} & \mathcal{H}_\sigma \end{bmatrix}$ and $E_{3\sigma i} = \begin{bmatrix} \bar{\mathcal{C}}_{w\sigma i} & 0 \end{bmatrix}$, the closed-loop form of system (1) can be obtained in the following compact form;

$$\begin{cases} \dot{x}(t) = & [\bar{\mathcal{A}}_{\sigma i}x(t) + \bar{\mathcal{B}}_{\sigma i}(\mathcal{G} - \bar{\mathcal{G}})\bar{\mathcal{K}}_{\sigma j}x(t-h) + \bar{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\bar{\mathcal{K}}_{\sigma j}x(t-h) + \bar{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) \\ & +\bar{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)], \\ \dot{e}_\chi(t) = & [\mathcal{W}_\sigma + \mathcal{L}_\sigma \bar{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma]e_\chi(t) + E_2\bar{\omega}(t), \\ z(t) = & [\bar{\mathcal{C}}_{\sigma i}x(t) + E_{3\sigma i}\bar{\omega}(t)], \end{cases} \tag{7}$$

where $\bar{\mathcal{K}}_{\sigma j} = \sum_{j=1}^{N_\sigma} \hbar_{\sigma j}(\xi(t))\mathcal{K}_{\sigma i}$. Further, the prime assumptions and definitions that pave a way in easing the analysis of the desired result are listed out as follows:

Assumption 1. *The external disturbance $\bar{w}(t)$ is supposed to be time-varying and hence, it fulfils $\int_0^{T_f} \bar{w}^T(s)\bar{w}(s)ds \leq w_f$, where w_f is a positive scalar.*

Definition 1. [42] System (1) is said to be stochastically finite time stable with respect to $(c_1, c_2, T_f, \alpha, F)$ if $\sup_{-\max(\epsilon,h)\leq t_0 \leq 0} \{\mathbb{E}\{x^T(t_0)Fx(t_0)\}, \{\mathbb{E}\{e^T_\chi(t_0)Fe_\chi(t_0)\}, \{\mathbb{E}\{\dot{x}^T(t_0)F\dot{x}(t_0)\}\} \leq c_1 \implies \mathbb{E}\{x^T(t)Fx(t)\} < c_2$ where $c_2 > c_1$ and $F > 0$ denotes the symmetric matrix.

Definition 2. [26] System (1) is said to be strictly (Q, S, R)-dissipative with respect to $(c_1, c_2, T_f, \alpha, F)$ if for any $t \geq 0$ and some scalar $\theta > 0$, under zero initial state, the following condition is satisfied:

$$\mathbb{E}\Big\{\int_0^{T_f} (z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t))ds\Big\} \geq \theta\, \mathbb{E}\Big\{\int_0^{T_f} w^T(t)w(t)ds\Big\}, \tag{8}$$

where $Q \leq 0$, S and R are given real matrices. In addition, the matrices Q and R are symmetric in nature.

Remark 1. Notably, the (Q, S, R)-dissipative performance comprises H_∞, passivity, mixed H_∞ and passivity as special cases. It is eminent that if we consider $Q = -I$, $S = 0$ and $R = \gamma^2 I$ then (Q, S, R) dissipativity is reduced into H_∞ performance. When $Q = 0$, $S = I$ and $R = \gamma I$ then it can be reduced to strictly passivity. Also, if $Q = \mho I$, $S = (1 - \mho)$ and $R = \gamma^2 \mho I$, where $\mho \in (0, 1)$ such that (Q, S, R) dissipativity is deduced into mixed H_∞ and passivity case.

3. Main results

3.1. Stochastic finite-time boundedness analysis

Now, we focus to solve finite-time stability problem for the considered TSFSS (1) by using the devised control strategy (5). To be particular, in the context of linear matrix inequalities, a set of sufficient conditions to guarantee the desired outcomes of the closed-loop system (7) is obtained. This is accomplished by constructing appropriate Lyapunov-Krasovskii functional.

Primarily, in Theorem 1, adequate criterion is developed in a way to assure the stochastic finite-time boundedness of addressed closed-loop system having controller and observer gain matrices as known. Further, by taking into account the dissipative performance, we extend the results established in Theorem 1 to obtain Theorem 2 with unknown controller and observer gain matrices. To be precise, the main difference between two theorems is that in Theorem 1, stochastic finite-time boundedness with known gain matrices is established, whereas in Theorem 2, the stochastic finite-time dissipativity analysis with unknown gain matrices is procured.

Theorem 1. *Let the control and observer gain matrices $\mathcal{K}_{\sigma i}$ and $\mathcal{L}_\sigma$, respectively be known. For given positive scalars ϵ, h, μ, c_1, c_2, w_f, α, T_f and symmetric matrix F, the closed-loop form of the considered system (2) is said to be stochastically finite-time bounded such that if there exist positive definite matrices $\mathcal{P}_{r\sigma}$ $(r = 1, \ldots, 7)$ such that the following constraints hold:*

$$\begin{cases} \mathbf{\Pi^{ii}} < 0, & (i = 1, 2, \ldots N_\sigma) \\ \mathbf{\Pi^{ij}} + \mathbf{\Pi^{ji}} < 0, & (i, j = 1, \ldots N_\sigma), i \neq j \end{cases} \tag{9}$$

$$\mathcal{P}_{r\sigma} < \mu \mathcal{P}_{r\varsigma}, \quad \mu > 1, \quad \forall\, \sigma, \varsigma \in \mathbb{M}, \quad \sigma \neq \varsigma, \tag{10}$$

$$\varpi c_1 + w_f[e^{\alpha T_f} - 1] < e^{\alpha T_f} \lambda_1 c_2, \tag{11}$$

where

$$\mathbf{\Pi^{ij}} = \begin{bmatrix} [\Pi^{ij}]_{8\times 8} & \Pi_a^{ij\,T} & \Pi_b^{ij\,T} \\ * & \Pi_{a1}^{ij} & 0 \\ * & * & \Pi_{b1}^{ij} \end{bmatrix},$$

$$\Pi_a^{ij} = \begin{bmatrix} 0 & 0 & \delta_1 h \mathcal{B}_{\sigma i} \Delta_1 \mathcal{K}_{\sigma i} & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \delta_k h \mathcal{B}_{\sigma i} \Delta_k \mathcal{K}_{\sigma i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Pi_b^{ij} = \begin{bmatrix} 0 & 0 & \delta_1 \epsilon \mathcal{B}_{\sigma i} \Delta_1 \mathcal{K}_{\sigma i} & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \delta_k \epsilon \mathcal{B}_{\sigma i} \Delta_k \mathcal{K}_{\sigma i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{a1}^{ij} = diag\{\underbrace{-\mathcal{P}_{6\sigma}^{-1}, \ldots, \mathcal{P}_{6\sigma}^{-1}}_{k}\}, \qquad \Pi_{b1}^{ij} = diag\{\underbrace{-\mathcal{P}_{7\sigma}^{-1}, \ldots, \mathcal{P}_{7\sigma}^{-1}}_{k}\},$$

$\Pi_{1,1}^{ij} = 2\mathcal{P}_{1\sigma}\mathcal{A}_{\sigma i} + \mathcal{P}_{3\sigma} + \mathcal{P}_{4\sigma} + \mathcal{P}_{5\sigma} - \mathcal{P}_{6\sigma} - \mathcal{P}_{7\sigma} + \alpha \mathcal{P}_{1\sigma}$, $\Pi_{1,2}^{ij} = \mathcal{P}_{1\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Pi_{1,3}^{ij} = \mathcal{P}_{1\sigma}\mathcal{B}_{\sigma i}\bar{\mathcal{G}}\mathcal{K}_{\sigma i} + \mathcal{P}_6$, $\Pi_{1,4}^{ij} = \mathcal{P}_{1\sigma}\mathcal{A}_{d\sigma i} + \mathcal{P}_{7\sigma}$, $\Pi_{1,6}^{ij} = \mathcal{P}_{1\sigma}E_1$, $\Pi_{1,7}^{ij} = h\mathcal{A}_{\sigma i}^T$, $\Pi_{1,8}^{ij} = \epsilon \mathcal{A}_{\sigma i}^T$, $\Pi_{2,2}^{ij} = 2\mathcal{P}_{2\sigma}\mathcal{W}_\sigma + 2\mathcal{P}_{2\sigma}\mathcal{L}_\sigma\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Pi_{2,6}^{ij} = \mathcal{P}_{2\sigma}E_2$, $\Pi_{2,7}^{ij} = h\mathcal{V}_\sigma^T\mathcal{B}_{\sigma i}^T$, $\Pi_{2,8}^{ij} = \epsilon \mathcal{V}_\sigma^T\mathcal{B}_{\sigma i}^T$, $\Pi_{3,3}^{ij} = -\mathcal{P}_{3\sigma} - \mathcal{P}_{6\sigma}$, $\Pi_{3,7}^{ij} = h\mathcal{K}_{\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T$, $\Pi_{3,8}^{ij} = \epsilon \mathcal{K}_{\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T$, $\Pi_{4,4}^{ij} = -(1-\hat{\epsilon})\mathcal{P}_{4\sigma} - \mathcal{P}_{7\sigma}$, $\Pi_{5,5}^{ij} = -\mathcal{P}_{5\sigma}$, $\Pi_{6,6}^{ij} = -\alpha I$, $\Pi_{6,7}^{ij} = hE_1^T$, $\Pi_{6,8}^{ij} = \epsilon E_1^T$, $\Pi_{7,7}^{ij} = -\mathcal{P}_{6\sigma}^{-1}$, $\Pi_{8,8}^{ij} = -\mathcal{P}_{7\sigma}^{-1}$, $\check{\mathcal{P}}_{r\sigma} = F^{\frac{-1}{2}}\mathcal{P}_{r\sigma}F^{\frac{-1}{2}}$, $\lambda_1 = \lambda_{min}(\check{\mathcal{P}}_{1\sigma})$ *and* $\lambda_s = \lambda_{max}(\check{\mathcal{P}}_{r\sigma})$, $s = \{2, 3, \ldots, 7\}$, $r = \{1, 2, \ldots, 7\}$.
Further, the average dwell-time satisfies

$$\zeta_a > \zeta_a^* = \frac{T\, ln\mu}{ln(\lambda_1 c_2) - ln(\varpi c_1 + w_f[e^{\alpha T_f} - 1]) + \alpha T_f}, \tag{12}$$

where $\mu > 1$.

Proof. In order to prove the stochastic finite-time boundedness of the closed-loop system (7), we construct the Lyapunov-Krasovskii functional in the following form:

$$V_\sigma(t) = \sum_{\nu=1}^{3} \mathcal{V}_{\nu\sigma}(t), \tag{13}$$

where

$$V_{1\sigma}(t) = \begin{bmatrix} x(t) \\ e_\chi(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{P}_{1\sigma} & 0 \\ 0 & \mathcal{P}_{2\sigma} \end{bmatrix} \begin{bmatrix} x(t) \\ e_\chi(t) \end{bmatrix},$$

$$V_{2\sigma}(t) = \int_{t-h}^{t} x^T(s)\mathcal{P}_{3\sigma}x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)\mathcal{P}_{4\sigma}x(s)ds + \int_{t-\epsilon}^{t} x^T(s)\mathcal{P}_{5\sigma}x(s)ds,$$

$$V_{3\sigma}(t) = h\int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(u)\mathcal{P}_{6\sigma}\dot{x}(u)duds + \epsilon\int_{-\tau(t)}^{0}\int_{t+s}^{t} \dot{x}^T(u)\mathcal{P}_{7\sigma}\dot{x}(u)duds.$$

Using the infinitesimal operator £{.} defined in [39] and taking mathematical expectation on both sides, we obtain

$$\begin{aligned}
\mathbb{E}\{£V_{1\sigma}(t)\} =& \mathbb{E}\{2x^T(t)\mathcal{P}_{1\sigma}\dot{x}(t) + 2e_\chi^T(t)\mathcal{P}_{2\sigma}\dot{e}_\chi(t)\} \\
=& 2x^T(t)\mathcal{P}_{1\sigma}\Big[\tilde{\mathcal{A}}_{\sigma i}x(t) + \tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h) + \tilde{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)\Big] \\
&+ 2e_\chi^T(t)\mathcal{P}_{2\sigma}\Big[(\mathcal{W}_\sigma + \mathcal{L}_\sigma\tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma)e_\chi(t) + E_2\bar{\omega}(t)\Big],
\end{aligned} \tag{14}$$

$$\begin{aligned}
\mathbb{E}\{£V_{2\sigma}(t)\} \le& \mathbb{E}\{x^T(t)\mathcal{P}_{3\sigma}x(t) - x^T(t-h)\mathcal{P}_{3\sigma}x(t-h) + x^T(t)\mathcal{P}_{4\sigma}x(t) - (1-\hat{\epsilon})x^T(t-\tau(t))\mathcal{P}_{4\sigma}x(t-\tau(t)) \\
&+ x^T(t)\mathcal{P}_{5\sigma}x(t) - x^T(t-\epsilon)\mathcal{P}_{5\sigma}x(t-\epsilon)\} \\
=& \mathbb{E}\{x^T(t)(\mathcal{P}_{3\sigma} + \mathcal{P}_{4\sigma} + \mathcal{P}_{5\sigma})x(t) - x^T(t-h)\mathcal{P}_{3\sigma}x(t-h) - (1-\hat{\epsilon})x^T(t-\tau(t))\mathcal{P}_{4\sigma}x(t-\tau(t)) \\
&- x^T(t-\epsilon)\mathcal{P}_{5\sigma}x(t-\epsilon)\},
\end{aligned} \tag{15}$$

$$\mathbb{E}\{£V_{3\sigma}(t)\} = \mathbb{E}\{h^2\dot{x}^T(t)\mathcal{P}_{6\sigma}\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^T(s)\mathcal{P}_{6\sigma}\dot{x}(s)ds + \epsilon^2\dot{x}^T(t)\mathcal{P}_{7\sigma}\dot{x}(t) - \epsilon\int_{t-\tau(t)}^{t} \dot{x}^T(s)\mathcal{P}_{7\sigma}\dot{x}(s)ds\}. \tag{16}$$

As a continuation, by substituting the system portrayed in (7), the first and third terms of (16) is re-written as

$$\begin{aligned}
h^2\mathbb{E}\{\dot{x}^T(t)\mathcal{P}_{6\sigma}\dot{x}(t)\} =& \Big[\tilde{\mathcal{A}}_{\sigma i}x(t) + \tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h) + \tilde{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)\Big]^T h^2\mathcal{P}_{6\sigma} \\
&\Big[\tilde{\mathcal{A}}_{\sigma i}x(t) + \tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h) + \tilde{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)\Big] \\
&+ \mathbb{E}\{x^T(t-h)\tilde{\mathcal{K}}_{\sigma i}^T(\mathcal{G}-\bar{\mathcal{G}})^T\tilde{\mathcal{B}}_{\sigma i}^T h^2\mathcal{P}_{6\sigma}\tilde{\mathcal{B}}_{\sigma i}(\mathcal{G}-\bar{\mathcal{G}})\tilde{\mathcal{K}}_{\sigma i}x(t-h)\},
\end{aligned} \tag{17}$$

$$\begin{aligned}
\epsilon^2\mathbb{E}\{\dot{x}^T(t)\mathcal{P}_{7\sigma}\dot{x}(t)\} =& \Big[\tilde{\mathcal{A}}_{\sigma i}x(t) + \tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h) + \tilde{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)\Big]^T \epsilon^2\mathcal{P}_{7\sigma} \\
&\Big[\tilde{\mathcal{A}}_{\sigma i}x(t) + \tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h) + \tilde{\mathcal{A}}_{d\sigma i}x(t-\tau(t)) + \tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_\sigma e_\chi(t) + E_1\bar{\omega}(t)\Big] \\
&+ \mathbb{E}\{x^T(t-h)\tilde{\mathcal{K}}_{\sigma i}^T(\mathcal{G}-\bar{\mathcal{G}})^T\tilde{\mathcal{B}}_{\sigma i}^T \epsilon^2\mathcal{P}_{7\sigma}\tilde{\mathcal{B}}_{\sigma i}(\mathcal{G}-\bar{\mathcal{G}})\tilde{\mathcal{K}}_{\sigma i}x(t-h)\}.
\end{aligned} \tag{18}$$

Then, by using (6), the second term in (17) and (18) can be re-formulated as follows:

$$\begin{aligned}
&\mathbb{E}\{x^T(t-h)\tilde{\mathcal{K}}_{\sigma i}^T(\mathcal{G}-\bar{\mathcal{G}})^T\tilde{\mathcal{B}}_{\sigma i}^T h^2\mathcal{P}_{6\sigma}\tilde{\mathcal{B}}_{\sigma i}(\mathcal{G}-\bar{\mathcal{G}})\tilde{\mathcal{K}}_{\sigma i}x(t-h)\} \\
&\quad = x^T(t-h), \tilde{\mathcal{K}}_{\sigma i}^T \sum_{\ell=1}^{k} \delta_l^2\Delta_l^T\tilde{\mathcal{B}}_{\sigma i}^T h^2\mathcal{P}_{6\sigma}\tilde{\mathcal{B}}_{\sigma i}\Delta_l\tilde{\mathcal{K}}_{\sigma i}x(t-h),
\end{aligned} \tag{19}$$

$$\mathbb{E}\{x^T(t-h)\tilde{\mathcal{K}}_{\sigma i}^T(\mathcal{G}-\bar{\mathcal{G}})^T\tilde{\mathcal{B}}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\tilde{\mathcal{B}}_{\sigma i}(\mathcal{G}-\bar{\mathcal{G}})\tilde{\mathcal{K}}_{\sigma i}x(t-h)\}$$
$$=x^T(t-h)\tilde{\mathcal{K}}_{\sigma i}^T\sum_{\ell=1}^{k}\delta_l^2\Delta_l^T\tilde{\mathcal{B}}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\tilde{\mathcal{B}}_{\sigma i}\Delta_l\tilde{\mathcal{K}}_{\sigma i}x(t-h). \tag{20}$$

By applying Jensen's inequality, (16) is re-written as

$$-h\int_{t-h}^{t}\dot{x}^T(s)\mathcal{P}_{6\sigma}\dot{x}(s)ds\le\begin{bmatrix}x(t)\\x(t-h)\end{bmatrix}^T\begin{bmatrix}-\mathcal{P}_{6\sigma}&\mathcal{P}_{6\sigma}\\ *&-\mathcal{P}_{6\sigma}\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h)\end{bmatrix}, \tag{21}$$

$$-\epsilon\int_{t-\tau(t)}^{t}\dot{x}^T(s)\mathcal{P}_{7\sigma}\dot{x}(s)ds\le\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}-\mathcal{P}_{7\sigma}&\mathcal{P}_{7\sigma}\\ *&-\mathcal{P}_{7\sigma}\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}. \tag{22}$$

Further, by summing the inequalities (13)-(22), we can get

$$\mathbb{E}\{\pounds V(t)\}+\alpha\mathbb{E}\{V(t)\}-\bar{\omega}^T(t)\alpha\bar{\omega}(t)\le\sum_{i=1}^{N_\sigma}\sum_{j=1}^{N_\sigma}\hbar_{\sigma i}(\xi(t))\hbar_{\sigma j}(\xi(t))\Big[\Upsilon^T(t)\hat{\mathbf{\Pi}}^{\mathbf{ij}}\Upsilon(t)\Big], \tag{23}$$

where

$$\hat{\mathbf{\Pi}}^{\mathbf{ij}}=\begin{bmatrix}\hat{\Pi}_{1,1}^{ij}&\hat{\Pi}_{1,2}^{ij}&\hat{\Pi}_{1,3}^{ij}&\hat{\Pi}_{1,4}^{ij}&0&\hat{\Pi}_{1,6}^{ij}\\ *&\hat{\Pi}_{2,2}^{ij}&0&0&0&\hat{\Pi}_{2,6}^{i}\\ *&*&\hat{\Pi}_{3,3}^{ij}&0&0&0\\ *&*&*&\hat{\Pi}_{4,4}^{ij}&0&0\\ *&*&*&*&\hat{\Pi}_{5,5}^{ij}&0\\ *&*&*&*&*&\hat{\Pi}_{6,6}^{ij}\end{bmatrix}$$

$\Upsilon^T(t)=\begin{bmatrix}x^T(t) & e_\chi^T(t) & x^T(t-h) & x^T(t-\tau(t)) & x^T(t-\epsilon) & \bar{\omega}^T(t)\end{bmatrix}^T$, where $\hat{\Pi}_{1,1}^{ij}=2\mathcal{P}_{1\sigma}\mathcal{A}_{\sigma i}+\mathcal{P}_{3\sigma}+\mathcal{P}_{4\sigma}+\mathcal{P}_{5\sigma}+\mathcal{A}_{\sigma i}^Th^2\mathcal{P}_{6\sigma}\mathcal{A}_{\sigma i}-\mathcal{P}_6+\mathcal{A}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\mathcal{A}_{\sigma i}-\mathcal{P}_{7\sigma}+\alpha\mathcal{P}_{1\sigma}$, $\hat{\Pi}_{1,2}^{ij}=\mathcal{P}_{1\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\hat{\Pi}_{1,3}^{ij}=\mathcal{P}_{1\sigma}\mathcal{B}_{\sigma i}\bar{\mathcal{G}}\mathcal{K}_{\sigma i}+\mathcal{P}_{6\sigma}$, $\hat{\Pi}_{1,4}^{ij}=\mathcal{P}_{1\sigma}\mathcal{A}_{d\sigma i}+\mathcal{P}_{7\sigma}$, $\hat{\Pi}_{1,6}^{ij}=\mathcal{P}_{1\sigma}E_1$, $\hat{\Pi}_{2,2}^{ij}=2\mathcal{P}_{2\sigma}\mathcal{W}_\sigma+2\mathcal{P}_{2\sigma}\mathcal{L}_\sigma\mathcal{B}_{\sigma i}\mathcal{V}_\sigma+\mathcal{V}_\sigma^T\mathcal{B}_{\sigma i}^Th^2\mathcal{P}_{6\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma+\mathcal{V}_\sigma^T\mathcal{B}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\hat{\Pi}_{2,8}^{ij}=\mathcal{P}_{2\sigma}E_2$, $\hat{\Pi}_{3,3}^{ij}=-\mathcal{P}_{3\sigma}+\mathcal{K}_{\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^Th^2\mathcal{P}_{6\sigma}\mathcal{B}_{\sigma i}\bar{\mathcal{G}}\mathcal{K}_{\sigma i}+\mathcal{K}_{\sigma i}^T\sum_{i=1}^{k}\delta_l^2\Delta_l^T\mathcal{B}_{\sigma i}^Th^2\mathcal{P}_{6\sigma}\mathcal{B}_{\sigma i}\Delta_l\mathcal{K}_{\sigma i}+\mathcal{K}_{\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\mathcal{B}_{\sigma i}\bar{\mathcal{G}}\mathcal{K}_{\sigma i}+\mathcal{K}_{\sigma i}^T\sum_{\imath=1}^{k}\delta_l^2\Delta_l^T\mathcal{B}_{\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\mathcal{B}_{\sigma i}\Delta_l\mathcal{K}_{\sigma i}-\mathcal{P}_{6\sigma}$, $\hat{\Pi}_{4,4}^{ij}=-(1-\hat{\epsilon})\mathcal{P}_{4\sigma}+\mathcal{A}_{d\sigma i}^Th^2\mathcal{P}_{6\sigma}\mathcal{A}_{d\sigma i}+\mathcal{A}_{d\sigma i}^T\epsilon^2\mathcal{P}_{7\sigma}\mathcal{A}_{d\sigma i}-\mathcal{P}_{7\sigma}$, $\hat{\Pi}_{5,5}^{ij}=-\mathcal{P}_{5\sigma}$, $\hat{\Pi}_{6,6}^{ij}=E_1^Th^2\mathcal{P}_{6\sigma}E_1+E_1^T\epsilon^2\mathcal{P}_{7\sigma}E_1-\alpha I$.

To this we add the use of the Schur complement in the above inequality (23), which yields the matrix in (9). Consequently, if the inequality (9) holds, then

$$\mathbb{E}\{\pounds V_\sigma(t)\}+\alpha\mathbb{E}\{V_\sigma(t)\}\le\bar{\omega}^T(t)\alpha\bar{\omega}(t). \tag{24}$$

After integrating the previous inequality from t_n to t, one arrives at the following result:

$$\mathbb{E}\{V_\sigma(t)\}\le e^{-\alpha(t-t_n)}\Big[\mathbb{E}\{V_\sigma(t_n)\}+w_f[e^{\alpha(t-t_n)}-1]\Big]. \tag{25}$$

It can be seen from (10) that

$$\mathbb{E}\{V_\sigma(t_n)\}\le\mu\mathbb{E}\{V_\sigma(t_n^-)\},\ \text{where}\ \ \mu>1. \tag{26}$$

Now, from (25) and (26), we derive

$$\begin{aligned}\mathbb{E}\{V_\sigma(t)\}\le&\mu\ e^{-\alpha(t-t_n)}\mathbb{E}\{V_\sigma(t_n^-)+w_f[e^{\alpha(t-t_n)}-1]\}\\ \le&\mu^2e^{-\alpha(t-t_{n-1})}\mathbb{E}\{V_\sigma(t_{n-1}^-)+w_f[e^{\alpha(t-t_{n-1})}-1]\}\\ \le&\ldots\le\ \mu^{N_\sigma(0,T_f)}e^{-\alpha T_f}\mathbb{E}\{V_\sigma(0)+w_f[e^{\alpha T_f}-1]\},\end{aligned} \tag{27}$$

where $\frac{t_a - t_b}{\zeta_a^*} \geq N_\sigma(0, T_f)$ denotes the number of switchings in the σ-th subsystem over $[0, T_f)$ and ζ_a^* represents the average dwell-time.

Consider $\breve{\mathcal{P}}_{r\sigma} = F^{\frac{-1}{2}} \mathcal{P}_{r\sigma} F^{\frac{-1}{2}}$ for any symmetric matrix F and from (13), we obtain

$$\mathbb{E}\{V_\sigma(t)\} \geq \mathbb{E}[x^T(t)\mathcal{P}_{1\sigma}x(t)] \geq \lambda_{min}(\breve{\mathcal{P}}_{1\sigma})\mathbb{E}[x^T(t)Fx(t)] = \lambda_1 \mathbb{E}[x^T(t)Fx(t)]. \tag{28}$$

On the other hand, under zero initial condition, we have

$$\begin{aligned}
\mathbb{E}\{V(0)\} =& \mathbb{E}\Big\{x^T(0)\mathcal{P}_{1\sigma}x(0) + e_\chi^T(0)\mathcal{P}_{2\sigma}e_\chi(0) + \int_{-h}^{0} x^T(s)\mathcal{P}_{3\sigma}x(s)ds + \int_{-\epsilon(0)}^{0} x^T(s)\mathcal{P}_{4\sigma}x(s)ds \\
&+ \int_{-\epsilon}^{0} x^T(s)\mathcal{P}_{5\sigma}x(s)ds + h\int_{-h}^{0}\int_{s}^{0} \dot{x}^T(u)\mathcal{P}_{6\sigma}\dot{x}(u)duds + \epsilon \int_{-\epsilon(0)}^{0}\int_{s}^{0} \dot{x}^T(u)\mathcal{P}_{7\sigma}\dot{x}(u)duds\Big\} \\
\leq& \Big[\lambda_{max}(\breve{\mathcal{P}}_1) + \lambda_{max}(\breve{\mathcal{P}}_2) + h\lambda_{max}(\breve{\mathcal{P}}_3) + \epsilon\lambda_{max}(\tilde{\mathcal{P}}_4) + \epsilon\lambda_{max}(\breve{\mathcal{P}}_5) + \frac{h^3}{2}\lambda_{max}(\breve{\mathcal{P}}_6) + \frac{\epsilon^3}{2}\lambda_{max}(\breve{\mathcal{P}}_7)\Big] \\
&\times \sup_{-\max(\epsilon,h)\leq t_0 < 0} \mathbb{E}\Big\{x^T(t_0)Fx(t_0), e_\chi^T(t_0)Fe_\chi(t_0), \dot{x}^T(t_0)F\dot{x}(t_0)\Big\} \\
=& \Big[2\lambda_2 + h\lambda_3 + \epsilon(\lambda_4 + \lambda_5) + \frac{h^3}{2}\lambda_6 + \frac{\epsilon^3}{2}\lambda_7\Big]c_1 \\
=& \varpi\, c_1.
\end{aligned} \tag{29}$$

By combining (27) to (29), we have

$$\lambda_1 \mathbb{E}[x^T(t)Fx(t)] \leq \mu^{\frac{T_f}{\zeta_a}} e^{-\alpha T_f}\Big[\varpi\, c_1 + w_f[e^{\alpha T_f} - 1]\Big]. \tag{30}$$

If (11) and (12) hold, we can compute

$$\mathbb{E}[x^T(t)Fx(t)] \leq c_2. \tag{31}$$

In accordance with Definition 1, it is easy to conclude that the closed-loop system (7) is stochastically finite-time bounded with respect to $(c_1, c_2, T_f, \alpha, F)$. Thus, the theorem's proof is completed. $\square$

Further, by taking gain matrices to be unknown and considering the dissipative performance, the necessary conditions are developed in the following theorem which assures the stochastic finite-time boundedness of the closed-loop system (7) and satisfies the desired $(Q, S, R) - \theta$ dissipative performance index.

3.2. Stochastic finite-time dissipativity analysis

Theorem 2. *With Assumption 1, let us consider the closed-loop system (7). Then, given with the positive scalars ϵ, h, μ, c_1, c_2, w_f, α, T_f and θ, symmetric matrix F and matrices Q, S, R are constants such that $Q \leq 0$, $R = R^T$ and $S = S^T$, if there exist symmetric matrices $\mathcal{P}_r > 0$ $(r = 1, 2, \ldots, 7)$ and appropriate dimensioned matrices $\mathcal{Y}_{1\sigma i}$, $\mathcal{Y}_{2\sigma}$ and the below-listed inequalities hold:*

$$\begin{cases} \mathbf{\Lambda^{ii}} < 0, & (i = 1, 2, \ldots N_\sigma) \\ \mathbf{\Lambda^{ij}} + \mathbf{\Lambda^{ji}} < 0, & (i, j = 1, \ldots N_\sigma), i \neq j \end{cases} \tag{32}$$

$$\kappa F^{-1} < \mathcal{P}_\sigma < F^{-1}, \tag{33}$$

$$0 < \bar{\mathcal{P}}_{\jmath\sigma} < 2F^{-1}, \quad \jmath = \{3, 4, \ldots, 7\}, \tag{34}$$

$$\begin{bmatrix} \bar{\vartheta} & \sqrt{2\epsilon(1 + \frac{\epsilon^2}{2}) + 2h(1 + \frac{h^2}{2})c_1} \\ * & -\kappa \end{bmatrix} < 0, \tag{35}$$

where

$$\mathbf{\Lambda^{ij}} = \begin{bmatrix} [\Lambda^{ij}]_{9\times 9} & \Lambda_a^{ij\,T} & \Lambda_b^{ij\,T} \\ * & \Lambda_{a1}^{ij} & 0 \\ * & * & \Lambda_{b1}^{ij} \end{bmatrix},$$

$$\Lambda_a^{ij} = \begin{bmatrix} 0 & 0 & \delta_1 h\mathcal{B}_{\sigma i}\Delta_1\mathcal{Y}_{1\sigma i} & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \delta_k h\mathcal{B}_{\sigma i}\Delta_k\mathcal{Y}_{1\sigma i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Lambda_b^{ij} = \begin{bmatrix} 0 & 0 & \delta_1 \epsilon\mathcal{B}_{\sigma i}\Delta_1\mathcal{Y}_{1\sigma i} & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \delta_k \epsilon\mathcal{B}_{\sigma i}\Delta_k\mathcal{Y}_{1\sigma i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_{a1}^{ij} = diag\{\underbrace{-2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{6\sigma}, \ldots, -2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{6\sigma}}_{k}\}, \Lambda_{b1}^{ij} = diag\{\underbrace{-2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{7\sigma}, \ldots, -2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{7\sigma}}_{k}\},$$

$\Lambda_{1,1}^{ij} = 2\mathcal{A}_{\sigma i}\mathcal{P}_\sigma + \bar{\mathcal{P}}_{3\sigma} + \bar{\mathcal{P}}_{4\sigma} + \bar{\mathcal{P}}_{5\sigma} - \bar{\mathcal{P}}_{6\sigma} - \bar{\mathcal{P}}_{7\sigma} + \alpha\mathcal{P}_\sigma$, $\Lambda_{1,2}^{ij} = \mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Lambda_{1,3}^{ij} = \mathcal{B}_{\sigma i}\bar{\mathcal{G}}Y_{1\sigma i} + \bar{\mathcal{P}}_{6\sigma}$, $\Lambda_{1,4}^{ij} = \mathcal{A}_{d\sigma i}\mathcal{P}_\sigma + \bar{\mathcal{P}}_{7\sigma}$, $\Lambda_{1,6}^{ij} = E_1 - 2\mathcal{P}_\sigma\mathcal{C}_{\sigma i}^T S$, $\Lambda_{1,7}^{ij} = h\mathcal{P}_\sigma\mathcal{A}_{\sigma i}^T$, $\Lambda_{1,8}^{ij} = \epsilon\mathcal{P}_\sigma\mathcal{A}_{\sigma i}^T$, $\Lambda_{1,9}^{ij} = \mathcal{P}_\sigma\mathcal{C}_{\sigma i}^T\bar{Q}^{\frac{1}{2}}$, $\Lambda_{2,2}^{ij} = 2\mathcal{P}_{2\sigma}\mathcal{W}_\sigma + 2\mathcal{Y}_{2\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Lambda_{2,6}^{ij} = \mathcal{P}_{2\sigma}E_2$, $\Lambda_{2,7}^{ij} = h\mathcal{V}_\sigma\mathcal{B}_{\sigma i}$, $\Lambda_{2,8}^{ij} = \epsilon\mathcal{V}_\sigma\mathcal{B}_{\sigma i}$, $\Lambda_{3,3}^{ij} = -\bar{\mathcal{P}}_{3\sigma} - \bar{\mathcal{P}}_{6\sigma}$, $\Lambda_{3,7}^{ij} = h\mathcal{Y}_{1\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T$, $\Lambda_{3,8}^{ij} = \epsilon\mathcal{Y}_{1\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T$, $\Lambda_{4,4}^{ij} = -(1-\hat{\epsilon})\bar{\mathcal{P}}_{4\sigma} - \bar{\mathcal{P}}_{7\sigma}$, $\Lambda_{4,7}^{ij} = h\mathcal{P}_\sigma\mathcal{A}_{d\sigma i}^T$, $\Lambda_{4,8}^{ij} = \epsilon\mathcal{P}_\sigma\mathcal{A}_{d\sigma i}^T$, $\Lambda_{5,5}^{ij} = -\bar{\mathcal{P}}_{5\sigma}$, $\Lambda_{6,6}^{ij} = -2E_{3\sigma i}^T S - R + \theta I$, $\Lambda_{6,7}^{ij} = hE_1^T$, $\Lambda_{6,8}^{ij} = \epsilon E_1^T$, $\Lambda_{6,9}^{ij} = E_{3\sigma i}^T\bar{Q}^{\frac{1}{2}}$, $\Lambda_{7,7}^{ij} = -2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{6\sigma}$, $\Lambda_{8,8}^{ij} = -2\mathcal{P}_\sigma + \bar{\mathcal{P}}_{7\sigma}$, $\Lambda_{9,9}^{ij} = -I$ *and* $\bar{\vartheta} = 2c_1 + [(w_f - c_2)e^{\alpha T_f} - w_f]\kappa$, *then the closed-loop system* (7) *is stochastically finite-time bounded with a desired* $(Q, S, R) - \theta$ *dissipativity performance index. Moreover, the controller and observer gain matrices are calculated by utilizing the following relation* $\mathcal{K}_{\sigma i} = \mathcal{Y}_{1\sigma i}\mathcal{P}_\sigma^{-1}$ *and* $\mathcal{L}_\sigma = \mathcal{P}_{2\sigma}^{-1}\mathcal{Y}_{2\sigma}$.

Proof. By taking the same Lyapunov-Krasovskii functional and by following the similar lines of Theorem 1, the proof of Theorem 2 can be obtained. Precisely, for any non-zero disturbance $\bar{\omega}(t)$, we define

$$\mathrm{J}(t) = z^T(t)Qz(t) + 2z(t)^T S\bar{\omega}(t) + \bar{\omega}^T(t)(R - \theta I)\bar{\omega}(t). \tag{36}$$

From the above-mentioned relation, one has

$$\mathbb{E}\{\pounds V_\sigma(t)\} + \alpha\mathbb{E}\{V_\sigma(t)\} - \mathrm{J}(t) < \Upsilon^T(t)[\acute{\Pi}^{ij}]_{7\times 7}\Upsilon(t), \tag{37}$$

where $\acute{\Pi}_{1\times 1}^{ij} = \hat{\Pi}_{1,1}^{ij} - \mathcal{C}_{\sigma i}^T Q\mathcal{C}_{\sigma i}$, $\acute{\Pi}_{1\times 2}^{ij} = \Pi_{1,2}^{ij}$, $\acute{\Pi}_{1\times 3}^{ij} = \Pi_{1,3}^{ij}$, $\Pi_{1\times 4}^{ij} = \Pi_{1,4}^{ij}$, $\Pi_{1\times 6}^{ij} = \Pi_{1,6}^{ij} - 2\mathcal{C}_{\sigma i}^T S$, $\acute{\Pi}_{2\times 2}^{ij} = \Pi_{2,2}^{ij}$, $\Pi_{2\times 6}^{ij} = \Pi_{2,6}^{ij}$, $\Pi_{3\times 3}^{ij} = \Pi_{3,3}^{ij}$, $\Pi_{4\times 4}^{ij} = \Pi_{4,4}^{ij}$, $\Pi_{5\times 5}^{ij} = \Pi_{5,5}^{ij}$, $\Pi_{5\times 6}^{ij} = -E_{3\sigma i}^T QE_{3\sigma i} - 2E_{3\sigma i}^T S - R + \theta I$ and the remaining elements are same as in (23).

The matrix described above can be rewritten using the Schur complement lemma as follows:

$$\acute{\Pi}^{ij} = \begin{bmatrix} [\acute{\Pi}_{\aleph}^{ij}]_{9\times 9} & \Pi_a^{ij\,T} & \Pi_b^{ij\,T} \\ * & \Pi_{a1}^{ij} & 0 \\ * & * & \Pi_{b1}^{ij} \end{bmatrix},$$

where

$$\acute{\Pi}^{ij} = \begin{bmatrix} \acute{\Pi}_{\aleph}^{ij} & \Pi_a^{ij\,T} & \Pi_b^{ij\,T} \\ * & \Pi_{a1}^{ij} & 0 \\ * & * & \Pi_{b1}^{ij} \end{bmatrix},$$

where

$$\acute{\Pi}_{\aleph}^{ij}=\begin{bmatrix}\Pi_{1,1}^{ij} & \Pi_{1,2}^{ij} & \Pi_{1,3}^{ij} & \Pi_{1,4}^{ij} & 0 & \Pi_{1,6}^{ij}-2\mathcal{C}_{\sigma i}^{T}S & \Pi_{1,7}^{ij} & \Pi_{1,8}^{ij} & \mathcal{C}_{\sigma i}^{T}Q^{\frac{1}{2}}\\ * & \Pi_{2,2}^{ij} & 0 & 0 & 0 & \Pi_{2,6}^{ij} & 0 & 0 & 0\\ * & * & \Pi_{3,3}^{ij} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Pi_{4,4}^{ij} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \Pi_{5,5}^{ij} & 0 & 0 & 0 & 0\\ * & * & * & * & * & -2E_{3\sigma i}^{T}S-R+\theta I & \Pi_{6,7}^{ij} & \Pi_{6,8}^{ij} & E_{3\sigma i}^{T}Q^{\frac{1}{2}}\\ * & * & * & * & * & * & -\mathcal{P}_{6\sigma}^{-1} & 0 & 0\\ * & * & * & * & * & * & * & -\mathcal{P}_{7\sigma}^{-1} & 0\\ * & * & * & * & * & * & * & * & -I\end{bmatrix}.$$

Now, pre- and post-multiplying aforementioned matrix with $diag\{\mathcal{P}_{1\sigma}^{-1}, I, \mathcal{P}_{1\sigma}^{-1}, \mathcal{P}_{1\sigma}^{-1}, \mathcal{P}_{1\sigma}^{-1}, \underbrace{I, \dots, I}_{2m+4}\}$ and its transpose. Besides, by letting $\mathcal{P}_{\sigma}=\mathcal{P}_{1\sigma}^{-1}$, $\mathcal{P}_{\sigma}\mathcal{P}_{\ell\sigma}\mathcal{P}_{\sigma}=\bar{\mathcal{P}}_{\ell\sigma}$ and utilizing the relation $-2\mathcal{P}_{\sigma}+\mathcal{P}_{\ell\sigma}=-\mathcal{P}_{\sigma}\mathcal{P}_{\ell\sigma}\mathcal{P}_{\sigma}$, $\ell=\{3,\dots,7\}$, we acquire the matrix (32) given in theorem statement.

At the same time, we know that $\breve{\mathcal{P}}_{r\sigma}=F^{\frac{-1}{2}}\mathcal{P}_{r\sigma}F^{\frac{-1}{2}}$. The following relations can be acquired by applying the congruence transformation, $\breve{\mathcal{P}}_{\sigma}=F^{\frac{1}{2}}\mathcal{P}_{\sigma}F^{\frac{1}{2}}$ and $\lambda_{max}(\breve{\mathcal{P}}_{\sigma})=\frac{1}{\lambda_{min}(\breve{\mathcal{P}}_{1\sigma})}$, it ensures that $I<F^{\frac{-1}{2}}\mathcal{P}_{1\sigma}F^{\frac{-1}{2}}<\frac{1}{\kappa}I$, we have $\lambda_2<\frac{1}{\kappa}$ and $\lambda>1$. Further, $0<F^{\frac{-1}{2}}\mathcal{P}_{\upsilon\sigma}F^{\frac{-1}{2}}<2(F^{\frac{-1}{2}}\mathcal{P}_{1\sigma}F^{\frac{-1}{2}})^2<\frac{2}{\kappa^2}I$, $\upsilon=\{3,4,\dots,7\}$. Thus by employing the above relations, we deduce (33) and (34). Now by using the prior mentioned relations in (11), we get the following inequality;

$$\left[\frac{2}{\kappa}+\frac{2h}{\kappa^2}\left(1+\frac{h^2}{2}\right)+4\epsilon\left(1+\frac{\epsilon^2}{4}\right)\right]c_1+w_f\left[e^{\delta T_f}-1\right]<e^{\delta T_f}c_2, \tag{38}$$

wherein by taking Schur complement, it can be equivalently written as (35). The proof for this theorem is now complete. □

Suppose that there is no time-varying delay in TSFSS (1) under consideration, then we have the following closed-loop system:

$$\begin{cases}\dot{x}(t)= & [\tilde{\mathcal{A}}_{\sigma i}x(t)+\tilde{\mathcal{B}}_{\sigma i}(\mathcal{G}-\bar{\mathcal{G}})\tilde{\mathcal{K}}_{\sigma i}x(t-h)+\tilde{\mathcal{B}}_{\sigma i}\bar{\mathcal{G}}\tilde{\mathcal{K}}_{\sigma i}x(t-h)+\tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_{\sigma}e_{\chi}(t)+E_1\bar{\omega}(t)],\\ \dot{e}_{\chi}(t)= & [\mathcal{W}_{\sigma}+\mathcal{L}_{\sigma}\tilde{\mathcal{B}}_{\sigma i}\mathcal{V}_{\sigma}]e_{\chi}(t)+E_2\bar{\omega}(t),\\ z(t)= & [\tilde{\mathcal{C}}_{\sigma i}x(t)+E_{3\sigma i}\bar{\omega}(t)].\end{cases} \tag{39}$$

Corollary 1. *Let us consider the system (39) with Assumption 1. Suppose that we are given with the positive scalars α, h, c_1, c_2, w_f, T_f, μ, θ, symmetric matrix F, if there exist symmetric matrices $\mathcal{P}_r>0$ $(r=1,2,3,6)$, appropriate dimensioned matrices $\mathcal{Y}_{1\sigma i}$ and $\mathcal{Y}_{2\sigma}$ in a way that the below given conditions are satisfied;*

$$\begin{cases}\mathbf{\Gamma^{ii}} & <0, \quad (i=1,2,\dots N_{\sigma})\\ \mathbf{\Gamma^{ij}}+\mathbf{\Gamma^{ji}} & <0, \quad (i,j=1,\dots N_{\sigma}), i\neq j\end{cases} \tag{40}$$

$$\kappa F^{-1}<\mathcal{P}_{\sigma}<F^{-1}, \tag{41}$$

$$0<\bar{\mathcal{P}}_{q\sigma}<2F^{-1}, \quad q=\{3,6\}, \tag{42}$$

$$\begin{bmatrix}2c_1+[(w_f-c_2)e^{\alpha T_f}-w_f]\kappa & 2h(1+\frac{h^2}{2})c_1\\ * & -\kappa\end{bmatrix}<0, \tag{43}$$

where

$$\mathbf{\Gamma^{ij}}=\begin{bmatrix}[\Gamma^{ij}]_{6\times 6} & \Gamma_a^{ij\,T}\\ * & \Gamma_{a1}^{ij}\end{bmatrix},$$

$$\Gamma_a^{ij}=\begin{bmatrix}0 & 0 & \delta_1 h\mathcal{B}_{\sigma i}\Delta_1\mathcal{Y}_{1\sigma i} & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \delta_k h\mathcal{B}_{\sigma i}\Delta_k\mathcal{Y}_{1\sigma i} & 0 & 0 & 0\end{bmatrix}, \Gamma_{a1}^{ij}=diag\{\underbrace{-2\mathcal{P}_\sigma+\bar{\mathcal{P}}_{6\sigma},\ldots,-2\mathcal{P}_\sigma+\bar{\mathcal{P}}_{6\sigma}}_{k}\},$$

and $\Gamma_{1,1}^{ij}=2\mathcal{A}_{\sigma i}\mathcal{P}_\sigma+\bar{\mathcal{P}}_{3\sigma}-\bar{\mathcal{P}}_{6\sigma}+\alpha\mathcal{P}_\sigma$, $\Gamma_{1,2}^{ij}=\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Gamma_{1,3}^{ij}=\mathcal{B}_{\sigma i}\bar{\mathcal{G}}Y_{1\sigma i}+\bar{\mathcal{P}}_{6\sigma}$ $\Gamma_{1,4}^{ij}=E_1-2\mathcal{P}_\sigma\mathcal{C}_{\sigma i}^T S$, $\Gamma_{1,5}^{ij}=h\mathcal{P}_\sigma\mathcal{A}_{\sigma i}^T$, $\Gamma_{1,6}^{ij}=\mathcal{P}_\sigma\mathcal{C}_{\sigma i}^T\bar{Q}^{\frac{1}{2}}$, $\Gamma_{2,2}^{ij}=2\mathcal{P}_{2\sigma}\mathcal{W}_\sigma+2\mathcal{Y}_{2\sigma}\mathcal{B}_{\sigma i}\mathcal{V}_\sigma$, $\Gamma_{2,4}^{ij}=\mathcal{P}_{2\sigma}E_2$, $\Gamma_{2,5}^{ij}=h\mathcal{V}_\sigma\mathcal{B}_{\sigma i}$, $\Gamma_{3,3}^{ij}=-\bar{\mathcal{P}}_{3\sigma}-\bar{\mathcal{P}}_{6\sigma}$, $\Gamma_{3,7}^{ij}=h\mathcal{Y}_{1\sigma i}^T\bar{\mathcal{G}}^T\mathcal{B}_{\sigma i}^T$, $\Gamma_{4,4}^{ij}=-2E_{3\sigma i}^T S-R+\theta I$, $\Gamma_{4,5}^{ij}=hE_1^T$, $\Gamma_{4,6}^{ij}=E_{3\sigma i}^T\bar{Q}^{\frac{1}{2}}$, $\Gamma_{5,5}^{ij}=-2\mathcal{P}_\sigma+\bar{\mathcal{P}}_{6\sigma}$ $\Gamma_{6,6}^{ij}=-I$, *then the closed-loop system drafted in (39) is stochastically finite-time bounded. Moreover, the gain matrices are computed via* $\mathcal{K}_{\sigma i}=\mathcal{Y}_{1\sigma i}\mathcal{P}_\sigma^{-1}$ *and* $\hat{L}_\sigma=\mathcal{P}_{2\sigma}^{-1}\mathcal{Y}_{2\sigma}$.

Remark 2. The authors in [15] discussed the dynamic output feedback H_∞ control problem for TSFSSs. Meanwhile in [16], the problem of passivity and feedback passification for TSFSSs is investigated. It should be pinpointed out that in the above said words, the stabilization issue for TSFSSs has been discussed with one type of merged disturbance. However due to complex environment, multiple heterogeneous disturbances may exist in the system model. Thus, in our present work, we have considered the stabilization problem of TSFSSs with multiple disturbances, time-varying state delay, input delay in a single framework. Additionally, because the modes and patterns of actuator failures are basically random in nature, the actuator fault may manifest in a stochastic way. Nonetheless, in [37] and [38], the actuator fault is considered to be constant, which is not practically sufficient. Hence, we have examined into a model where actuator failures occur at random manner and fault rates are described using stochastic variables which follow a Bernoulli distribution. In addition, unlike asymptotic and exponential stability [4] and [14] which are defined across an indefinite length of time, the finite-time stability concept is explored in this study. Specifically, the finite-time stability prevents the states from exceeding a particular range within a predetermined time frame and it is distinguished from those other types of stability.

4. Simulation verification

In the subsequent part, simulation results of two numerical examples are provided to demonstrate the practical applicability and effectiveness of the theoretical conclusions stated in the previous sections. More specifically, by the virtue of Example 1, simultaneously stochastic finite-time boundedness of the considered system (7) is guaranteed. Furthermore, mass spring damper model is considered as an Example 2 to endorse the implementation of the designed control law in practice.

Example 1. In this example, a 2-rule TSFSS with two operating modes in the format of (1) is taken into account. The following are the system matrices that correspond to them:
Subsystem 1:

$$\mathcal{A}_{11}=\begin{bmatrix}-0.2 & 0.1\\ 1.2 & -0.2\end{bmatrix}, \mathcal{A}_{12}=\begin{bmatrix}-0.6 & 0.2\\ 1 & -0.5\end{bmatrix}, \mathcal{A}_{d11}=\begin{bmatrix}-0.1 & 0\\ -0.3 & 0.02\end{bmatrix}, \mathcal{A}_{d12}=\begin{bmatrix}0.23 & 0.1\\ 0.03 & -0.2\end{bmatrix},$$
$$\mathcal{B}_{11}=\mathcal{B}_{12}=\begin{bmatrix}0.5 & 0.4\\ 0.8 & 0.7\end{bmatrix}, \mathcal{B}_{w11}=\mathcal{B}_{w12}=\begin{bmatrix}0.12 & 0\\ 0 & 0.13\end{bmatrix}, \mathcal{C}_{11}=\begin{bmatrix}0.3 & 0.1\\ 0.3 & 0.1\end{bmatrix}, \mathcal{C}_{12}=\begin{bmatrix}0.1 & 0.2\\ 0.1 & 0.2\end{bmatrix}.$$

Subsystem 2:

$$\mathcal{A}_{21}=\begin{bmatrix}-0.5 & 0.7\\ 1 & -0.3\end{bmatrix}, \mathcal{A}_{22}=\begin{bmatrix}-1 & 0.4\\ 0.9 & -0.4\end{bmatrix}, \mathcal{A}_{d21}=\begin{bmatrix}-0.3 & -0.1\\ 0.3 & 0.1\end{bmatrix}, \mathcal{A}_{d22}=\begin{bmatrix}-0.13 & 0\\ 0.1 & 0.35\end{bmatrix},$$
$$\mathcal{B}_{21}=\mathcal{B}_{22}=\begin{bmatrix}0.5 & 0.4\\ 0.8 & 0.7\end{bmatrix}, \mathcal{B}_{w21}=\mathcal{B}_{w22}=\begin{bmatrix}0.12 & 0\\ 0 & 0.13\end{bmatrix}, \mathcal{C}_{21}=\begin{bmatrix}0.4 & 0.2\\ 0.3 & 0.1\end{bmatrix}, \mathcal{C}_{22}=\begin{bmatrix}0.1 & 0.3\\ 0.1 & 0.2\end{bmatrix}.$$

Moreover, the associated matrices of the exogenous system (3) with two operating modes are taken as follows:
Subsystem 1:

$$\mathcal{W}_1=\begin{bmatrix}-7.5 & 7.5\\ 7.5 & -7.5\end{bmatrix}, \mathcal{H}_1=\begin{bmatrix}1.3 & 7\\ 1 & 4\end{bmatrix}, \mathcal{V}_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

Table 1
Gain matrices with its corresponding index γ for distinct disturbance attenuation performances.

	$(Q,S,R)-\theta$ dissipativity	H_∞	$Passivity$	$Mixed\ H_\infty\ and\ Passivity$
$\mathcal{K}_{11}$	$-\begin{bmatrix}1.2506 & 0.4961\\ 0.5325 & 0.4677\end{bmatrix}$	$-\begin{bmatrix}1.0585 & 0.2636\\ 0.2662 & 0.8742\end{bmatrix}$	$-\begin{bmatrix}1.3318 & 0.7310\\ 0.6676 & 0.1633\end{bmatrix}$	$-\begin{bmatrix}1.0380 & 0.2880\\ 0.2934 & 0.8991\end{bmatrix}$
$\mathcal{K}_{12}$	$-\begin{bmatrix}1.0524 & 0.5443\\ 0.6929 & 0.6332\end{bmatrix}$	$-\begin{bmatrix}0.9744 & 0.4415\\ 0.4833 & 0.5284\end{bmatrix}$	$-\begin{bmatrix}1.0158 & 0.6159\\ 0.7402 & 0.5956\end{bmatrix}$	$-\begin{bmatrix}0.9417 & 0.4633\\ 0.4934 & 0.5649\end{bmatrix}$
$\mathcal{K}_{21}$	$-\begin{bmatrix}0.4629 & 0.4931\\ 0.7051 & 0.6555\end{bmatrix}$	$-\begin{bmatrix}0.6065 & 0.4994\\ 0.5055 & 0.9005\end{bmatrix}$	$-\begin{bmatrix}0.2635 & 0.5618\\ 1.0142 & 0.4784\end{bmatrix}$	$-\begin{bmatrix}0.6245 & 0.5167\\ 0.5275 & 0.9271\end{bmatrix}$
$\mathcal{K}_{22}$	$-\begin{bmatrix}0.1758 & 0.2955\\ 0.7386 & 1.1232\end{bmatrix}$	$-\begin{bmatrix}0.3792 & 0.3719\\ 0.4071 & 1.0816\end{bmatrix}$	$\begin{bmatrix}0.2028 & -0.2481\\ -1.1936 & -1.1577\end{bmatrix}$	$-\begin{bmatrix}0.3877 & 0.3926\\ 0.4261 & 1.1214\end{bmatrix}$
$\mathcal{L}_1$	$-\begin{bmatrix}9.4007 & 12.7214\\ 6.9777 & 12.7767\end{bmatrix}$	$-\begin{bmatrix}3.5321 & 5.6167\\ 3.0749 & 5.2843\end{bmatrix}$	$-\begin{bmatrix}9.3500 & 10.7956\\ 7.0326 & 10.8117\end{bmatrix}$	$-\begin{bmatrix}3.0666 & 3.5115\\ 2.0375 & 3.8128\end{bmatrix}$
$\mathcal{L}_2$	$-\begin{bmatrix}8.7273 & 17.4028\\ 9.3504 & 22.8026\end{bmatrix}$	$-\begin{bmatrix}4.2397 & 6.7100\\ 3.6601 & 5.8052\end{bmatrix}$	$-\begin{bmatrix}7.8715 & 11.4720\\ 6.7381 & 18.1956\end{bmatrix}$	$-\begin{bmatrix}4.0953 & 6.0902\\ 3.5542 & 5.8185\end{bmatrix}$
γ	0.5	0.71	0.62	0.95

Subsystem 2:

$$\mathcal{W}_2=\begin{bmatrix}-7.6 & 8\\ 8 & -7.6\end{bmatrix}, \mathcal{H}_2=\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}, \mathcal{V}_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

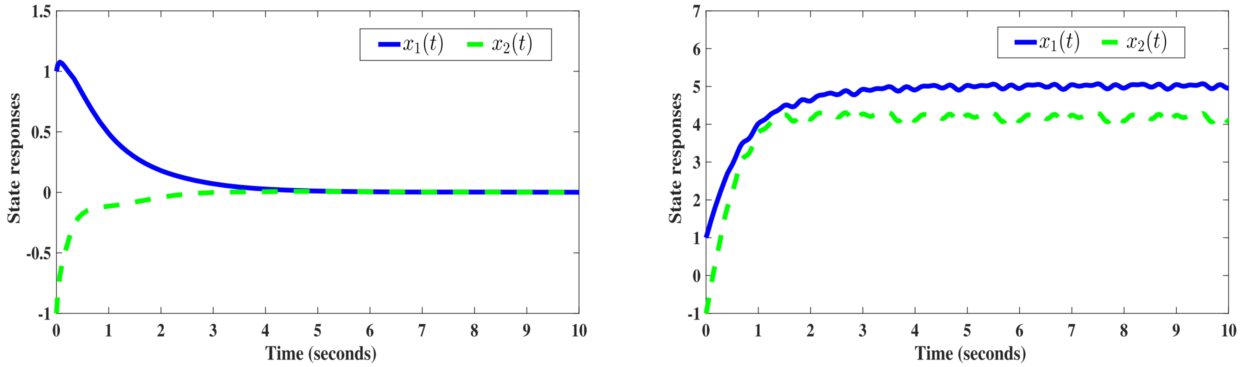
On the flip side, the disturbance signals are given by $w_1(t)=0.01\,sint$, $w_2(t)=0.01\,cos(\pi t)$, $\eta_1(t)=2\,sin(5\pi t)+3\,cos(7\pi t)+0.4\,cos(2\pi t)$ and $\eta_2(t)=cos(5\pi t)+0.7\,sin(\pi t)$. The membership functions are chosen as $\hbar_{11}(x_1(t))=\hbar_{21}(x_1(t))=\frac{sin^2(x_1(t))}{25}$ and $\hbar_{12}(x_1(t))=\hbar_{22}(x_1(t))=1-\frac{sin^2(x_1(t))}{25}$. The time-varying state delay function is taken as $0.2+0.2sin(t)$. In addition, we suppose that the stochastic behavior corresponding to the fault model obeys Bernoulli distributed white sequence, we have

$$\begin{cases}Prob\{\beta_\ell=1\}=\mathbb{E}\{\beta_\ell\}=\varrho_\ell,\\ Prob\{\beta_\ell=0\}=1-\mathbb{E}\{\beta_\ell\}=1-\varrho_\ell,\end{cases}$$

where ϱ_ℓ are non-negative real valued scalars. Notably, if the value of ϱ_ℓ increases, then the range of possible actuator failure will be decreased, that is, the value of $\varrho_\ell=1$ ensures the good condition of actuators. Further, the expectation and variance of fault are calculated by making use of the following relation $\vartheta=\varrho_\ell$ and $\delta_k^2=\varrho_\ell(1-\varrho_\ell)$, $\ell=1,\dots,k$. Take $k=2$ for simulation purpose and as a consequence, we consider $\varrho_1=0.8$ and $\varrho_2=0.9$.

The remaining parameters are chosen as $\epsilon=0.4$, $\hat{\tau}=0.2$, $h=0.3$, $c_1=0.5$, $c_2=2.2$, $w_f=0.4$, $T_f=10$, $\alpha=0.1$, $F=I$, $Q=-1.5$, $S=0.8$, $R=1.4I$ and $\theta=0.5$. Based on the values considered above and with initial conditions $x(t_0)=[1\ -1]^T$ and $\chi(t_0)=[50\ -50]^T$, $t_0\in[-max(\epsilon,h),0]$, the necessary criterion given in Theorem 2 is solved. The gain matrices for the controller and observer are tabulated in Table 1 based on the feasible solutions obtained. Precisely, Table 1 displays the desired gain matrices and its corresponding disturbance attenuation index γ under $(Q,S,R)-\theta$ dissipative, H_∞, passivity and mixed H_∞ and passivity performance.

In addition, we obtain the average dwell time as $\zeta_a^*=1.1095$. Then, with respect to the gain values mentioned in Table 1, the corresponding graphs of the addressed TSFSS (1) in response of developed composite anti-disturbance controller (5) are plotted in Figs. 1-8. To be particular, in Fig. 1, response of state trajectories $x_1(t)$ and $x_2(t)$ in the presence and absence of term $\hat{d}(t)$ in the developed controller's design (5) is exhibited. As seen in Fig. 1, we observe that the proposed controller stabilized the system states. Further, accuracy of the constructed disturbance observer is validated by plotting $d(t)$, its estimated term $\hat{d}(t)$ and error $d(t)-\hat{d}(t)$ depicted in Fig. 2. Additionally, the devised anti-disturbance observer-based reliable control signal's trajectory is pictured in Fig. 3. In respect of the optimal values of the finite-time parameters c_1 and c_2, the trajectories of $x^T(t)Fx(t)$ are given in Fig. 4 which signifies the finite-time boundedness of the addressed system. The corresponding switching signal is pictured in Fig. 5. Additionally, the developed control method is compared to the $(Q,S,R)-\theta$ dissipative approach to demonstrate its advantages over existing methods. Hence, in Fig. 6, the comparative plot between the developed controller and traditional $(Q,S,R)-\theta$



(a) with $\hat{d}(t)$ (b) without $\hat{d}(t)$

Fig. 1. Response of state dynamics of the closed-loop system (7) with and without $\hat{d}(t)$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

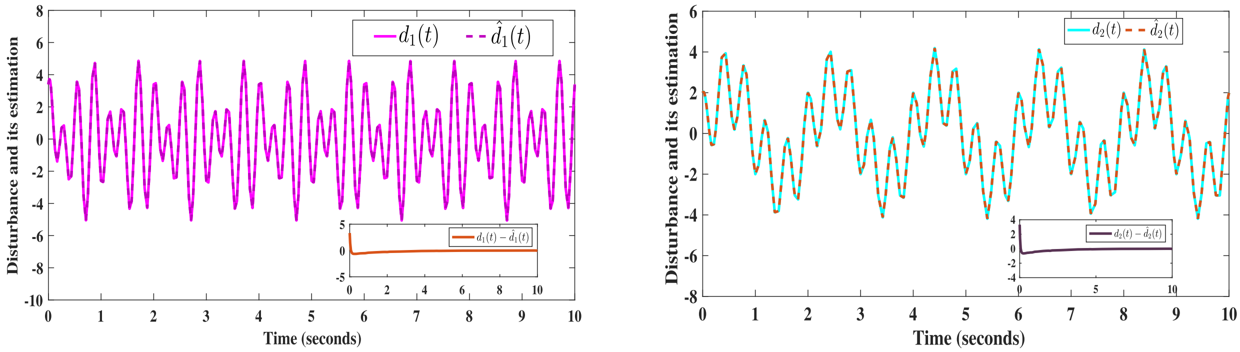


Fig. 2. Disturbance estimation and its error response.

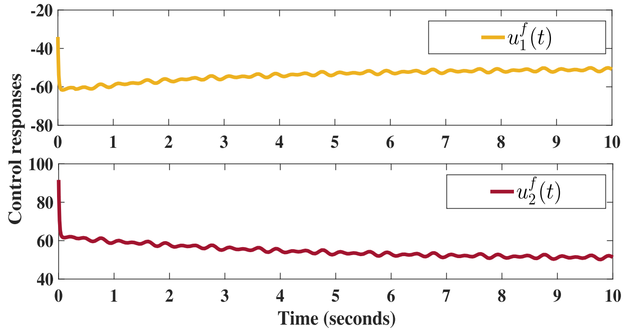


Fig. 3. Control responses.

dissipative controller is provided. From this figure, we can conclude that the satisfactory results for the undertaken system are attained by using the method provided in this work.

Now, to distinguish the efficiency between the proposed control scheme and its deduced schemes for attenuating the norm-bounded disturbances which are analyzed by selecting the values of Q, S, R as specified in Remark 1. Based on the three sets of gain values given in Table 1, the simulations are carried out and as a result, the state responses of the considered TSFSS (1) under distinct disturbance attenuation performances, namely H_∞, passivity and mixed H_∞ and passivity, are presented in Fig. 7. Further, to show the relationship between the coefficient matrices of exogenous system (3) and the accuracy of disturbance rejection, the gain matrices corresponding to different parameters of $\mathcal{W}_\sigma$

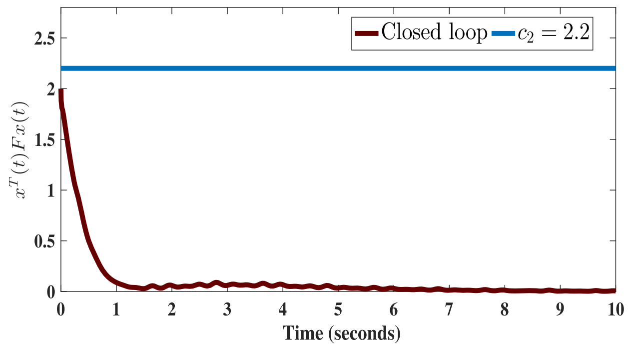


Fig. 4. Trajectory of $x^T(t)Fx(t)$.

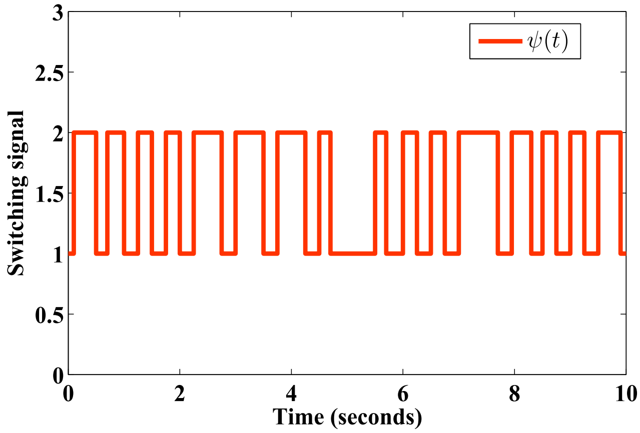


Fig. 5. Switching signal.

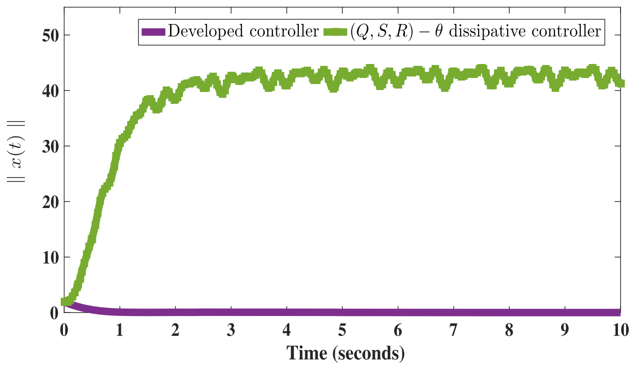


Fig. 6. Evolution of $||x(t)||$.

and $\mathcal{V}_\sigma$ (refer Table 2) are computed and given with Table 3. Based on the obtained gain matrices (refer Table 3), the disturbance estimation error is plotted in Fig. 8.

In this way, it is worthy to conclude that the results established in this work assure the simultaneous disturbance estimation and finite-time boundedness of the system under consideration in the existence of stochastic faults in actuators and multiple disturbances.

Example 2. Precisely, this example illustrates the practical applicability of the established result given in Corollary 1. In this context, the well-known mass-spring damping model is adopted from [45], whose dynamics are given as:

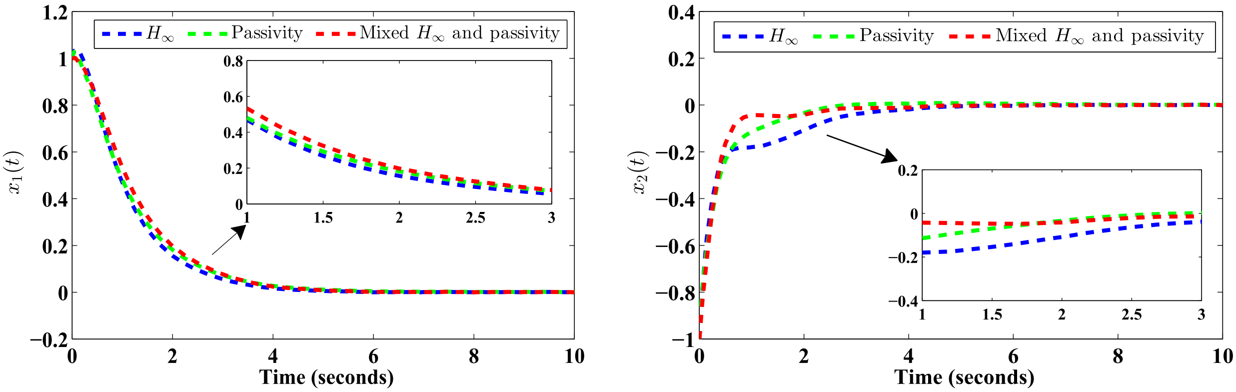


Fig. 7. State responses.

Table 2
Different values of $\mathcal{W}_\sigma$ and $\mathcal{V}_\sigma$ for distinct exogenous systems (3).

	$\mathcal{W}_1$	$\mathcal{W}_2$	$\mathcal{V}_1$	$\mathcal{V}_2$
Case 1	$\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$
Case 2	$\begin{bmatrix} -3 & 4.25 \\ -4.5 & 4 \end{bmatrix}$	$\begin{bmatrix} -3 & 2.5 \\ -2.8 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & 8 \\ -1 & 4 \end{bmatrix}$	$\begin{bmatrix} 2.6 & 1 \\ -3 & 5 \end{bmatrix}$
Case 3	$\begin{bmatrix} 1 & 5 \\ -5 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
Case 4	$\begin{bmatrix} 2 & 7 \\ -7 & 2 \end{bmatrix}$	$\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$	$\begin{bmatrix} 5 & -0.1 \\ 0 & 5 \end{bmatrix}$	$\begin{bmatrix} 6 & -0.2 \\ 0 & 6 \end{bmatrix}$

Table 3
Observer and controller gain matrices for different values W_σ and V_σ in Table 2.

	Case 1	Case 2	Case 3	Case 4
$\mathcal{K}_{11}$	$-\begin{bmatrix} 1.2506 & 0.4961 \\ 0.5325 & 0.4677 \end{bmatrix}$	$-\begin{bmatrix} 1.0439 & 0.2849 \\ 0.2887 & 0.9078 \end{bmatrix}$	$-\begin{bmatrix} 1.0382 & 0.2893 \\ 0.2936 & 0.8898 \end{bmatrix}$	$-\begin{bmatrix} 1.0468 & 0.2773 \\ 0.2807 & 0.9066 \end{bmatrix}$
$\mathcal{K}_{12}$	$-\begin{bmatrix} 1.0524 & 0.5443 \\ 0.6929 & 0.6332 \end{bmatrix}$	$-\begin{bmatrix} 0.9404 & 0.4634 \\ 0.4892 & 0.5676 \end{bmatrix}$	$-\begin{bmatrix} 0.9513 & 0.4632 \\ 0.4959 & 0.5492 \end{bmatrix}$	$-\begin{bmatrix} 0.9501 & 0.4587 \\ 0.4877 & 0.5618 \end{bmatrix}$
$\mathcal{K}_{21}$	$-\begin{bmatrix} 0.4629 & 0.4931 \\ 0.7051 & 0.6555 \end{bmatrix}$	$-\begin{bmatrix} 0.6294 & 0.5217 \\ 0.5299 & 0.9339 \end{bmatrix}$	$-\begin{bmatrix} 0.6209 & 0.5177 \\ 0.5264 & 0.9222 \end{bmatrix}$	$-\begin{bmatrix} 0.6269 & 0.5153 \\ 0.5230 & 0.9289 \end{bmatrix}$
$\mathcal{K}_{22}$	$-\begin{bmatrix} 0.1758 & 0.2955 \\ 0.7386 & 1.1232 \end{bmatrix}$	$-\begin{bmatrix} 0.3937 & 0.3978 \\ 0.4273 & 1.1222 \end{bmatrix}$	$-\begin{bmatrix} 0.3876 & 0.3896 \\ 0.4228 & 1.1098 \end{bmatrix}$	$-\begin{bmatrix} 0.3938 & 0.3912 \\ 0.4226 & 1.1206 \end{bmatrix}$
$\mathcal{L}_1$	$-\begin{bmatrix} 9.4007 & 12.7214 \\ 6.9777 & 12.7767 \end{bmatrix}$	$-\begin{bmatrix} 15.8852 & 31.8991 \\ 1.9394 & 43.5853 \end{bmatrix}$	$\begin{bmatrix} 5.3339 & -23.6550 \\ 32.1781 & -67.0403 \end{bmatrix}$	$\begin{bmatrix} 11.4260 & -47.8228 \\ 51.176 & -109.7251 \end{bmatrix}$
$\mathcal{L}_2$	$-\begin{bmatrix} 8.7273 & 17.4028 \\ 9.3504 & 22.8026 \end{bmatrix}$	$\begin{bmatrix} -2.6398 & -36.3609 \\ 15.1905 & -69.9072 \end{bmatrix}$	$\begin{bmatrix} -9.3269 & -16.1178 \\ 14.8586 & -46.8133 \end{bmatrix}$	$\begin{bmatrix} -5.1744 & -40.9465 \\ 32.2876 & -89.4340 \end{bmatrix}$

$$m\ddot{x} + F_f + F_s = u(t),$$

where m denote spring's mass, F_f and F_s represents spring's friction force and restoring force with nonlinear or uncertain terms, respectively and $u(t)$ represents the control input vector. Prompted by [45], the above system is equivalently re-drafted in the form of (39) as a 2-rule TSFSS associated with stochastic actuator faults, input delay and multiple disturbances. Then, the related system matrices with two switching modes borrowed from [45] are given as follows:

$$\mathcal{A}_{11} = \begin{bmatrix} 0 & 1 \\ -0.02 & 0 \end{bmatrix}, \mathcal{A}_{12} = \begin{bmatrix} 0 & 1 \\ -0.02 & -0.225 \end{bmatrix}, \mathcal{A}_{21} = \begin{bmatrix} 0 & 1 \\ -1.5275 & 0 \end{bmatrix}, \mathcal{A}_{22} = \begin{bmatrix} 0 & 1 \\ -1.5275 & -0.225 \end{bmatrix},$$

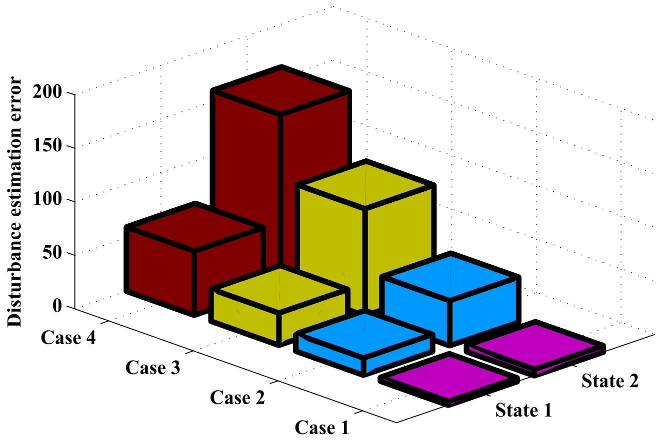


Fig. 8. Disturbance estimation error for the distinct cases given in Table 2.

Table 4
Gain matrices with its corresponding index γ for distinct disturbance attenuation performances.

	$(Q, S, R)-\theta$ dissipativity	H_∞	$Passivity$	$Mixed\ H_\infty\ and\ Passivity$
K_{11}	$-\begin{bmatrix}1.7433 & 2.1111\end{bmatrix}$	$-\begin{bmatrix}1.5457 & 2.0163\end{bmatrix}$	$-\begin{bmatrix}1.5548 & 2.0107\end{bmatrix}$	$-\begin{bmatrix}1.5501 & 2.0055\end{bmatrix}$
K_{12}	$-\begin{bmatrix}1.9042 & 2.1038\end{bmatrix}$	$-\begin{bmatrix}1.5704 & 1.9755\end{bmatrix}$	$-\begin{bmatrix}1.5887 & 1.9741\end{bmatrix}$	$-\begin{bmatrix}1.5841 & 1.9673\end{bmatrix}$
K_{21}	$\begin{bmatrix}0.4202 & -1.8332\end{bmatrix}$	$\begin{bmatrix}0.5458 & -1.7832\end{bmatrix}$	$\begin{bmatrix}0.5715 & -1.7842\end{bmatrix}$	$\begin{bmatrix}0.5691 & -1.7776\end{bmatrix}$
K_{22}	$\begin{bmatrix}0.1357 & -1.8467\end{bmatrix}$	$\begin{bmatrix}0.5336 & -1.7504\end{bmatrix}$	$\begin{bmatrix}0.5432 & -1.7605\end{bmatrix}$	$-\begin{bmatrix}0.5409 & -1.7530\end{bmatrix}$
L_1	$\begin{bmatrix}6.4863 & -14.1735\\ -4.1048 & -26.0232\end{bmatrix}$	$-\begin{bmatrix}1.1528 & 3.9306\\ 1.8218 & 7.9508\end{bmatrix}$	$-\begin{bmatrix}1.0169 & 5.5050\\ 2.3066 & 10.9606\end{bmatrix}$	$\begin{bmatrix}1.1166 & -6.2000\\ -2.3447 & -11.4913\end{bmatrix}$
L_2	$-\begin{bmatrix}0.1096 & 11.5006\\ 3.9423 & 22.2345\end{bmatrix}$	$\begin{bmatrix}0.8899 & -3.8021\\ -1.6010 & -7.0122\end{bmatrix}$	$\begin{bmatrix}0.6633 & -5.1808\\ -2.0370 & -9.7269\end{bmatrix}$	$-\begin{bmatrix}0.6759 & -5.4608\\ -2.0914 & -10.2343\end{bmatrix}$
γ	0.4	0.82	0.6	0.87

$$\mathcal{B}_{11} = \mathcal{B}_{12} = \mathcal{B}_{21} = \mathcal{B}_{22} = \begin{bmatrix}0\\1\end{bmatrix},$$

The membership functions are selected as $\hbar_{11}(x_1(t)) = 1 - \frac{x_1^2(t)}{2.25}$; $\hbar_{21}(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25}$; $\hbar_{12}(x_1(t)) = \frac{x_1^2(t)}{2.25}$ and $\hbar_{22}(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25}$.

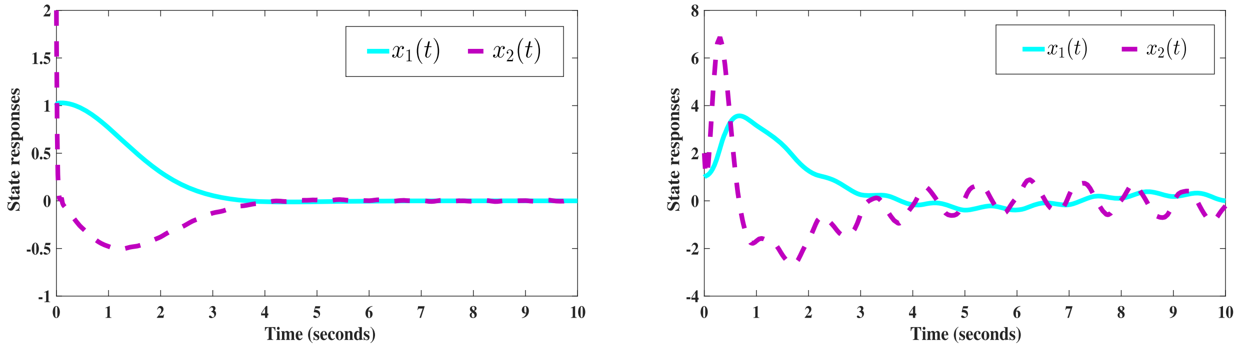
Further, the external disturbances and its corresponding coefficient matrices are selected as $w(t) = 0.001\,tant$, $\eta(t) = 0.4\,cos(3\pi t) + 0.7\,sin(5\pi t)$. $\mathcal{B}_{w11} = \mathcal{B}_{w12} = \mathcal{B}_{w21} = \mathcal{B}_{w22} = \begin{bmatrix}0\\1\end{bmatrix}$, $\mathcal{C}_{11} = \begin{bmatrix}0.3 & 0.1\end{bmatrix}$, $\mathcal{C}_{12} = \begin{bmatrix}0.3 & 0.4\end{bmatrix}$, $\mathcal{C}_{21} = \begin{bmatrix}0.3 & 0.1\end{bmatrix}$, $\mathcal{C}_{22} = \begin{bmatrix}0.3 & 0.4\end{bmatrix}$, $\mathcal{C}_{w11} = 0.2$, $\mathcal{C}_{w12} = 0.3$, $\mathcal{C}_{w21} = 0.2$, $\mathcal{C}_{w22} = 0.3$.

In addition to that, the coefficient matrices of the exogenous system (3) with two switching modes are considered as follows:

$$\mathcal{W}_1 = \begin{bmatrix}1 & 12\\ -12 & 1\end{bmatrix}, \mathcal{H}_1 = \begin{bmatrix}0.01\\0.02\end{bmatrix}, \mathcal{V}_1 = \begin{bmatrix}1 & 2\end{bmatrix};$$
$$\mathcal{W}_2 = \begin{bmatrix}1 & 13\\ -13 & 1\end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix}0.01\\0.02\end{bmatrix}, \mathcal{V}_2 = \begin{bmatrix}1 & 2\end{bmatrix}.$$

In order to solve the obtained sufficient stability conditions (40)-(43), the rest of the values are selected as $h = 0.1$, $c_1 = 0.2$, $c_2 = 5.2$, $w_f = 0.4$, $T_f = 10$, $\alpha = 0.1$, $F = I$, $Q = -1.2$, $S = 0.8$, $R = 1.8I$ and $\theta = 0.4$. The assumption regarding the stochastic faults in actuators is same as in previous example along with $k = 1$ and $\varrho = 0.8$. Then, by solving the conditions in Corollary 1, the gain matrices are computed and displayed in Table 4.

In addition to the above, the average dwell time is obtained as $\zeta_a^* = 0.4961$. The outcomes are depicted in Figs. 9-16 under these calculated controller and observer gain matrices together with the same initial conditions considered in the



(a) with $\hat{d}(t)$

(b) without $\hat{d}(t)$

Fig. 9. Evolution of state trajectories with and without $\hat{d}(t)$.

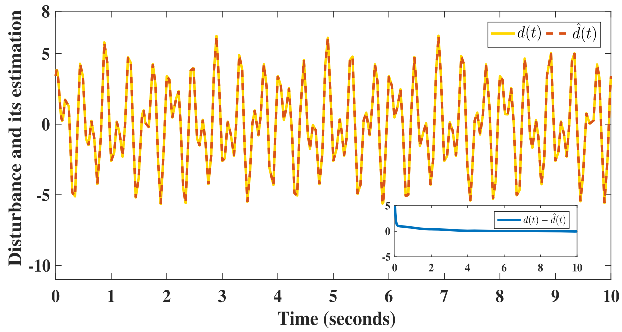


Fig. 10. Disturbance estimation and its error.

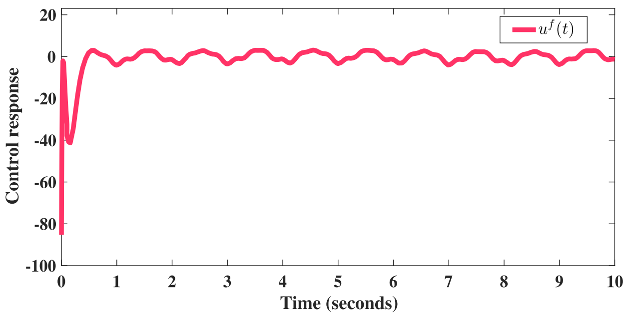


Fig. 11. Control responses.

previous example. Precisely, Fig. 9 presents state responses of (39) in the presence and absence of the term $\hat{d}(t)$ in the developed controller (5). From this figure, the efficiency and importance of the proposed composite anti-disturbance observer-based control scheme is revealed. In Fig. 10, the response of the disturbance $d(t)$ and its estimation is presented, wherein the exact estimation is accomplished. Fig. 11 signifies the control input response. Further, the evolution of all the system states converge to zero within the optimal bound values c_1 and c_2, which is clearly revealed from Fig. 12. The corresponding switching mode and norm-bounded disturbance signal of the considered system are respectively doodled in Fig. 13 and 14. Therefore, it is concluded that the considered mass-spring damper system represented by (39) is stochastically finite-time bounded. In addition to that, similar to the prior example, Fig. 15 is drawn to demonstrate the supremacy of the proposed control protocol with the traditional $(Q, S, R) - \theta$ dissipative method, where the satisfactory performance is accomplished via the developed control law. Besides, the response of state trajectories $x_1(t)$ and $x_2(t)$ under H_∞, passivity, mixed H_∞ and passivity performances are plotted in Fig. 16.

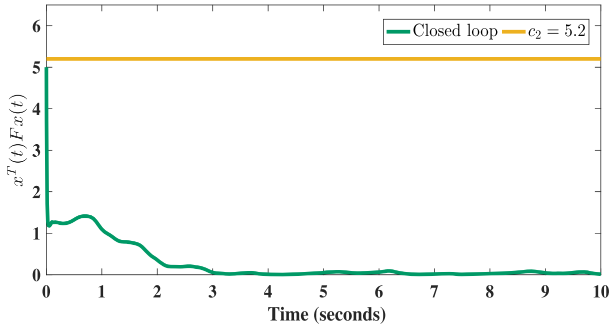


Fig. 12. Evolution of $x^T(t)Fx(t)$.

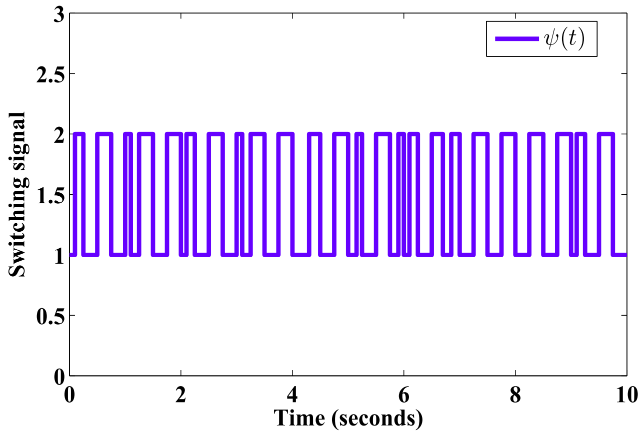


Fig. 13. Switching signal.

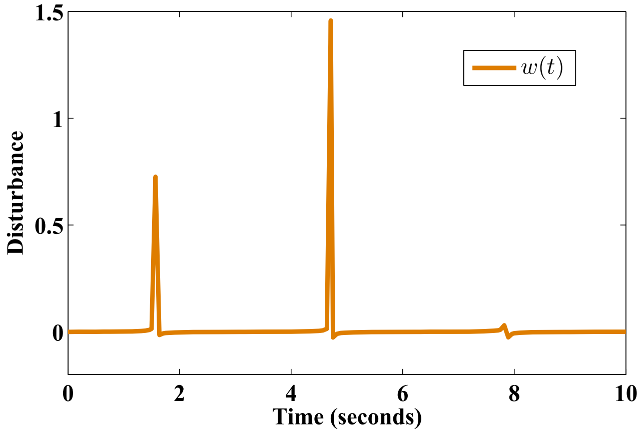


Fig. 14. Disturbance.

As a next step, we'll verify the proposed method with the existing results as in [46]. It is seen from Fig. 17 that, in comparison to the methodology in [46], the response of $\| x(t) \|$ using the developed control technique quickly converges to zero. Moreover, from Table 5, it is clear that the minimum disturbance attenuation level computed in this article is comparatively is lesser than that of in [46]. Overall, these simulation results reveal the practical significance of the proposed control law and also its robustness against the traditional disturbance attenuation method.

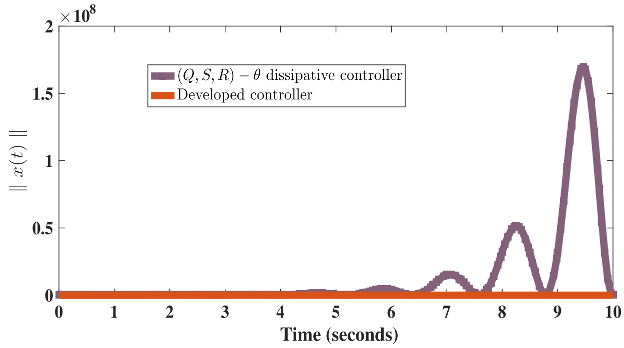


Fig. 15. Response of $||x(t)||$.

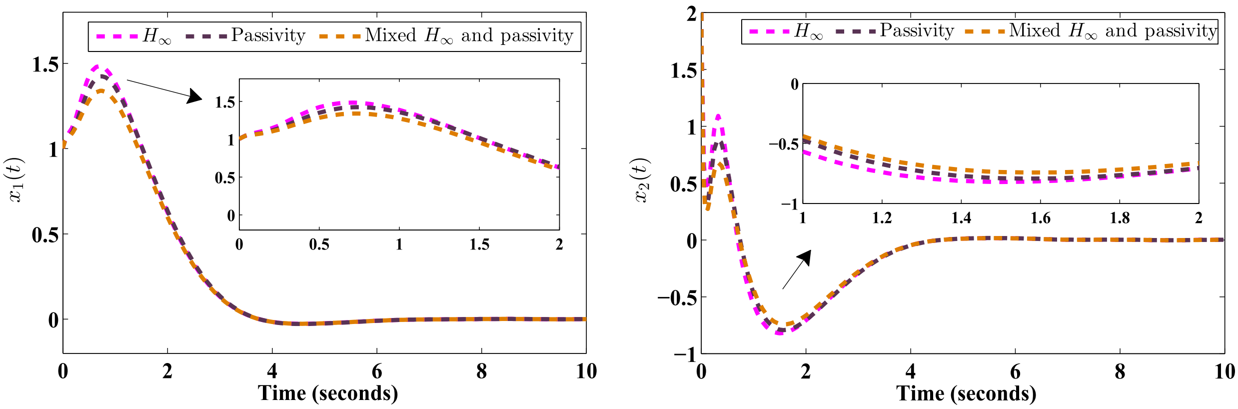


Fig. 16. State responses.

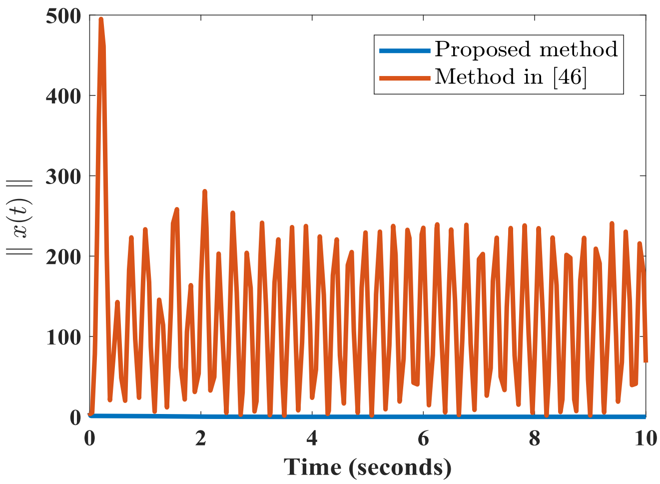


Fig. 17. Evaluation of $\| x(t) \|$.

As a result of the above study, we can conclude that the considered mass-spring damper system in the form of (39) is stabilized by virtue of the designed composite anti-disturbance controller (5).

Table 5
Minimum disturbance attenuation level.

Methods	γ_{min}	Improvement
Reference [46]	0.67	40.29%
Corollary 1	0.4	

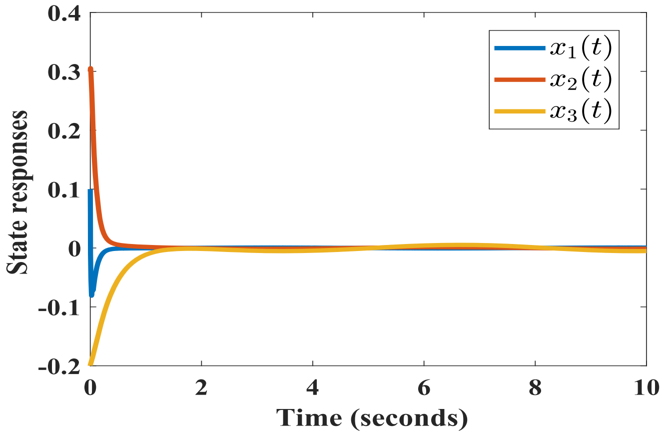


Fig. 18. Evolution of state trajectories with $d(t)$.

Example 3. In order to demonstrate the superiority of the proposed control scheme in comparison to the existing literature, a comparative analysis is presented in this example. In particular, we take into consideration Lorenz system given by the following equations [47]:

$$\begin{aligned}
\dot{x}_1(t) &= -px_1(t) + px_2(t) + u(t) \\
\dot{x}_2(t) &= rx_1(t) - x_2(t) - x_1(t)x_3(t) \\
\dot{x}_3(t) &= x_1(t)x_2(t) - qx_3(t)
\end{aligned} \tag{44}$$

where $p = 10$, $q = 8/3$, $r = 28$ and $v = 25$.

Having one switched system and two fuzzy rules, following is the matrix representation of the system (39) with regard to the above system:

$$\mathcal{A}_1 = \begin{bmatrix} -p & p & 0 \\ r & -1 & v \\ 0 & v & -q \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} -p & p & 0 \\ r & -1 & v \\ 0 & -v & -q \end{bmatrix}, \mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Moreover, the membership functions are chosen as $\hbar_1(x_1(t)) = \frac{1}{2}(1 + \frac{x_1(t)}{2})$; $\hbar_2(x_1(t)) = 1 - \hbar_1(x_1(t))$. Now, the exogenous system matrices are given by

$$\mathcal{W} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}, \mathcal{V} = \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}; \mathcal{H} = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}.$$

The remaining matrices are chosen as

$$\mathcal{C}_1 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}, \mathcal{C}_2 = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.1 \end{bmatrix}, \mathcal{B}_{w1} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \end{bmatrix}^T, \mathcal{B}_{w2} = \begin{bmatrix} 0.1 & 0.2 & 0.2 \end{bmatrix}^T.$$

The rest of parameter values are selected as same in Example 2. Then, we solve the LMIs in Corollary 1 to obtain the desired gain matrices:

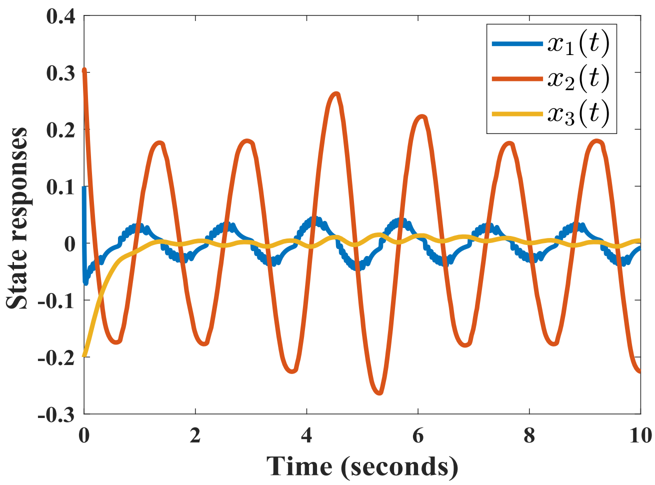


Fig. 19. Evolution of state trajectories without $\hat{d}(t)$.

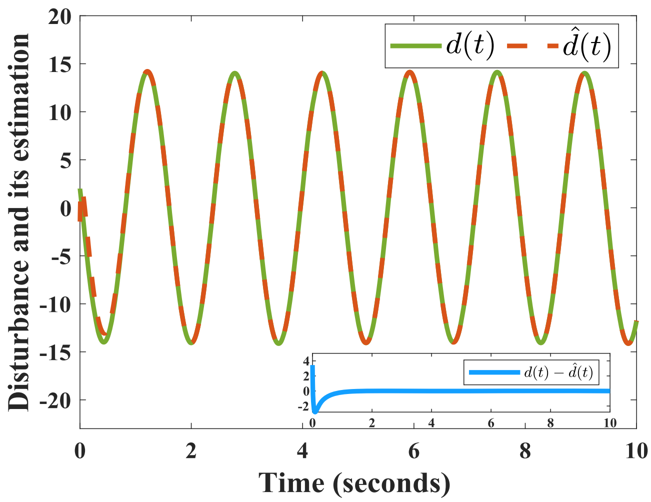


Fig. 20. Disturbance and its estimation with error.

$K_1 = \begin{bmatrix} -161.1512 & -64.0852 & -0.5785 \end{bmatrix}$, $K_2 = \begin{bmatrix} -161.8399 & -63.7862 & -11.4287 \end{bmatrix}$ and

$$L = \begin{bmatrix} -12.6407 & 8.2961 & -1.5277 \\ -23.3395 & 15.2994 & -2.7372 \end{bmatrix}.$$

For the purpose of simulation, the disturbances are picked by $w(t) = 0.1 * \sin(t)$ and $\eta(t) = \sin(t) + \cos(t)$. Further, the initial conditions are $x(0) = \begin{bmatrix} 0.1 & 0.3 & -0.2 \end{bmatrix}$. To be specific, Fig. 18 depicts the system (44)'s state response, which illustrates that the underlying system is stabilised by means of the designed controller. Moreover, the considered system's state outcomes without the estimated term $\hat{d}(t)$ in the developed controller are shown in Fig. 19. As seen in these figures, the efficacy of the developed control scheme is revealed. Further, the disturbance $d(t)$ and its estimation $\hat{d}(t)$ with the disturbance estimation error is plotted in Fig. 20. In addition to this, the proposed control technique is compared with that of the existing method in [47]. Precisely, the comparison plot for the response of $\| x(t) \|$ of the developed controller and the existing controller [47] is plotted in Fig. 21. From this graph, we infer that the approach presented in this paper yields satisfactory results for the considered system.

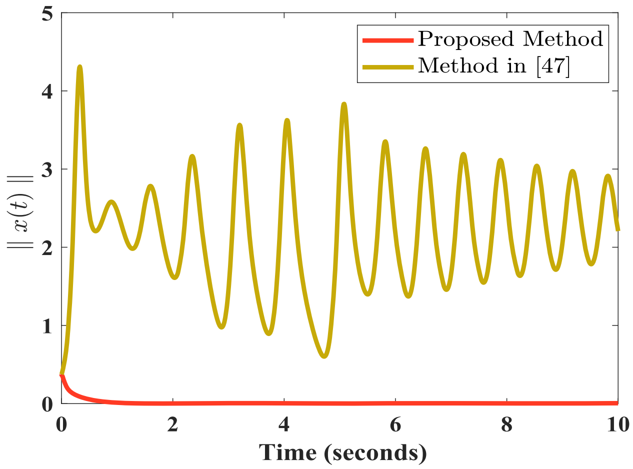


Fig. 21. Response of $\| x(t) \|$.

5. Conclusion

This study focused on solving the disturbance rejection based stabilization problem for TSFSS in conjunction with multiple disturbances, state/input delays and stochastic actuator faults. Specifically, to account the stochastic behavior of actuator faults, the random variables that obey Bernoulli distribution are examined. The composite disturbance observer approach based $(Q, S, R) - \theta$ dissipative performance is designed to achieve both the disturbance rejection and attenuation, respectively. To be precise, a unified control protocol is proposed for obtaining the required result by combining the output of anti-disturbance observer and parallel distributed compensation based reliable controller with input delay. Based on these settings, the stochastic finite-time boundedness of the states of the system (1) is confirmed by the virtue of Lyapunov stability theory. Further, numerical examples with simulation results are given to verify the efficiency of the proposed control design. Specifically, mass-spring-damper model is considered to authenticate the applicability of the theoretical outcomes. Further, the anti-disturbance control problem for stochastic switched IT2 fuzzy systems with multiple disturbances, time-varying delays and actuator saturation is an unexplored work, that will be studied as our future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

References

[1] Y. Jin, Y. Zhang, Y. Jing, J. Fu, An average dwell-time method for fault-tolerant control of switched time-delay systems and its application, IEEE Trans. Ind. Electron. 66 (Jun. 2018) 3139–3147.
[2] W. Xiang, J. Lam, J. Shen, Stability analysis and $L1$-gain characterization for switched positive systems under dwell-time constraint, Automatica 85 (Nov. 2017) 1–8.
[3] Z. Guangdeng, D. Yang, J. Lam, X. Song, Fault-tolerant control of switched LPV systems: a bumpless transfer approach, IEEE/ASME Trans. Mechatron. 27 (Jul. 2021) 1436–1446.
[4] L. Liu, Q. Zhou, H. Liang, L. Wang, Stability and stabilization of nonlinear switched systems under average dwell time, Appl. Math. Comput. 298 (Apr. 2017) 77–94.
[5] S. Li, C.K. Ahn, Z. Xiang, Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching, IEEE Trans. Fuzzy Syst. 27 (Nov. 2018) 200–205.

[6] X. Zhao, P. Shi, X. Zheng, L. Zhang, Adaptive tracking control for switched stochastic nonlinear systems with unknown actuator dead-zone, Automatica 60 (Oct. 2015) 193–200.

[7] C. Ge, Y. Shi, J.H. Park, C. Hua, Robust H_∞ stabilization for TS fuzzy systems with time-varying delays and memory sampled-data control, Appl. Math. Comput. 346 (Apr. 2019) 500–512.

[8] Y. Wei, J. Qiu, X. Peng, H.K. Lam, TS fuzzy-affine-model-based reliable output feedback control of nonlinear systems with actuator faults, Circuits Syst. Signal Process. 37 (Jan. 2018) 81–97.

[9] M. Wang, J. Qiu, M. Chadli, M. Wang, A switched system approach to exponential stabilization of sampled-data TS fuzzy systems with packet dropouts, IEEE Trans. Cybern. 46 (Nov. 2015) 3145–3156.

[10] Y. Wang, H. Shen, H.R. Karimi, D. Duan, Dissipativity-based fuzzy integral sliding mode control of continuous-time TS fuzzy systems, IEEE Trans. Fuzzy Syst. 26 (Jun. 2017) 1164–1176.

[11] L. Wang, H.K. Lam, Further study on observer design for continuous-time Takagi-Sugeno fuzzy model with unknown premise variables via average dwell time, IEEE Trans. Cybern. 50 (Aug. 2019) 4855–4860.

[12] X. Zhao, Y. Yin, L. Zhang, H. Yang, Control of switched nonlinear systems via TS fuzzy modeling, IEEE Trans. Fuzzy Syst. 24 (Jun. 2015) 235–241.

[13] H. Shen, M. Xing, Z.G. Wu, S. Xu, J. Cao, Multiobjective fault-tolerant control for fuzzy switched systems with persistent dwell time and its application in electric circuits, IEEE Trans. Fuzzy Syst. 28 (Aug. 2019) 2335–2347.

[14] J. Han, H. Zhang, Y. Wang, X. Sun, Robust fault detection for switched fuzzy systems with unknown input, IEEE Trans. Cybern. 48 (Oct. 2017) 3056–3066.

[15] X. Su, L. Wu, P. Shi, C.P. Chen, Model approximation for fuzzy switched systems with stochastic perturbation, IEEE Trans. Fuzzy Syst. 23 (Oct. 2014) 1458–1473.

[16] H. Sang, J. Zhao, Passivity and passification for switched T-S fuzzy systems with sampled-data implementation, IEEE Trans. Fuzzy Syst. 28 (Jul. 2019) 1219–1229.

[17] S. Kuppusamy, Y.H. Joo, Nonfragile retarded sampled-data switched control of T-S fuzzy systems and its applications, IEEE Trans. Fuzzy Syst. 28 (Sep. 2019) 2523–2532.

[18] J. Yang, W.H. Chen, S. Li, L. Guo, Y. Yan, Disturbance/uncertainty estimation and attenuation techniques in PMSM drives-a survey, IEEE Trans. Ind. Electron. 64 (Jun. 2017) 3273–3285.

[19] W.H. Chen, K. Ding, X. Lu, Disturbance-observer-based control design for a class of uncertain systems with intermittent measurement, J. Franklin Inst. 354 (Sep. 2017) 5266–5279.

[20] J. Zhang, X. Liu, Y. Xia, Z. Zuo, Y. Wang, Disturbance observer-based integral sliding-mode control for systems with mismatched disturbances, IEEE Trans. Ind. Electron. 63 (Jun. 2016) 7040–7048.

[21] X. Wei, L. Dong, H. Zhang, J. Han, X. Hu, Composite anti-disturbance control for stochastic systems with multiple heterogeneous disturbances and input saturation, ISA Trans. 100 (May. 2020) 436–445.

[22] D. Yang, J. Zhao, Composite anti-disturbance control for switched systems via mixed state-dependent and time-driven switching, IET Control Theory Appl. 10 (Jun. 2016) 1981–1990.

[23] X. Yao, J.H. Park, L. Wu, L. Guo, Disturbance-observer-based composite hierarchical antidisturbance control for singular Markovian jump systems, IEEE Trans. Autom. Control 64 (Aug. 2018) 2875–2882.

[24] H. Zhang, X. Wei, H.R. Karimi, J. Han, Anti-disturbance control based on disturbance observer for nonlinear systems with bounded disturbances, J. Franklin Inst. 355 (Aug. 2018) 4916–4930.

[25] G. Zong, Y. Li, H. Sun, Composite anti-disturbance resilient control for Markovian jump nonlinear systems with general uncertain transition rate, Sci. China Inf. Sci. 62 (Feb. 2019) 1–18.

[26] H. Shen, Z. Wang, X. Huang, J. Wang, Fuzzy dissipative control for nonlinear Markovian jump systems via retarded feedback, J. Franklin Inst. 351 (Jul. 2014) 3797–3817.

[27] H.D. Choi, C.K. Ahn, P. Shi, L. Wu, M.T. Lim, Dynamic output-feedback dissipative control for TS fuzzy systems with time-varying input delay and output constraints, IEEE Trans. Fuzzy Syst. 25 (May. 2016) 511–526.

[28] J. Zhang, D. Liu, Y. Ma, Finite-time dissipative control of uncertain singular TS fuzzy time-varying delay systems subject to actuator saturation, Comput. Appl. Math. 39 (Sep. 2020) 1–22.

[29] S. Li, C.K. Ahn, Z. Xiang, Adaptive fuzzy control of switched nonlinear time-varying delay systems with prescribed performance and unmodeled dynamics, Fuzzy Sets Syst. 371 (Sep. 2019) 40–60.

[30] S. Wang, J. Feng, H. Zhang, Robust fault-tolerant control for a class of networked control systems with state delay and stochastic actuator failures, Int. J. Adapt. Control Signal Process. 28 (Sep. 2014) 798–811.

[31] O.M. Kwon, M.J. Park, J.H. Park, S.M. Lee, Stability and stabilization of TS fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals, Inf. Sci. 372 (Dec. 2016) 1–15.

[32] L. Wang, H.K. Lam, New stability criterion for continuous-time Takagi–Sugeno fuzzy systems with time-varying delay, IEEE Trans. Cybern. 49 (Feb. 2018) 1551–1556.

[33] Q. Zhang, J. Dong, Disturbance-observer-based adaptive fuzzy control for nonlinear state constrained systems with input saturation and input delay, Fuzzy Sets Syst. 392 (Aug. 2020) 77–92.

[34] J. Na, Y. Huang, X. Wu, S.F. Su, G. Li, Adaptive finite-time fuzzy control of nonlinear active suspension systems with input delay, IEEE Trans. Cybern. 50 (Feb. 2019) 2639–2650.

[35] Y. Wang, P. Shi, H. Yan, Reliable control of fuzzy singularly perturbed systems and its application to electronic circuits, IEEE Trans. Circuits Syst. I, Regul. Pap. 65 (Jun. 2018) 3519–3528.

[36] H. Shen, M. Xing, Z.G. Wu, J.H. Park, Fault-tolerant control for fuzzy switched singular systems with persistent dwell-time subject to actuator fault, Fuzzy Sets Syst. 392 (Aug. 2020) 60–76.

[37] J. Luo, X. Liu, W. Tian, S. Zhong, K. Shi, M. Li, Finite-time H_∞ reliable control for T-S fuzzy systems with variable sampling, Phys. A, Stat. Mech. Appl. 538 (Jan. 2020) 122697.

[38] H. Yan, J. Wang, F. Wang, Z. Wang, S. Zou, Observer-based reliable passive control for uncertain T-S fuzzy systems with time-delay, Int. J. Syst. Sci. 50 (Apr. 2019) 905–918.

[39] S. Harshavarthini, R. Sakthivel, C.K. Ahn, Finite-time reliable attitude tracking control design for nonlinear quadrotor model with actuator faults, Nonlinear Dyn. 96 (Jun. 2019) 2681–2692.

[40] S. Hu, D. Yue, Z. Du, J. Liu, Reliable H_∞ non-uniform sampling tracking control for continuous-time non-linear systems with stochastic actuator faults, IET Control Theory Appl. 6 (Jan. 2012) 120–129.

[41] H. Liu, Y. Shen, X. Zhao, Delay-dependent observer-based H_∞ finite-time control for switched systems with time-varying delay, Nonlinear Anal. Hybrid Syst. 6 (Aug. 2012) 885–898.

[42] H. Liu, X. Zhao, Finite-time H_∞ control of switched systems with mode-dependent average dwell time, J. Franklin Inst. 351 (Mar. 2014) 1301–1315.

[43] J. Cheng, D. Zhang, W. Qi, J. Cao, K. Shi, Finite-time stabilization of TS fuzzy semi-Markov switching systems: a coupling memory sampled-data control approach, J. Franklin Inst. 357 (Nov. 2020) 11265–11280.

[44] G. Zong, H. Ren, H.R. Karimi, Event-triggered communication and annular finite-time H_∞ filtering for networked switched systems, IEEE Trans. Cybern. 51 (Aug. 2020) 309–317.

[45] W. Yang, S. Tong, Adaptive output feedback fault-tolerant control of switched fuzzy systems, Inf. Sci. 329 (Feb. 2016) 478–490.

[46] D. Zhai, A.Y. Lu, J. Dong, Q.L. Zhang, Dynamic output feedback H_∞ control for switched T-S fuzzy systems via discretized Lyapunov function technique, Neurocomputing 177 (Feb. 2016) 651–669.

[47] P. Tang, Y. Ma, Exponential stabilization and non-fragile sampled-date dissipative control for uncertain time-varying delay TS fuzzy systems with state quantization, Inf. Sci. 4 (Feb. 2021) 513–536.