

Research Article

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Aspects of thermal radiation for the second law analysis of magnetized Darcy–Forchheimer movement of Maxwell nanomaterials with Arrhenius energy effects

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Abstract: The present mathematical model investigates convective heat transfer in the Darcy–Forchheimer flow of a magneto-Maxwell nanofluid over a porous stretched sheet. The impacts of convective boundary conditions, activation energy, Joule heating, thermal radiation, and energy production are also considered in the flow model. Additionally, the heat transmission process is described using the Cattaneo–Christov heat flow model. A system of highly non-linear ordinary differential equations is built using suitable similarity transformations. The model has been

numerically tackled utilizing MATLAB's `bvp4c` package. Computational findings are calculated for liquid velocity, liquid temperature, nanoparticle concentration, and total entropy creation as functions of transverse displacement for investigating the velocity, mass, and thermal properties of the Maxwell nanoliquid under suitable assumptions and boundary conditions. Oscillations in friction factor, mass, and heat transfer rates have also been examined. It is observed that the velocity distribution improves as the thermal and concentration Grashof numbers increase, whereas it decreases as the suction parameter and Darcy–Forchheimer parameter increase. It is further reported that the concentration upsurges with larger estimates of activation energy in the central component of the geometry. This research has practical applications in escalating thermal transfer systems employed in industrial procedures, freezing technologies, and energy systems. It supports heightening thermal management in reactors, geothermal structures, and polymer processing.

Keywords: numerical simulation, Maxwell nanofluid, Darcy–Forchheimer, chemical reaction, Arrhenius energy, entropy generation

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Nomenclature

Letters

p	stretching rate constant
B_0	uniform magnetic field ($\text{kg} \cdot \text{S}^{-2} \cdot \text{A}^{-1}$)
E	activation energy
Ec	Eckert number
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
Fr	Darcy–Forchheimer number
Gr	thermal Grashof number
Gc	concentration Grashof number

K_r	rate of reaction constant
K	porosity parameter
M	magnetic field
N_b	Brownian motion
N_t	thermophoresis parameter
Nu	rate of heat transfer
Pr	Prandtl number
S	suction parameter
Bi	Biot number
Sc	Schmidt number
Sh	rate of mass transfer
T	temperature of the fluid (K)
T_w	temperature near the sheet (K)
T_∞	free stream temperature (K)
u, v	velocity components in x and y directions ($m \cdot s^{-1}$)
x, y	Cartesian coordinates (m)

Greek symbols

α	thermal diffusivity coefficient ($m^2 \cdot s^{-1}$)
β	Maxwell fluid parameter
γ	chemical reaction parameter
δ	thermal relaxation parameter
λ_1	relaxation time of the heat flux
η	similarity variable
ρ	fluid density ($kg \cdot m^{-3}$)
ρC_p	fluid thermal capacity ($J \cdot m^{-3} \cdot K^{-1}$)
ν	kinematic viscosity ($m^2 \cdot s^{-1}$)
μ	dynamic viscosity ($kg \cdot ms^{-1}$)

1 Introduction

Non-Newtonian fluids exhibit variable viscosity under stress, which has significant applications across various sectors, including biological fluid motion, food processing, rock crystal growth, microelectronic microchips, crystal fiber production, and electronic generators. The Maxwell framework is among the most significant models studied for these fluids. The Maxwell fluid is a viscoelastic material model that consists of a dashpot (a viscous component) and a spring (an elastic component) connected in series. Because it can deform like a solid and flow like a fluid, it is perfect for simulating time-dependent behavior and stress relaxation. The Maxwell fluid is frequently employed in geophysical sciences, biological mechanics, and polymer science to explain challenging material reactions to applied stress. Sara *et al.* [1] analyzed the mass and energy transport in Maxwell fluid over an

expanding porous sheet. Amna *et al.* [2] investigated the Darcy–Forchheimer flow of non-Newtonian fluids through a permeable medium. Khan *et al.* [3] modeled the MHD irregular radiating Maxwell nanofluid in porous media, while Jawad *et al.* [4] explored the Darcy–Forchheimer law and hydromagnetic Maxwell fluid transport involving chemical reactions. Other researchers [5–8] have discussed non-Newtonian fluids in various contexts, further highlighting their broad implications.

Nanofluids are fluids containing nanoparticles, with particle size on a nanometer scale. These fluids are created by evenly dispersing nanoparticles in a host fluid to maintain a stable suspension, aiming to enhance thermal properties even at low concentrations. Nanofluids exhibit exceptional characteristics, making them vital to heat transfer applications. They are engineered colloidal suspensions composed of nanomaterials such as oxides, metals, carbides, or carbon nanotubes within a base liquid. Common base fluids include water, ethylene glycol, and oil. The nanoscale particles used in these fluids are typically oxides, metals, or carbides. Due to their unique properties, nanofluids are crucial in various fields, including environmental, chemical, nuclear, industrial, electrical, and medical sciences, contributing to advancements in heat management and efficiency across these domains. Buongiorno [9] explored nanoparticle mobility, including thermophoresis and Brownian movement effects. Also, it provided essential applications and models for nanomaterials. Kumar *et al.* [10] addressed the 2D hydromagnetic fluid of nanomaterial flow toward a stretched sheet with a chemical reaction. Mahabaleshwar *et al.* [11] examined the impacts of heat radiation on the flow of a Boussinesq couple stress nanofluid via a porous nonlinear stretching sheet. Saidulu and Sreeram Reddy [12] investigated the impact of dissipative heat transfer on the movement of a micropolar nanofluid across an exponentially stretched sheet. Saleem *et al.* [13] examined the influence of radiative energy transfer on hybrid nanomaterials across a sheet with viscous dissipation. Salahuddin *et al.* [14] considered the implications of activation energy for the stagnation point movement of radiative energy and heat generation with Darcy–Forchheimer. Gurejala *et al.* [15] investigated the magnetized Maxwell nanomaterial toward a permeable stretched sheet with a thermal source and a convective boundary condition. Mahabaleshwar *et al.* [16] explored the influence of radiation on the laminar movement of a dusty ternary nanofluid via a porous stretching/shrinking sheet incorporating mass transpiration.

Activation energy is the minimum energy required by a reactive molecule to transform into a product. It represents the lowest energy needed to activate atoms or

molecules for a chemical reaction or change. The concept was introduced in 1889 by the Swedish physicist Svante Arrhenius, who coined the term “activation energy.” A key application of this concept lies in hyperthermia treatment, an essential component of cancer chemotherapy, where activation energy principles aid in the generation and enhancement of therapeutic effects. It is also utilized in dose measurement, which tracks the temperature range at which tumor cells are most susceptible to dying from warmth-induced death of cells during treatment with hyperthermia and counts the total quantity of thermal energy that the tumor cell has received. The activation energy of Arrhenius is most often used cumulatively in the development of medicine, stability assessment, and formulation. It probably has a significant impact on maintaining food quality. Ahmed et al. [17] explored the consequences of activation energy to study slanted upward plates in chemically reacting the Maxwell fluid flow. Bafe et al. [18] addressed the thermal bioconvection of 3D rotational movement of Williamson nanomaterials with activation energy in a Darcy–Forchheimer flow along a sheet. A study by Ali et al. [19] examined the Darcy–Forchheimer nanofluid circulation over a vertical plate that included a permeable medium and radiation. The Darcy–Forchheimer motion of Maxwell tiny fluids across a permeable stretched surface with buoyancy forces and Arrhenius energy was examined by Jawad et al. [20]. The passage of Prandtl nanofluids and heat radiation via a porous extended surface contained in a Darcy–Forchheimer media was examined by Zafar et al. [21]. Li et al. [22] discussed the impact of Darcy law on a micropolar fluid flow across a channel.

Ghazwani et al. [23] studied the energy and fluid motion within the framework of non-radiative conditions and Cattaneo–Christov energy flux across a slanted extended cylinder. Yasir et al. [24] explored the influence of mass transfer on Maxwell fluid flow along a stretched cylinder using the innovative Cattaneo–Christov thermal transport model. Jabeen et al. [25] investigated mass and energy characteristics in Maxwell fluid motion, incorporating Cattaneo–Christov fluxes under linear stretching conditions. The Cattaneo–Christov model in magnetized Maxwell fluid across a permeable stretched surface with thermal generation was examined by Jawad and Nisar [26]. Awais et al. [27] addressed the entropy generation process in Darcy–Forchheimer flow, considering the effects of viscous dissipation, variable fluid properties, and activation energy. Khan et al. [28] analyzed the entropy in the hydrodynamic transport of non-Newtonian fluids, accounting for viscous dissipation and Arrhenius activation energy. Hamid and Khan [29] examined the magneto-Williamson nanofluid flow over a circular cylinder,

incorporating the effects of activation energy. Zeeshan et al. [30] focused on entropy generation in non-Newtonian fluid flow past a stretched cylinder. Sharma et al. [31] investigated entropy analysis in the electro-magnetohydrodynamic flow of Jeffrey fluid, including the effects of radiation and magnetohydrodynamics.

In light of the aforementioned significant research openings, the present effort investigates the impact of the convectional heat transfer effect on the Darcy–Forchheimer movement of a Maxwell nanofluid across a porous stretched surface in the presence of hydrodynamics. Important components, including heat radiation, energy creation, and viscous dissipation, are taken into consideration in the mathematical model. The study investigates the flow and thermal characteristics of Maxwell nanoliquid across a porous stretched sheet under the influence of relevant physical factors by applying numerical simulations using MATLAB’s *bvp4c* tool. The discoveries demonstrate that when the thermal and concentration Grashof numbers grow, the distribution of velocity gets stronger, whereas it decreases as the suction parameter and Darcy–Forchheimer factor increase. The study reports that the concentration increases with higher activation energy estimates in the core region of the geometry.

Novelty:

The novelty of this research lies in investigating Maxwell nanofluid movement over a porous stretched surface by integrating convectional thermal transport, Darcy–Forchheimer impacts, radiation, and viscous dissipation, offering an inclusive and accurate model of intricate thermal-fluid behavior. This investigation has practical applications in elevating thermal transfer systems used in industrial procedures, freezing technologies, and energy systems. It supports optimizing heat management in reactors, geothermal structures, and polymer processing.

2 Mathematical modeling

This work examines the entropy analysis in Darcy–Forchheimer motion of magneto-Maxwell nanomaterial over a porous stretching surface, considering the effects of radiative heat transfer, activation energy, Joule heating, energy generation, and convective boundary conditions. Further, the heat transmission process is described using the Cattaneo–Christov thermal flow concept. The movement of nanoparticle effects is witnessed with the help of Buongiorno’s model. In the analysis, the y -axis is considered to be perpendicular to the extended sheet, which has a velocity u along the x -axis. In Figure 1, the flow phenomena and the magnetic field intensity B_0 positioned perpendicular to it are depicted. In the

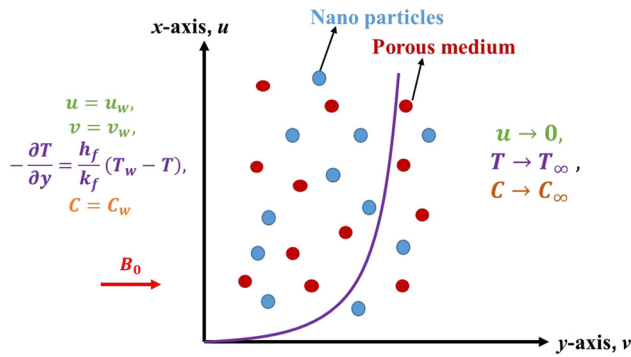


Figure 1: Pictorial demonstration of the flow model.

suggested Maxwell nanofluid flow model, the proper set of partial differential equations is written in the following manner [15,26]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & \vartheta \frac{\partial^2 u}{\partial y^2} - \lambda_1 \left[u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] \\ & - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu}{\rho k_1} u - \frac{c_b}{\sqrt{k_1}} u^2 \\ & + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty), \end{aligned} \quad (2)$$

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \left(\frac{k}{\rho C_p} \right) \frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu}{\rho C_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 \\ & + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \\ & - \lambda_2 \left[u \frac{\partial T}{\partial x} \frac{\partial u}{\partial x} + v \frac{\partial T}{\partial y} \frac{\partial v}{\partial y} + u \frac{\partial T}{\partial y} \frac{\partial v}{\partial x} \right. \\ & + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} + 2uv \frac{\partial^2 T}{\partial x \partial y} + u^2 \frac{\partial^2 T}{\partial x^2} \\ & + v^2 \frac{\partial^2 T}{\partial y^2} \left. \right] + \frac{1}{\rho C_p} \left[\mu_f \left(\frac{\partial u}{\partial y} \right)^2 \right. \\ & - \lambda_1 \left[v^2 \left(\frac{\partial u}{\partial y} \right)^2 + 2uv \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right] \\ & + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{\sigma B_0^2}{\rho C_p} u^2, \end{aligned} \quad (3)$$

$$\begin{aligned} u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = & D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \\ & - K_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n \exp \left(\frac{-E_a}{kT} \right). \end{aligned} \quad (4)$$

The considered boundary conditions are as follows:

$$\begin{aligned} u = u_w, \quad v = v_w, \quad C = C_w, \\ \frac{\partial T}{\partial y} = \frac{-h_f}{k_f} (T_w - T) \quad \text{at } y = 0 \\ u \rightarrow 0, \quad C \rightarrow C_\infty, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (5)$$

Introducing the similarity transformations

$$\begin{aligned} u = pxf'(\eta), \quad v = -(p\vartheta)^{\frac{1}{2}} f(\eta), \\ N = px\sqrt{\frac{p}{\vartheta}}, \quad \eta = \sqrt{\frac{p}{\vartheta}} y, \end{aligned} \quad (6)$$

$$T - T_\infty = \theta(\eta)(T_w - T_\infty), \quad C - C_\infty = \varphi(\eta)(C_w - C_\infty).$$

The subsequent dimensionless equations were obtained through application of the similarity transforms found in equations (2)–(5) as follows:

$$\begin{aligned} f''' + ff'' + \beta(2ff'f'' - f^2f''') - (M + K)f' + Gr\theta \\ + Gc\phi - Frf'^2 - f'^2 = 0, \end{aligned} \quad (7)$$

$$\begin{aligned} ((1 + R) - Pr\delta f^2)\theta'' + Pr[f\theta' - \delta ff'\theta' + Nb\theta'\varphi' \\ + Nt\theta'^2 + MEcf'^2] \\ + \beta(2ff'^2f'' - f^2f''^2) + PrQ\theta + PrEc\varphi'^2 = 0, \end{aligned} \quad (8)$$

$$\varphi'' + Scf\varphi' + \frac{Nt}{Nb}\theta'' - Sc\gamma(1 + \alpha_1\theta)^n \exp\left(\frac{-E}{1 + \alpha_1\theta}\right)\varphi = 0. \quad (9)$$

The corresponding boundary conditions are as follows:

$$\begin{aligned} f(0) = S, \quad f'(0) = 1, \quad \varphi(0) = 1, \\ \theta'(0) = -Bi(1 - \theta(0)) \\ f'(\infty) = 0, \quad \varphi(\infty) = 0, \quad \theta(\infty) = 0. \end{aligned} \quad (10)$$

where

$$\begin{aligned} M = \frac{\sigma B_0^2}{\rho p}, \quad K = \frac{v}{pk_1}, \quad Fr = \frac{c_b}{\sqrt{k_1}}, \\ Gc = \frac{(C_w - C_\infty)gx\beta_C}{u_w^2}, \quad Gr = \frac{g(T_w - T_\infty)\beta_T x}{u_w^2}, \\ \delta = p\lambda_2, \quad R = \frac{16\sigma^*T_\infty^3}{3k^*k}, \quad Pr = \frac{v}{\alpha}, \quad Q = \frac{Q_0}{\rho C_p p}, \\ Ec = \frac{u_w^2}{(T_w - T_\infty)C_p}, \\ Nb = \frac{\tau D_B(C_w - C_\infty)}{v}, \quad Nt = \frac{\tau D_T(T_w - T_\infty)}{T_\infty v}, \\ Sc = \frac{v}{D_B}, \quad \gamma = \frac{K_r^2}{p}, \quad E = \frac{E_a}{kT_\infty}, \\ S = -\frac{v_w}{\sqrt{pv}}, \quad Bi = \frac{h_f}{k} \sqrt{\frac{v}{p}}. \end{aligned} \quad (11)$$

The rates of mass, heat, and skin friction are defined as follows:

$$\frac{\text{Sh}}{\sqrt{\text{Re}_x}} = -\varphi'(0), \quad \frac{\text{Nu}}{\sqrt{\text{Re}_x}} = -(1+R)\theta'(0), \quad (12)$$

$$\sqrt{\text{Re}_x} C_f = f''(0),$$

where $\text{Re}_x = \frac{\rho x^2}{\mu}$ is the local Reynolds number.

3 Examination of entropy generation

The following is the mathematical representation of the Maxwell fluid's volumetric entropy:

$$S_G = \frac{k}{T_\infty^2} \left(1 + \frac{16\sigma^* T_\infty^3}{3k^* k} \right) \left(\frac{\partial T}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_\infty} u^2 + \frac{RD}{C_\infty} \left(\frac{\partial C}{\partial y} \right)^2$$

$$+ \frac{RD}{T_\infty} \left(\frac{\partial C}{\partial y} \right) \left(\frac{\partial T}{\partial y} \right) + \frac{\mu_f}{k_1 T_\infty} u^2 \quad (13)$$

$$+ \frac{1}{T_\infty} \left[-\lambda_1 \left(\frac{\partial u}{\partial y} \right)^2 v^2 + 2uv \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right] + \mu_f \left(\frac{\partial u}{\partial y} \right)^2.$$

Substituting equation (6) in equation (13), we obtain

$$\text{Ns} = \alpha_1(1+R)\theta'^2 + \text{Br}(f''^2 + \beta(2ff'^2f'' - f'^2f''^2))$$

$$+ L \left(\frac{\alpha_2}{\alpha_1} \right) \varphi'^2 + \text{Br}(M+K)f'^2 + L\theta'\varphi', \quad (14)$$

where

$$L = \frac{D(C_w - C_\infty)R}{k}, \quad \text{Br} = \frac{\mu_f u_w^2}{\Delta T k}, \quad \alpha_2 = \frac{\Delta C}{C_\infty}, \quad \alpha_1 = \frac{\Delta T}{T_\infty} \quad (15)$$

are the diffusion parameter, Brinkman number, concentration, and thermal difference parameters, respectively.

4 Methodology

Let us introduce the variables $f_1 = f$, $f_2 = f'$, $f_3 = f''$, $f_4 = \theta$, $f_5 = \theta'$, $f_6 = \varphi$, $f_7 = \varphi'$. Now, the governing equations and the boundary conditions can be written as follows:

$$f_1' = f_2, \quad (16)$$

$$f_2' = f_3, \quad (17)$$

$$f_3' = \frac{1}{(1 - \beta f_1^2)} ((M+K)f_2 - f_1 f_3 - 2\beta f_1 f_2 f_3 - \text{Gr} f_4$$

$$- \text{Gc} f_6 + \text{Fr} f_2^2 + f_2^2), \quad (18)$$

$$f_4' = f_5, \quad (19)$$

$$f_5' = \frac{1}{[(1+R) - \text{Pr} \delta f_1 f_1]} (\text{Pr} [\delta f_1 f_2 f_5 - f_1 f_5 - \text{Nb} f_5 f_7$$

$$- \text{Nt} f_5 f_5 - M \text{Ec} f_2 f_2 + \beta(2f_1 f_2^2 f_3 - f_1^2 f_3^2) - Q f_4$$

$$- \text{Ec} f_3 f_3]), \quad (20)$$

$$f_6' = f_7, \quad (21)$$

$$f_7' = -\frac{\text{Nt}}{\text{Nb}} \left[\frac{1}{[(1+R) - \text{Pr} \delta f_1 f_1]} (\text{Pr} [\delta f_1 f_2 f_5 - f_1 f_5$$

$$- \text{Nb} f_5 f_7 - \text{Nt} f_5 f_5 - M \text{Ec} f_2 f_2 + \beta(2f_1 f_2^2 f_3 - f_1^2 f_3^2)$$

$$- Q f_4 - \text{Ec} f_3 f_3]) \right] + \text{Sc} \gamma (1 + \alpha_1 f_4)^n \exp \left(\frac{-E}{1 + \alpha_1 f_4} \right) f_6$$

$$- \text{Sc} f_1 f_7. \quad (22)$$

With the boundary conditions:

$$f_1 = S, f_2 = 1, f_5 = -\text{Bi}(1 - f_4), f_6 = 1$$

$$\text{at } \eta = 0, f_2 = 0, f_4 = 0, f_6 = 0 \quad \text{as } \eta \rightarrow \infty. \quad (23)$$

The numerical solutions for the above equations are obtained by the Runge–Kutta method with a shooting technique and implemented via the BVP4C MATLAB package.

5 Results and discussion

In this section, the numerical calculation for the nonlinear ODEs (7), (8), and (9) with the pertinent boundary circumstances (10) is thoroughly scrutinized. In order to resolve the fluid flow equation computationally, the fourth-order Runge–Kutta method and the well-known shooting method are combined. Additionally, using the specified boundary conditions, graphs and tabular representations of the numerical results of the current flow problem have been made possible by MATLAB's bvp4c solver. For this analysis, each parameter is numerically computed by assigning fixed values to other associated parameters such as $M = 0.5$, $K = 0.1$, $\text{Fr} = 0.3$, $\text{Bi} = 0.3$, $\text{Nb} = 0.2$, $\text{Nt} = 0.2$, $\text{Sc} = 0.5$, $Q = 0.2$, $\gamma = 0.5$, $\text{Pr} = 0.72$, $R = 0.5$, $\text{Br} = 0.5$, $\beta = 0.1$, $E = 0.5$, $\text{Lb} = 1$, $\text{Pe} = 0.15$, $\delta_1 = 0.2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, and $S = 0.1$.

The effect of the magnetic factor (M) on the outline of velocity ($f'(\eta)$) is demonstrated in Figure 2. The ascending levels of M show the velocity curve's declining behavior. Since M is the ratio of Lorentz forces to viscosity forces, physically speaking, an increase in the magnetic field strengthens the Lorentz force and decreases the fluid's velocity. The effect of the magnetic component (M) on the thermal field ($\theta(\eta)$) is described by Figure 3. It reveals the increasing profile of thermal distribution $\theta(\eta)$ due to

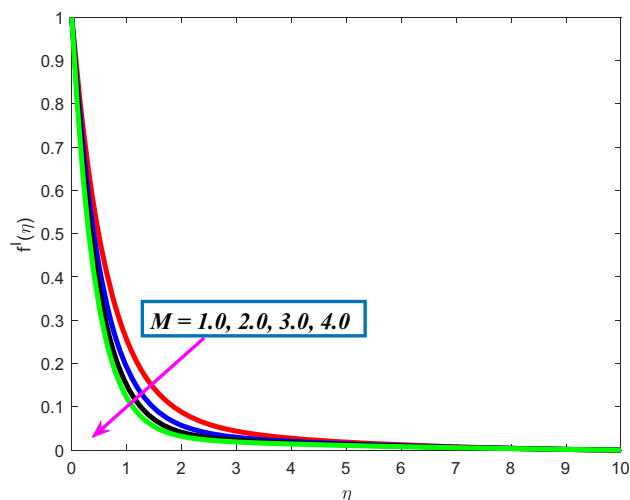


Figure 2: $f'(\eta)$ for various M values.

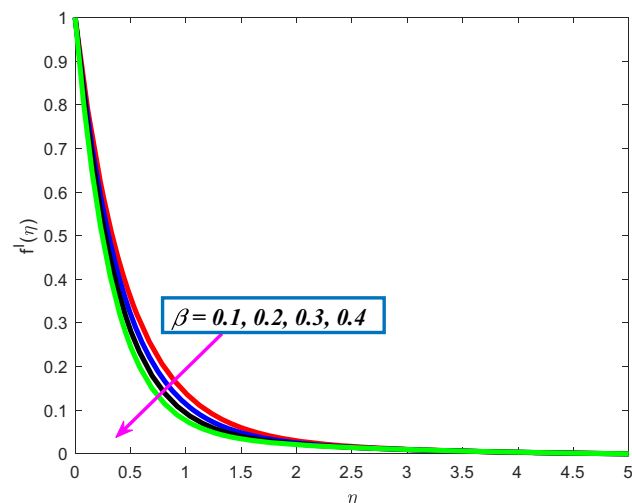


Figure 4: $f'(\eta)$ for various β values.

the dominant magnetic parameter. Figure 4 describes the outputs of the velocity region ($f'(\eta)$) for larger values of Maxwell parameter, denoted as β . It is noted that the liquid speed and the overall thickness of the mobility boundary layer diminished as we increased Maxwell parameter quantities. Increasing the Maxwell factor physically causes viscoelastic effects to appear in the fluid movement, resulting in greater friction within the system and creating a barrier. Due to its viscoelastic properties, the formation of molecular groupings restricts the free movement of the fluid, resulting in a reduction in its speed. Figure 5 depicts the effect of the porous factor (K) on the velocity ($f'(\eta)$). It was observed that when the values of the porosity component increase, the velocity distribution falls. In physical terms, boosting the porosity parameter lowers the speed

because the fluid flow is resisted by the medium's porosity, which leads to obstructions and higher friction. Figure 6 shows how $f'(\eta)$ varies as the Forchheimer number Fr increases. Physically, a porous material is one kind of opposing factor that seeks to impede the flow of a liquid flowing across it, hence reducing the flow's speed. Figure 7 depicts the behavior of velocity for various values of thermal Grashof number (Gr). It is noted that the increase in velocity is higher for the higher values of Gr . Figure 8 demonstrates the effect of concentration Grashof number (Gc) on the velocity profile. It provides that the increased Gc declines the velocity. Figure 9 draws attention to the thermal distribution $\theta(\eta)$ for greater temperature relaxation factor (δ). It is demonstrated that the energy field diminishes as δ is increased. Figure 10 reveals the impact

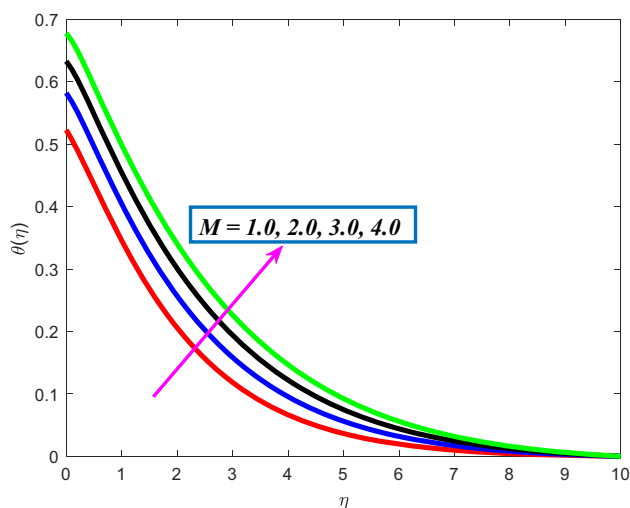


Figure 3: $\theta(\eta)$ for various M values.

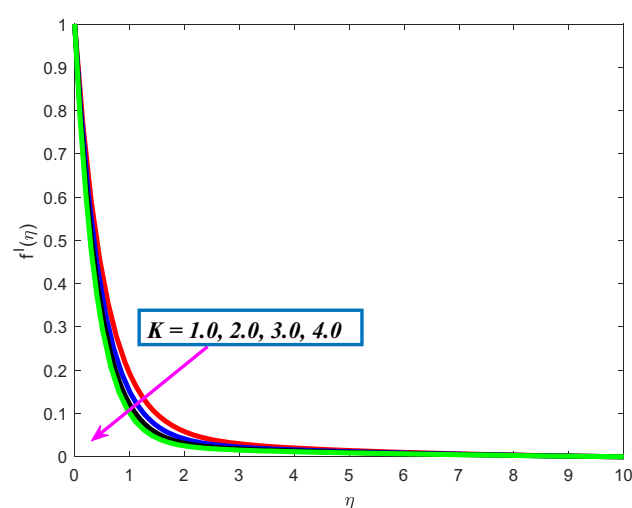


Figure 5: $f'(\eta)$ for various K values.

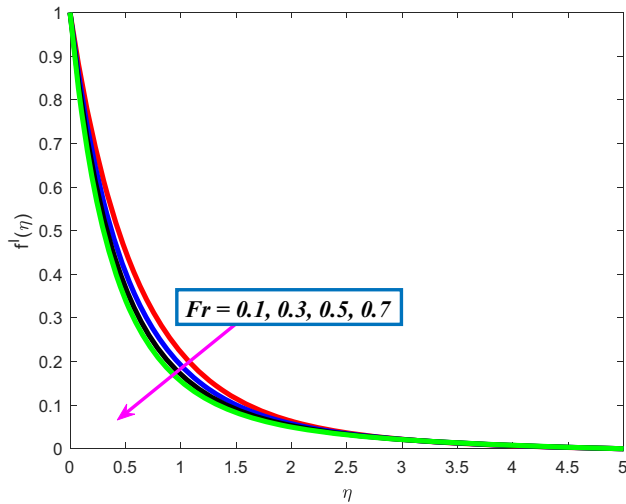


Figure 6: $f'(\eta)$ for various Fr values.

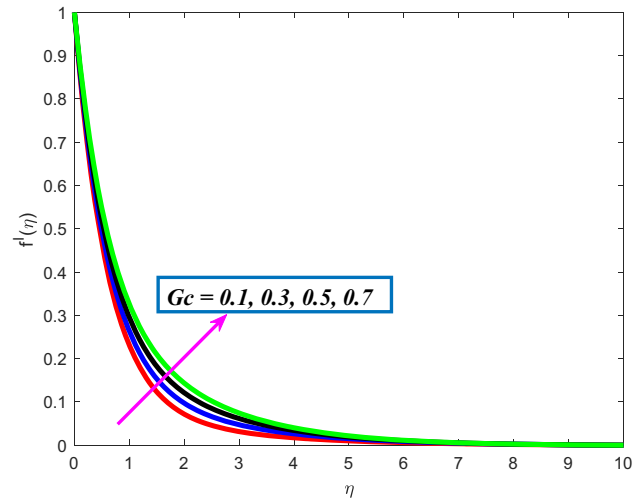


Figure 8: $f'(\eta)$ for various Gc values.

of the radiation parameter (R) on the thermal field ($\theta(\eta)$). It is noticed that θ is enhanced with a strengthened amount of R . In reality, additional energy is created inside the liquid due to a boost in the parameter for radiation (R), which therefore improves the energy profile. Figures 11 and 12 depict the behavior of mass and thermal profiles for disparate values of the Brownian motion parameter (Nb). The parameter Nb tends to increase the temperature and reduce the concentration. Brownian motion refers to the random and unpredictable movement of particles in a liquid due to frequent collisions with fast-moving molecules. These collisions generate heat, leading to a rise in temperature. Similarly, the concentration of species among nanoparticles decreases as repeated collisions

cause dispersion, highlighting the dynamic interaction between particles and their surroundings.

Figure 13 shows the changes in the solutal field with the thermophoresis parameter (Nt). The behavior of concentration distribution is increased with strengthened Nt . Figure 14 elaborates the concentration field $\phi(\eta)$ for a wide range of values in activation energy (E). The concentration profile improves as a result of the supplementing values of E . Figure 15 presents a sketch for the concentration profile $\phi(\eta)$ against different values of Schmidt number (Sc). Generally, the dominating Sc diminishes the mass diffusion coefficient, which reduces the concentration field. Figure 16 reveals how the concentration field is affected by the chemical reaction parameter (γ). It is found that the concentration field is reduced by raising chemical reactions.

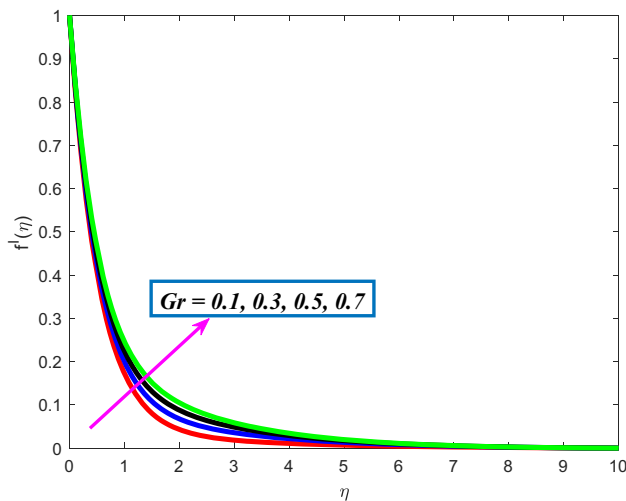


Figure 7: $f'(\eta)$ for various Gr values.

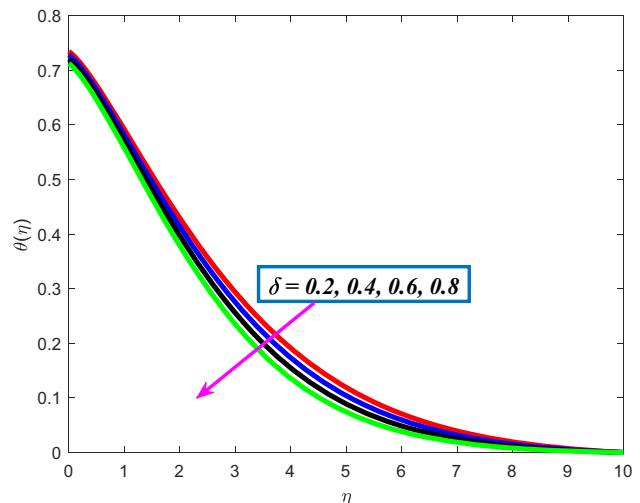
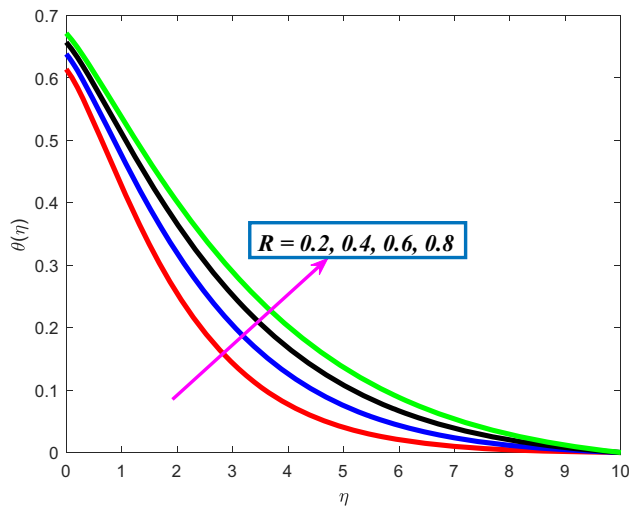
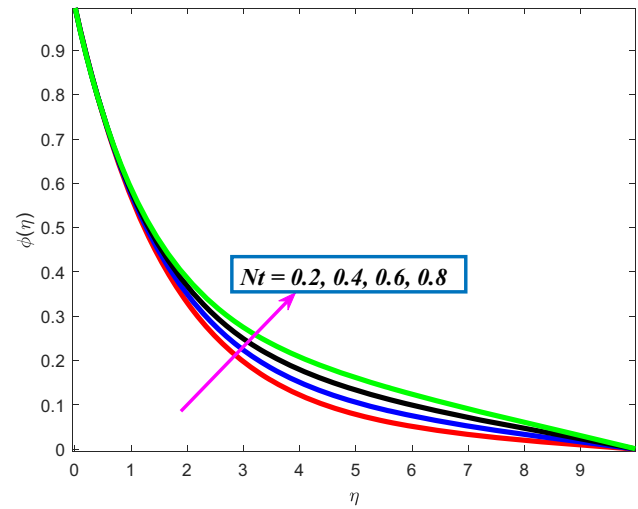
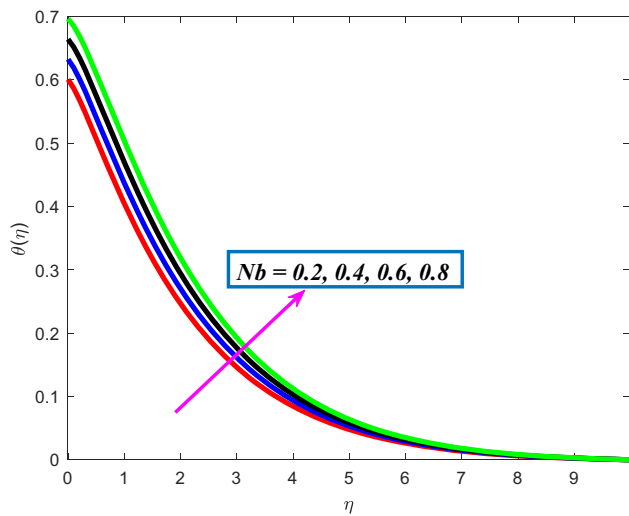
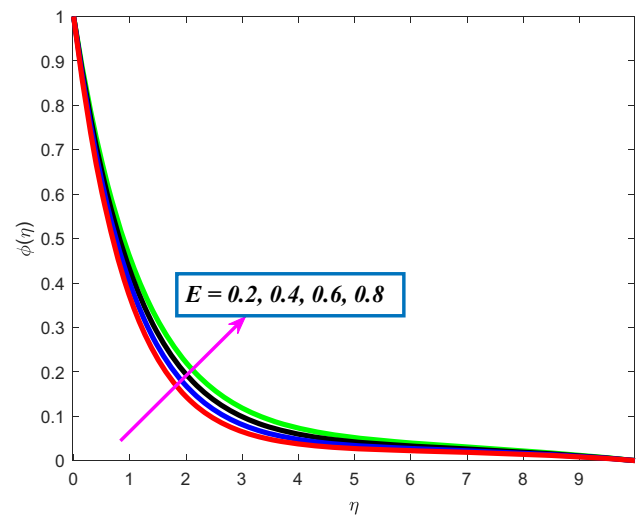
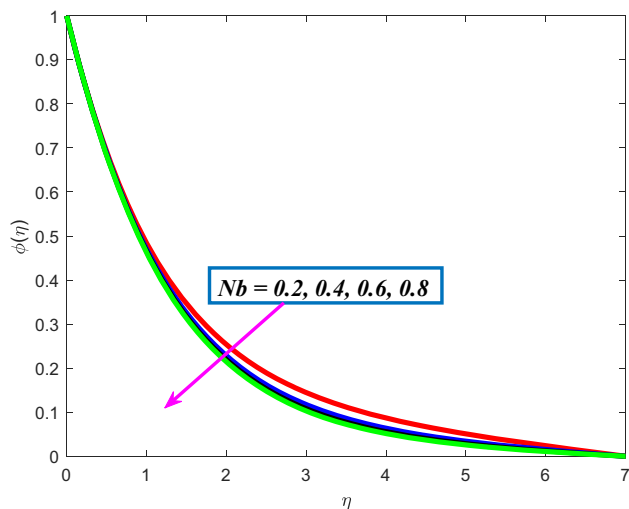
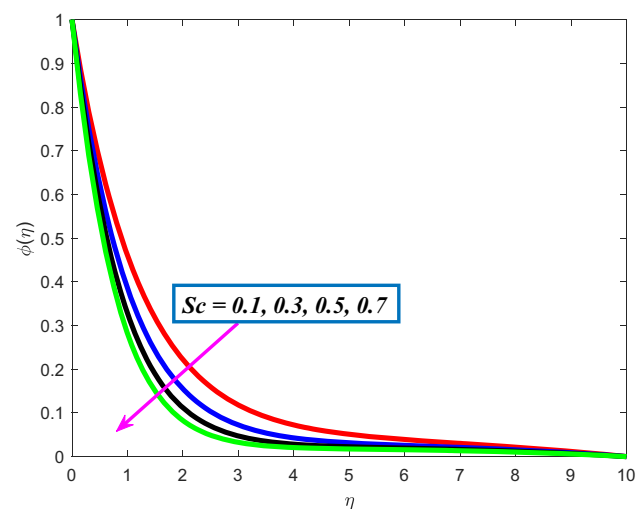
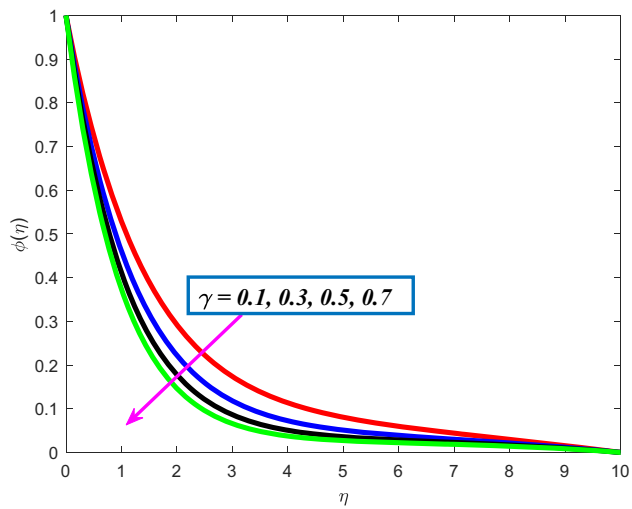
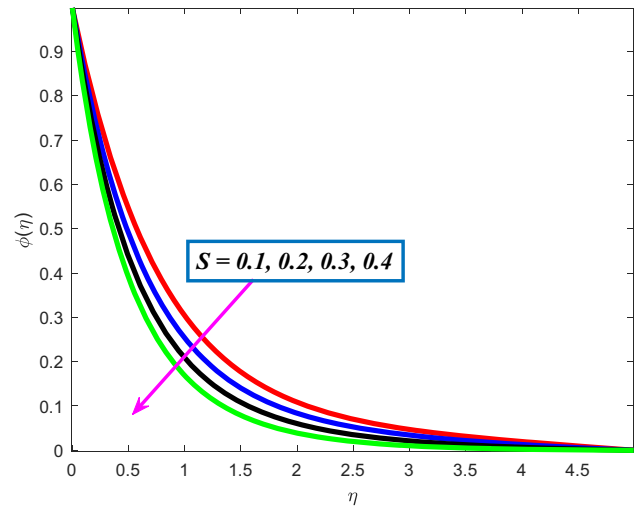
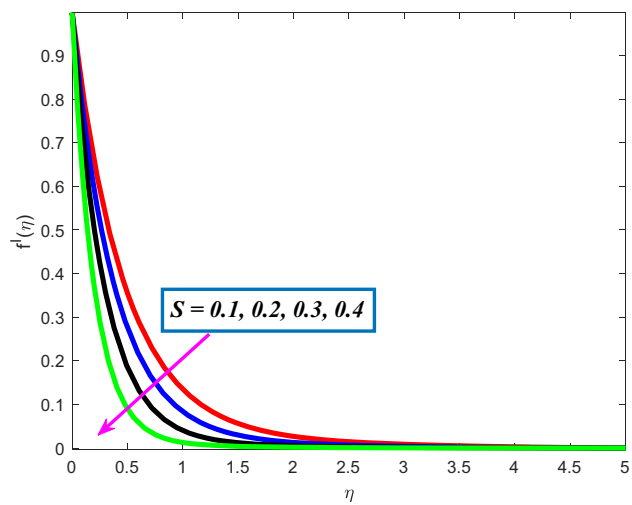
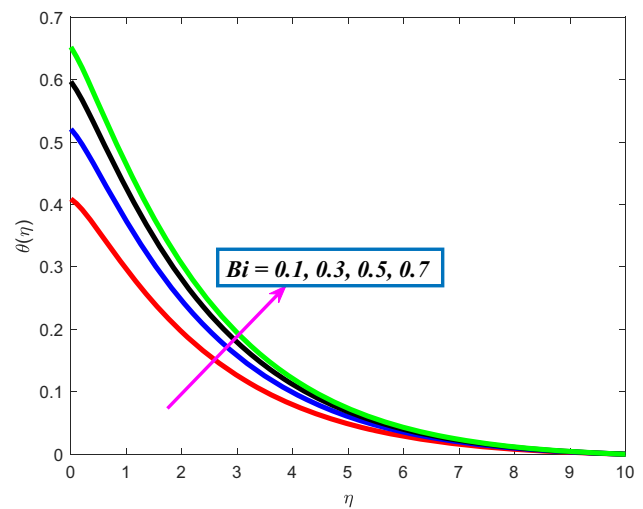
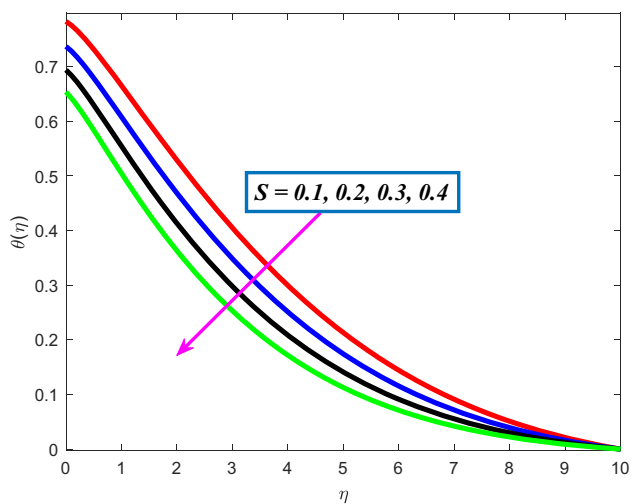
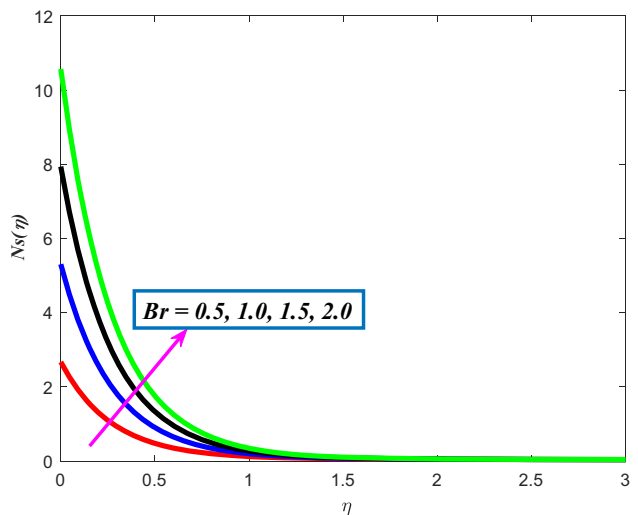
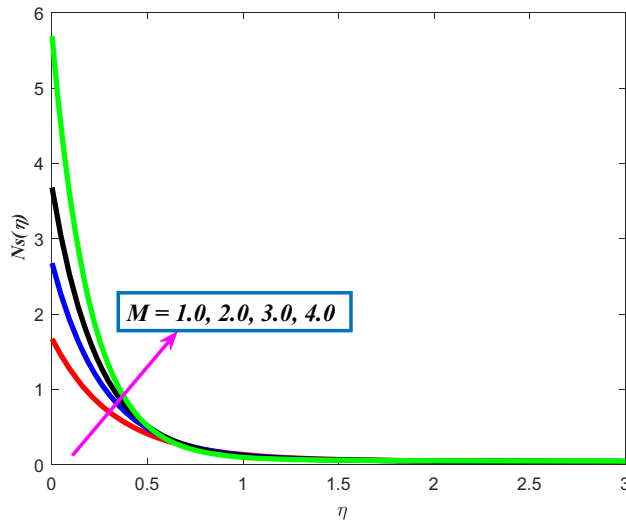
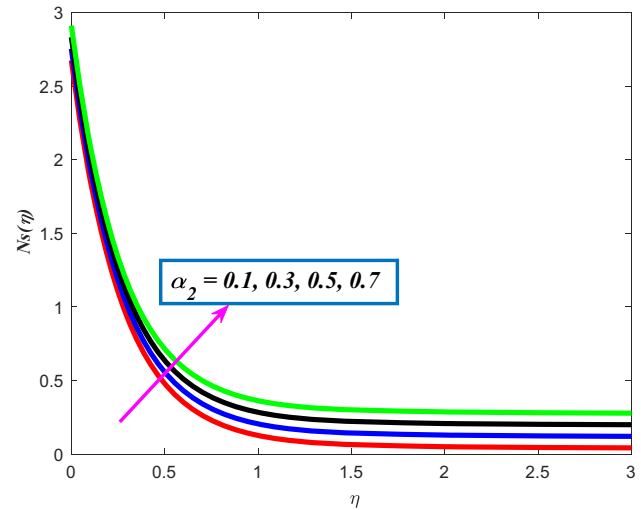
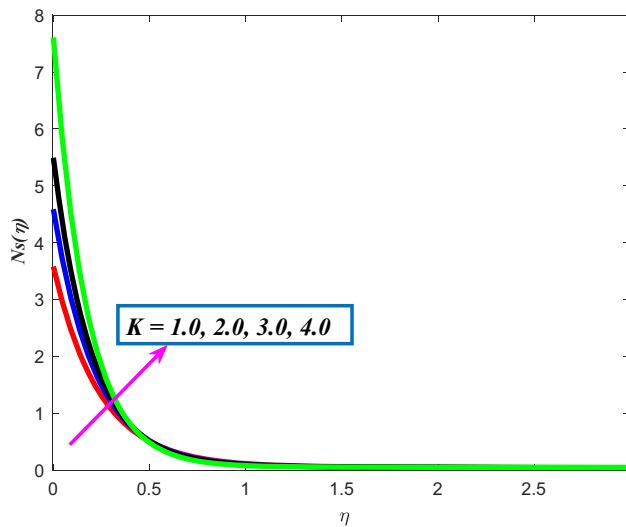
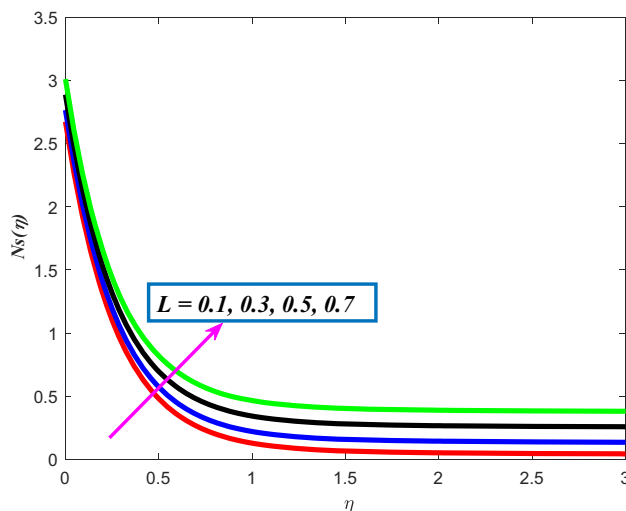
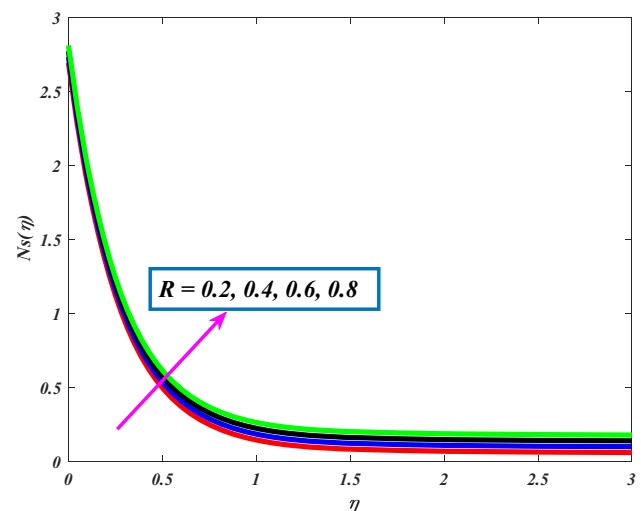


Figure 9: $\theta(\eta)$ for various δ values.

Figure 10: $\theta(\eta)$ for various R values.Figure 13: $\phi(\eta)$ for various Nt values.Figure 11: $\theta(\eta)$ for various Nb values.Figure 14: $\phi(\eta)$ for various E values.Figure 12: $\phi(\eta)$ for various Nb values.Figure 15: $\phi(\eta)$ for various Sc values.

Figure 16: $\varphi(\eta)$ for various γ values.Figure 19: $\varphi(\eta)$ for various S values.Figure 17: $f'(\eta)$ for various S values.Figure 20: $\theta(\eta)$ for various Bi values.Figure 18: $\theta(\eta)$ for various S values.Figure 21: $Ns(\eta)$ for various Br values.

Figure 22: $Ns(\eta)$ for various M values.Figure 25: $Ns(\eta)$ for various α_2 values.Figure 23: $Ns(\eta)$ for various K values.Figure 24: $Ns(\eta)$ for various L values.Figure 26: $Ns(\eta)$ for various R values.

Figures 17–19 illustrate the consequence of the suction factor (S) on the liquid velocity ($f'(\eta)$), thermal profile ($\theta(\eta)$), and solute profile ($\phi(\eta)$). The thermal, velocity, concentration, and the corresponding boundary layer thickness profiles show a drop over the whole boundary layer area for higher values of the suction parameter. In physical terms, a higher suction parameter results in a greater amount of fluid being removed, which reduces the fluid's velocity at the surface and slows down the flow. Figure 20 shows the Biot number (Bi) effects on the thermal field ($\theta(\eta)$). It is noticed that the increment in Bi improves the temperature. Figures 21–26 depict the entropy generation ($Ns(\eta)$) under the effects of Brinkman number (Br), magnetic field parameter (M), porosity parameter (K), diffusion parameter (L), concentration difference parameter (α_2), and radiation (R). It is found that the $Ns(\eta)$ enhanced with enhancement in Br , M , K , L , α_2 , and R .

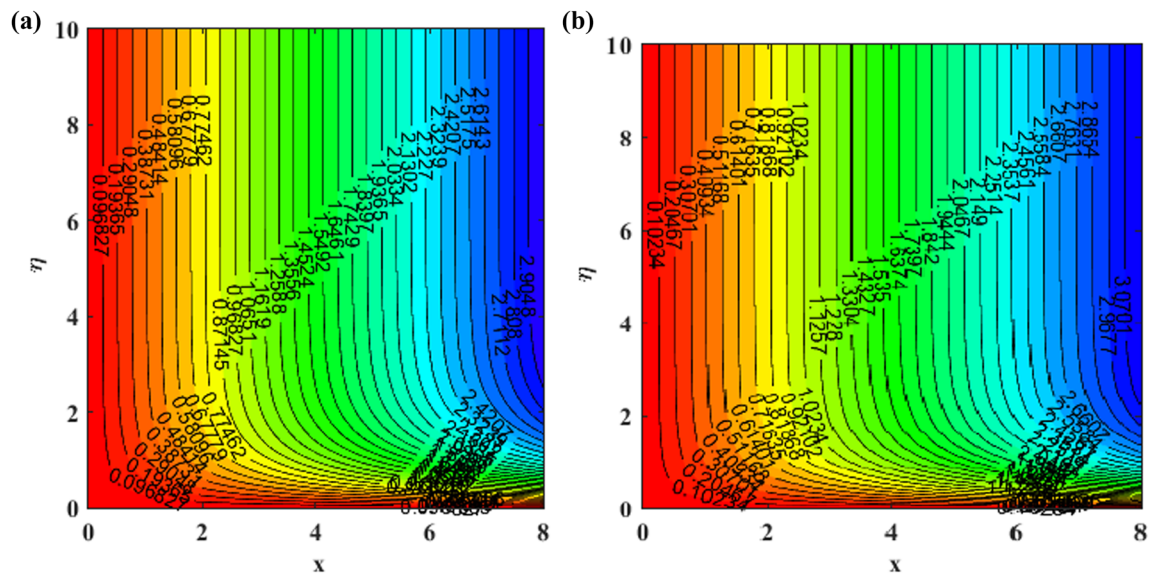


Figure 27: (a and b) Streamlines profile.

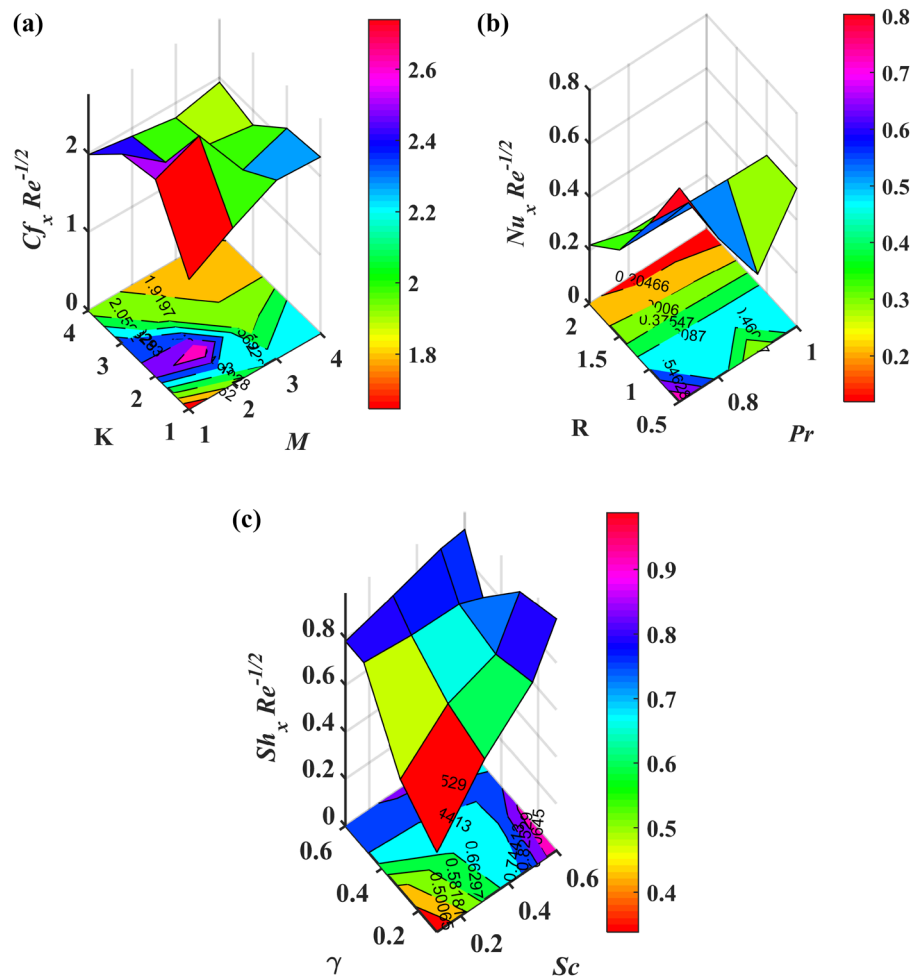


Figure 28: (a)–(c) Surface plot of skin friction coefficient, Nusselt number and Sherwood number.

Table 1: Variations in skin friction for various parameters

M	K	β	Gr	Gc	Fr	S	$C_f(\text{Re}_x)^{\frac{1}{2}}$
1.0							-1.646613
2.0							-1.983847
3.0							-2.271701
	1.0						-2.245651
	2.0						-2.504542
	3.0						-2.738965
		0.1					-1.983847
		0.2					-2.193957
		0.3					-2.409157
			0.1				-1.983847
			0.3				-1.894008
			0.5				-1.814466
				0.1			-1.983847
				0.3			-1.897335
				0.5			-1.810797
					0.1		-1.942817
					0.3		-1.983847
					0.5		-2.024084
						0.1	-1.983847
						0.2	-2.057192
						0.3	-2.135494

Contour plots are examined for different values of M and K , as shown in Figure 27(a) and (b). Figure 28(a)–(c) shows a sketch to view the skin friction, heat, and mass transmission rate against M , K , R , Pr , Sc , and γ . Figure 28(a) shows that the skin friction diminishes with an elevation in the M and K due to the inverse connection between them, as the ratio of inertia to viscous forces increases. Moreover, higher values of M and K cause an escalation in friction forces, thus leading to an increase in skin friction. The effect of Pr and R on heat transfer is shown in Figure 28(b). It is clear that higher Pr and R result in a greater Nusselt number, which in turn causes a higher temperature profile, and so on. Furthermore, increases in the Sc and γ increase in the Sherwood number.

Table 1 depicts that the friction factor declines with enhancement in M , K , Fr , β , and S , while opposite behavior is seen with enhancement in Gr and Gc . Table 2 presents the heat transfer rate increases for increasing values of M , Q , Ec , Nb , and Nt , but a reverse phenomenon is observed in the case of Pr , and Bi . Table 3 depicts that the larger values of Sc , γ , Nt , and E enhance the mass transfer rate. Moreover, Table 4 is constructed to present the individuality of this study.

Table 2: Deviations in the rate of mass transmission for various parameters

M	Pr	R	Q	Ec	Nb	Nt	δ	Bi	$(\text{Re}_x)^{-\frac{1}{2}} \text{Nu}$
1.0									0.080249
2.0									0.051925
3.0									0.028946
	0.7								0.051863
	0.8								0.052540
	0.9								0.054044
		0.5							0.055279
		1.0							0.051925
		1.5							0.052367
			0.1						0.051925
			0.2						-0.107686
			0.3						-0.694490
				0.1					0.051925
				0.2					0.017309
				0.3					-0.016940
					0.2				0.051925
					0.4				0.046719
					0.6				0.041765
						0.2			0.051925
						0.4			0.048915
						0.6			0.045768
							0.2		0.051925
							0.4		0.045171
							0.6		0.041477
								0.1	0.031962
								0.3	0.051925
								0.5	0.059454

Table 3: Variations in the rate of mass transmission for various parameters

Sc	γ	Nb	Nt	E	$(\text{Re}_x)^{-\frac{1}{2}} \text{Sh}$
0.1					0.338323
0.3					0.600189
0.5					0.786415
	0.1				0.470914
	0.3				0.654791
	0.5				0.786415
		0.2			0.786415
		0.4			0.765138
		0.6			0.758990
			0.2		0.786415
			0.4		0.842619
			0.6		0.903393
				0.2	0.786415
				0.4	0.848210
				0.6	0.916710

Table 4: Comparison of the current study with the existing studies in the literature

	Khan et al. [3]	Kumar et al. [10]	Jawad and Nisar [26]	Awais et al. [27]	Gurejala et al. [15]	Present study
Magnetic field	Studied	Studied	Studied	Studied	Studied	Studied
Non-Newtonian fluid	Studied	Not studied	Studied	Studied	Studied	Studied
Nanofluid	Not studied	Studied	Studied	Not studied	Studied	Studied
Darcy–Forchheimer	Not studied	Not studied	Not studied	Studied	Not studied	Studied
Heat source	Studied	Not studied	Studied	Studied	Studied	Studied
Cattaneo–Christov model	Studied	Not studied	Studied	Not studied	Not studied	Studied
Radiation	Not studied	Studied	Not studied	Studied	Not studied	Studied
Activation energy	Not studied	Studied	Not studied	Studied	Not studied	Studied
Biot number	Not studied	Not studied	Not studied	Not studied	Studied	Studied

6 Conclusions

To represent Maxwell nanofluid flow across an expanding surface, this study utilizes the Cattaneo–Christov framework in a novel manner. This investigation evaluates the Darcy–Forchheimer dissipative flow of hydromagnetic Maxwell nanofluid over an expanding surface, incorporating radiation energy transfer, convective boundaries, activation energy, Joule heating, and energy generation. The heat transfer process is analyzed using the Cattaneo–Christov heat flux framework. This also provides the uniqueness of this study. Numerical calculations are conducted using MATLAB's `bvp4c` solver to analyze the flow and thermal characteristics under these conditions. The following are the analysis's key findings:

- The thermal Grashof number and solute Grashof number serve to increase the Maxwell nanofluid velocity over an expanding surface.
- The greater suction parameter exhibits diminishing patterns in both fluid velocity and fluid temperature profiles.
- Higher thermal Biot parameter values lead to increased fluid temperature, while larger values of factor for temperature relaxation decrease the liquid thermal field.
- The concentration distribution drops as a consequence of development in the Brownian movement and Schmidt number.
- Increased Brinkman number and diffusion parameter strengthen the entropy generation.
- The coefficient of friction falls with respect to the rise in the parameter for the magnetic field, Darcy–Forchheimer parameter, and Maxwell fluid parameter.
- Higher Biot number values lead to more convective heat transport, which increases the Nusselt number overall. Conversely, as the magnetic and radiation parameters increase, the Nusselt number falls.

The results provide useful information for improving heat management in engineering systems, including MHD

devices, energy converters, and cooling technologies. Designers may maximize heat transfer, reduce friction, and regulate entropy formation by adjusting parameters such as the Biot number, magnetic field, and Brownian motion. These properties are advantageous for utilization in nanofluid-based thermal exchangers, electronics cooling, polymer processing, and biomedical flows. The future scope will involve adding changing fluid characteristics and various nanoparticle forms, as well as expanding it to unstable, three-dimensional, or rotating flows. Combining machine learning and optimization approaches can improve design processes and support implementation in advanced industrial, healthcare, and energy systems.

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