Towards low-noise tunable terahertz waves generation via photomixing

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ABSTRACT

We demonstrate a low-cost, stable, and tunable laser system by phase-locking a commercial telecom-grade tunable laser (ITLA) to an Optical Frequency Comb (OFC) within the C-band. Using minimal optical hardware and sub-mW OFC power, we achieved short-term integrated phase noise of 10 mrad and long-term frequency stability of ± 0.01 Hz over 10 h. This system enables scalable, OFC-locked tunable lasers and holds promise for applications like tunable THz wave generation and comb-locked transmitters in DWDM systems, supporting scalable phase-locking of multiple lasers with OFC power in the nW regime, making it highly adaptable for various OFC generators.

1. Introduction

Optical frequency combs, or OFCs, offer stable frequency spacing between comb tones. They are, therefore, applicable in several domains, including frequency physics [1], terahertz signal synthesis [2], and telecommunications [3]. Only a few chosen comb tones are needed in many of these applications, necessitating their isolation and frequently substantial amplification. Phase-lock loops [4] or optical injection locking [5] are the most energy-efficient ways to phase-lock laser(s) to the desired OFC tone(s).

In these non-metrological applications, there is frequently a requirement to phase-lock several lasers and/or to have tunable laser wavelengths. For terahertz signal creation, the separation between the two lasers must be adjustable by up to 8 nm for the generation of 0–1 THz. In telecommunications, the wavelength of signaling lasers is sometimes required to be tunable to facilitate the flexible allocation of DWDM or flexi-grid channels. These applications are frequently subject to stringent cost limitations, necessitating economical optical fiber communication (OFC), affordable tunable lasers, minimal and inexpensive optical components, and cost-effective, low-power electronics (i.e., integrated, low-bandwidth) for phase locking.

Through photo mixing [6], two lasers can be phase-locked to an optical frequency comb for low-noise terahertz signal generation. Phase

locking of tunable lasers is essential for tunability. A cost-effective solution necessitates low-cost tunable lasers synchronized to an optical frequency comb utilizing economical electronics. We recently demonstrated the phase locking of commercial telecom tunable lasers to optical frequency combs (OFC) utilizing a low-cost FPGA platform [7]. This enabled phase locking across the entire bandwidth of the tunable laser, specifically within the C-band range in our demonstration. The study examined the minimum power per OFC tone necessary for phase locking, facilitating the transmission of the full comb to the photodiode without causing saturation. This paved the development of flexible, cost-effective, and energy-efficient OFC-locked systems. Low-noise terahertz signal generation through photo mixing involves the phase locking of two lasers to an optical frequency comb. Phase locking of tunable lasers is essential for tunability. A cost-effective solution necessitates low-cost tunable lasers synchronized to an optical frequency comb utilizing economical electronics.

This study systematically investigates ITLA laser locking within an optical frequency comb (OFC). We demonstrate that using double-PI^2 (proportional-double-integrator), PI^4, and phase lock loop filters, in contrast to the commonly utilized single PI^2, results in a fivefold increase in loop bandwidth, achieving the ITLA laser control bandwidth. This enhancement yields substantial improvements in phase noise, residual jitter, and variations in frequency error. Additionally, we examine the

minimum optical frequency comb (OFC) tone power required for stable, low-noise phase locking and optimize the detection circuit to utilize the entire OFC comb without pre-filtering, thereby removing the necessity for tunable optical filters. This optimization reduces the power requirements of the optical frequency comb (OFC) necessary for phase locking, allowing for lower OFC power usage per locked laser. Consequently, this increases the number of lasers that can be locked with a single OFC by passively splitting its power using a single $1 \times N$ coupler.

2. Practical considerations

In the proposed configurations, the requirements for the optical frequency comb (OFC) regarding frequency spacing, flatness (variation in OFC tone powers), and, to some extent (as discussed later), output power is not critical. One promising OFC candidate, particularly for operations in the telecom C-band, is detailed in references [8]. It employs a limited number of passive fiber components spliced together and a single 1480-nm telecom-grade pump laser rated at 200 mW. Since the OFC is not the focus of our study, we utilized an OFC available in our laboratory, specifically the Menlo FC1500 oscillator, which delivers 0.5 mW over a 77 nm FWHM with a comb spacing of 250 MHz. Additional details will be provided later. The performance of the previously discussed compact comb offers a viable alternative.

We utilize telecom-grade ITLA-packaged lasers for ITLA tunable lasers. These lasers feature a maximum output power of 40 mW and wavelength configuration managed via their software. The phase section includes a modulation input utilized for feedback purposes. Feedback control was implemented using an FPGA-based Xilinx Zynq 7010 board on the Red Pitaya evaluation platform, which was programmed to facilitate two parallel PI^2 controllers [4]. The clock speed was 64 MHz, resulting in low cost and power consumption. Subsequent analysis demonstrates that its bandwidth aligns effectively with the modulation bandwidth of the employed tunable lasers.

Phase locking was executed with a 30 MHz offset, however, this can be adjusted using the FPGA boards within the 10 to 60 MHz range. As demonstrated in [5], it can be extended to an arbitrary number by employing additional RF synthesizers and mixers.

3. Experiments

In our vision, the optical fiber coupler (OFC) is initially divided passively to accommodate multiple phase-locked lasers. The reduction in optical frequency comb (OFC) power necessary for phase locking each tunable laser would facilitate the phase locking of more lasers using the same OFC. In situations where only a single laser or a limited number of lasers require locking, this approach decreases the optical power demands of the optical frequency comb, lowering both costs and power consumption. Our initial focus was to examine the minimum optical power necessary to lock our tunable lasers while characterizing their short- term and long-term performance. Initially, we phase locked two identical tunable lasers using the configuration illustrated in Fig. 1, as this arrangement yields a signal unaffected by interference from multiple comb tones. This establishes the standard for optimal performance. We subsequently verified that comparable performance and minimum power levels can be attained using OFC as the master signal, characterizing the performance with the setup illustrated in Fig. 2.

We used Low Pass Filter (LPF) with cut-off frequency of 50 MHz, 24 dB gain and 0.1–500 MHz bandwidth of RF Amplifier, Photodetector with bandwidth 1 GHz and responsivity of 0.9 A/W. 50/50 Coupler with excess loss of 0.5 dB.

The utilized C-band tunable ITLA lasers delivered an output power of 40 mW. The phase section regulating the laser carrier frequency was utilized for feedback purposes. The measured bandwidth was 100 kHz. The utilized OFC (total power: 0.5 mW, comb tone spacing: 250 MHz, 10-dB bandwidth: 77 nm) has its spectrum measured using an Optical Spectrum Analyzer (OSA), as depicted in Fig. 3, where individual tones
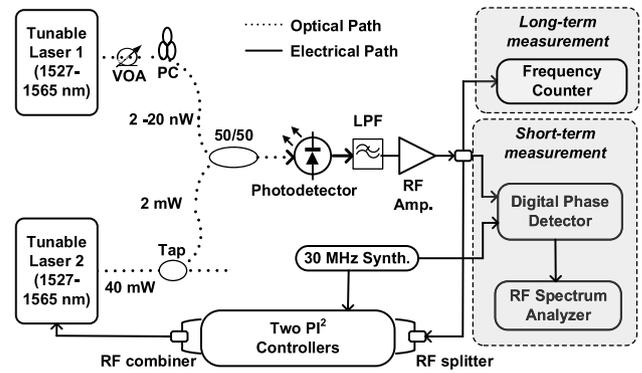

Fig. 1. Schematics of phase locking of two ITLA tunable lasers, one with power of 2 mW (1 mW reaching the photodetector after passing through the 50/50 coupler) and the other with low power, ranging from 1 nW (lowest power at which we achieved stable lock) to 10 nW (2–20 nW prior to the 50/50 coupler). VOA: Variable Optical Attenuator, PC: Polarization Controller, LPF: Low Pass Filter.
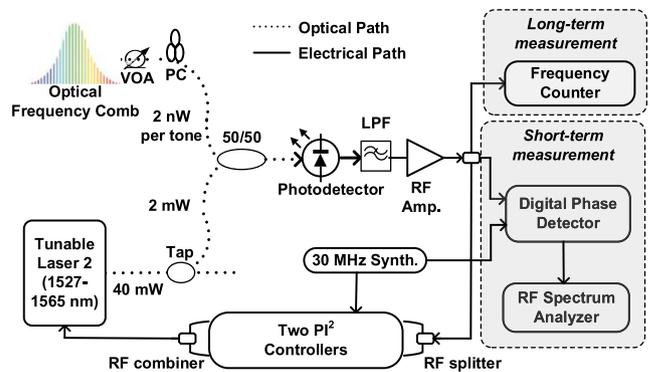

Fig. 2. Schematics of phase locking of ITLA tunable laser to OFC with power set at 2 nW for comb tone used for locking (1 nW reaching the photodetector). VOA: Variable Optical Attenuator, PC: Polarization Controller, LPF: Low Pass Filter.
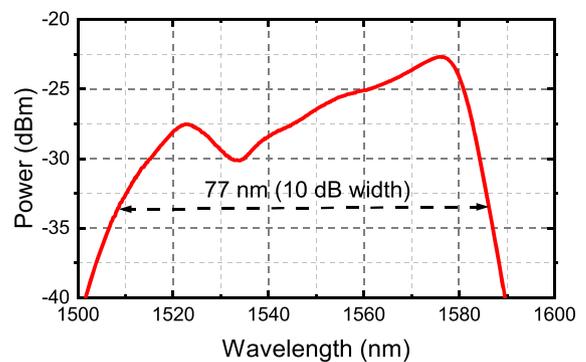

Fig. 3. Spectrum on the used OFC with total power of 0.5 mW and per-comb powers of 1.1 nW at 1530 nm, 2.3 nW at 1550 nm, and 3.7 nW at 1565 nm.

remain unresolved due to their proximity in spacing. The per-tone power measurements were 1.1 nW at 1530 nm, 2.3 nW at 1550 nm, and 3.7 nW at 1565 nm.

Combining the two signals using a 50/50 coupler, the resulting beating signal was detected with an amplified detector (Menlo FPD310), characterized by a bandwidth of 10–1000 MHz and a maximum input power of 2 mW. Given that our optical frequency comb (OFC) power was

below 1 mW, we opted to maintain the tunable laser power at the photodetector input at a constant 1 mW. This approach prevents detector saturation while utilizing the full OFC power for photodetection. Simultaneously, it ensures an optimized signal-to-noise ratio, as the total power incident on the photodetector consistently exceeded 1 mW, which is above half of its maximum capacity. The 2 mW extracted from the tunable laser, which corresponds to 1 mW incident on the photodetector after passing through the 50/50 combiner, constitutes a small fraction of the total tunable laser power of 40 mW. This arrangement allows most of the laser power to be utilized in various applications.

The photodetector signal underwent low-pass filtering using a 50-MHz filter, effectively isolating the 30-MHz beat frequency of the two lasers, as previously indicated, with a target lock at a 30 MHz offset. This additionally eliminates high-frequency noise and the interference of comb tones with other comb tones and the signal. The beating signal was then amplified using an RF amplifier (Mini-Circuits ZFL-500HLN+) and subsequently split in three: for long-term measurement, short-term measurement, and for providing the feedback, Figs. 1 and Fig. 2. The phase lock controller utilizing the Red Pitaya evaluation board featured two independent inputs and outputs, with each configured as a proportional-double-integrator (PI^2) controller, as outlined in [4]. The control bandwidth was mainly constrained by the delays linked to the input analog-to-digital and output digital-to-analog conversions, estimated at 225 kHz [4]. The error signal was input into both controllers, and their outputs were subsequently combined, utilizing two double-integrators gain (PI4) controller. This can be achieved through a single input and output configuration and reprogramming the FPGA. This approach is intended for future implementation, allowing one Red Pitaya to lock two lasers. The feedback bandwidth is anticipated to be constrained primarily by the laser modulation input bandwidth of 100 kHz, while all other components exhibit bandwidths exceeding 200 kHz.

3.1. Short-term stability measurement

Short-term stability, defined by observation times under 0.1 s (corresponding to frequencies exceeding 10 Hz), was assessed using a digital phase detector followed by spectrum analysis of the resulting signal. The digital phase detector offers multiple advantages compared to the frequently utilized frequency mixer. It exhibits minimal cross-sensitivity to amplitude noise. Additionally, it possesses a more extensive dynamic range not constrained by the periodic 2p variations in the response.

We initially examined the phase locking of two lasers, as illustrated in Fig. 1. Fig. 4 presents the phase noise measurements for the minimum power of 1 nW, which achieved stable phase lock at the photodetector input, along with measurements for slightly elevated powers of 5 nW and 10 nW. Without contrary indications, the PI4 controller was utilized for feedback purposes.
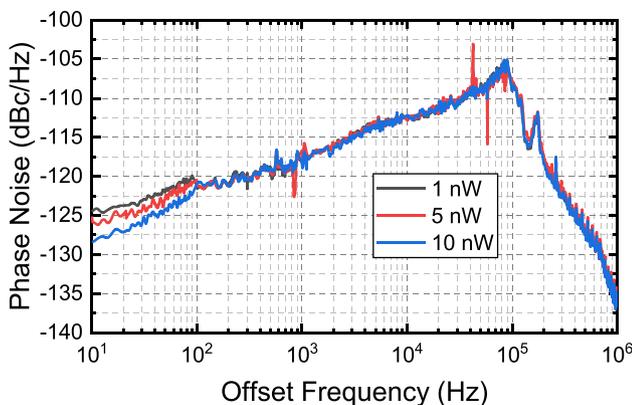
The phase noise at offset frequencies exceeding 80 kHz aligns with that of free-running lasers, as demonstrated subsequently. Below 80 kHz, the feedback generates phase noise that decreases inversely with offset frequency, consistent with the behavior of a well-designed phase locking system. The variation in laser power (1–10 nW) results in differences at low frequencies, specifically below 100 Hz, with a few decibels of difference noted at 10 Hz. This minor difference is attributed to environmentally related noise, as the signal in this low-frequency region exhibited slight fluctuations over time. The primary conclusion is that 1 nW (-60 dBm) delivers adequate power for phase locking with minimal impact on performance.

We subsequently replaced one tunable laser with our optical frequency comb (OFC), as shown in Fig. 2, and executed the locking of a tunable laser to the OFC using the same per-tone power as that employed in the locking of two lasers at 1–10 nW. In contrast to earlier research, this study employed the full comb signal (77-nm bandwidth) for photodetection, eliminating the need for tunable optical bandpass filters. This approach significantly reduces costs while maintaining optimal per-tone optical frequency comb power for photodetection. Fig. 5 presents the result. The performance observed closely resembles that achieved when locking two lasers, as illustrated in Fig. 4. This indicates a minimal penalty resulting from the substantial number of comb tones incident on the photodetector.

This study presents a detailed characterization of the OFC-locked tunable laser setup (Fig. 2) utilizing 1 nW per-tone OFC power. Fig. 6 compares phase noise measured from the beating of the free-running laser with the optical frequency comb tone alongside the noise obtained using PI2 and PI4 controllers. The optimal parameters, determined by observing phase noise during variations, are presented in Tables 1 and Table 2. Fig. 6 illustrates the noise floor obtained by linking the measurement input port of the digital phase detector to a replica of the reference signal.

The results indicate that PI4 facilitated an increase in control bandwidth from 20 kHz to 100 kHz, along with improved noise suppression at low offset frequencies compared to the PI2 controller. Additionally, PI4 demonstrates reductions of 13 dB and 20 dB in phase noise at 1 kHz and 10 kHz from the carrier frequency, respectively, when compared to PI2. The PI2 and PI4 reduce phase noise by 78 dB to 87 dB at a 10 Hz offset frequency relative to the free-running laser. The phase noise at 10 Hz measured with the PI4 is nearing the measurement noise floor, as illustrated in Fig. 6.

Additionally, we examined the relationship between performance and wavelength. Fig. 7 illustrates the phase noise measurements taken across the C-band, specifically at wavelengths of 1530, 1550, and 1565 nm. The performance remains consistent across all wavelengths, including 1530 nm, where the corresponding comb tone exhibits lower power compared to other wavelengths (e.g., over 6 dB weaker than at
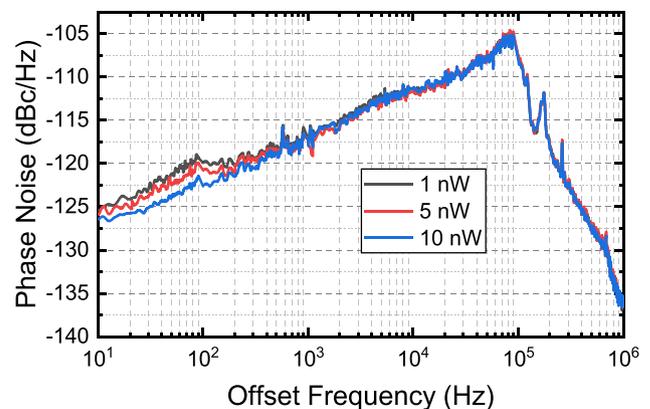

Fig. 4. Phase noise of beat signal between two tunable lasers with powers of 1 mW and 1–10 nW, respectively.

Fig. 5. Phase noise of the beat signal between a tunable laser (power of 1 mW) and OFC with per-tone power of 1–10 nW at the wavelength of 1550 nm.
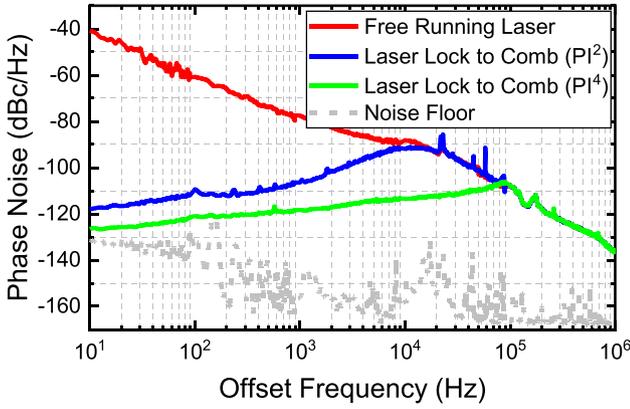
Fig. 6. Phase noise of the beat signal between a tunable laser (power of 1 mW) and OFC with per-tone power of 1 nW at the wavelength of 1550 nm for free-running laser (blue), when phaselocked using one double-integrator (PI², blue), and using two double-integrators (PI⁴, green). Measurement noise floor is also shown (grey, dashed).

Table 1
Optimum parameters for single PI^2.

Quantity	Value
Proportional gain, k_p	3.1
Integator gain k_i	2.14×10^3
2nd Integrator gain k_{ii}	3.66×10^2

Table 2
Optimum parameters for two PI^2 (PI^4).

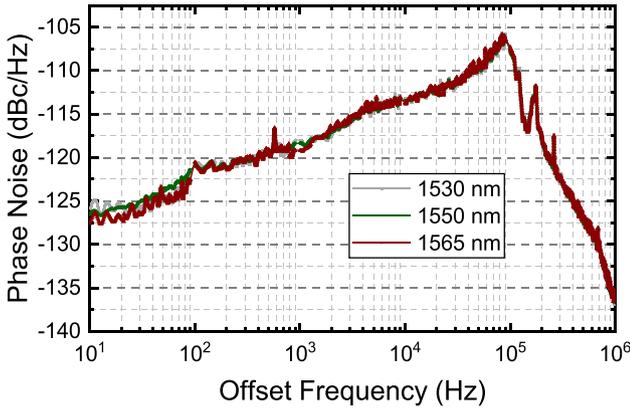| Quantity	First PI^2	Second PI^2
Proportional gain, k_p	1.6	1.5
Integator gain, k_i	3.2×10^4	2.58×10^4
2nd Integrator gain k_{ii}	1.02×10^3	5.42×10^3

Fig. 7. Phase Noise of beat signal of locking tunable laser to the OFC at different wavelengths with relevant OFC power set to 1 nW.

1575 nm, Fig. 3). This indicates the robustness of the technique against non-uniformity in the optical frequency comb power throughout its spectrum.

We calculated phase noise jitter by integrating the phase noise measured with the PI² and PI⁴ controllers, as shown in Fig. 8. Integration over the entire measurement bandwidth (10 Hz – 1 MHz) yielded 35 mrad for PI² and 10 mrad for PI⁴, respectively. Considering limited bandwidth constraints and low-cost electronics, this demonstrates commendable performance, with jitter exceeding 100 times lower than
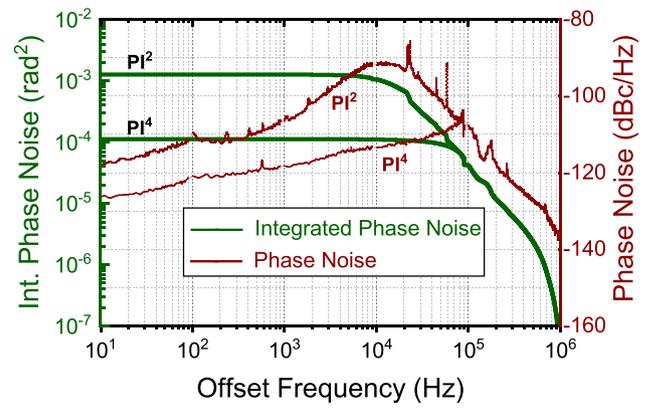

Fig. 8. Phase noise jitter derived from the measured phase noise of OFC-locked tunable laser (with per-tone power of 1 nW) when locked using PI² and PI⁴ feedback controller.

reported in [7].

3.2. Long-term stability

Long-term stability was assessed over observation times exceeding 100 μs or 1 s, corresponding to frequencies below 10 kHz and 1 Hz, respectively, using a frequency counter (Keysight 53230A). Initially, we assessed frequency error over time with a sampling interval set to 1 s. This measurement was conducted over 10 h to demonstrate robust and stable locking. Fig. 9 presents the results for both PI² and PI⁴ control. The peak-to-peak frequency error remained within ±0.01 Hz for PI⁴, representing an improvement of approximately five times compared to PI², indicating a substantial advantage of PI⁴ over extended observation periods. The performance attained is notable, exhibiting a frequency variation that is 100 times lower (with PI⁴) compared to the results presented in [9].

In order to acquire Alan Deviation data over average times of 100 μs to 1000 s, we also computed the Alan Deviation from the data shown in Fig. 9 and supplemented them with data taken at a shorter gating time (4 μs) (Fig. 11). The slope of the Allan deviation is −1 between 100 μs and 100 ms. The slope decreases with longer observation intervals, most likely due to low-frequency drift in our configuration. Crucially, the PI4 result is almost an order of magnitude better than the PI2 result for all average times. The stability with PI4 at 1 s averaging time is 2×10^{-14}.

4. Discussion

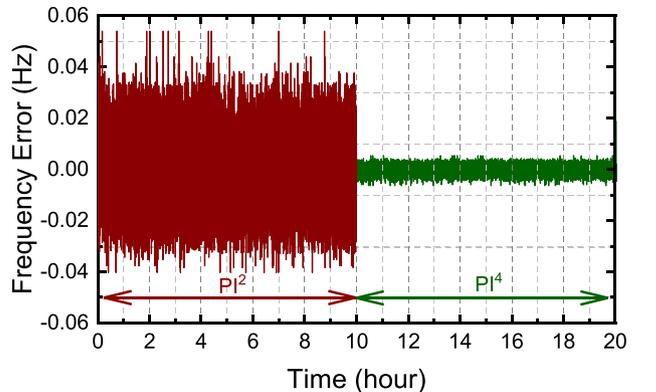The main focus of our work was the performance characterization of

Fig. 9. Frequency error of OFC-laser beat signal with one double-integrator (PI²) and two double-integrators (PI⁴) using 1 s gate time, measured over 10 h.

a single-tuneable laser locked to the OFC. As our vision, in practice, more lasers might be locked to the same OFC by passively splitting it. RF signals can be produced by locking two lasers to distinct regions of the OFC spectrum and then photomixing them [10]. This can produce signals up to 4.5 THz when using C-band tuneable lasers or 9 THz when combining one C-band and one L-band tunable laser. The main benefit of OFC-locking is that the resultant beat signal has far less phase noise compared to beats produced by two separate lasers. Let's calculate the performance that such a configuration can achieve. The fractional frequency stability of this locking may be computed by re-normalizing the data in Fig. 10 to the frequency spacing between the two lasers and multiplying by $\sqrt{2}$ to account for the two lasers. Each laser is locked to its corresponding comb tone. Fig. 11 displays the outcome for averaging time of 1 s with varying laser spacing. As anticipated, as the frequency difference increases but the locking stability stays constant, the fractional frequency instability improves with the frequency gap between the lasers.

Additionally, we plot our OFC's fractional frequency instability at an average period of 1 s, limited by its RF reference (Timetech 5.10) of 5×10^{-13}. Since the stability of the two laser beatings is less than that of our OFC, we can conclude that we are not constrained by it. The beating would be limited by the OFC rather than our locking, though, if the OFC had a lower-quality RF reference (for example, instability in the 10^{-11} level at 1 s). It is anticipated that the OFC performance will usually be the limiting factor because this is the level of RF reference found in typical crystal oscillators (as opposed to our laboratory-grate high-quality RF reference). Our laser phase locking will contribute less to the overall stability, demonstrating a more than sufficient level of phase locking performance.

We also include fractional frequency instability from additional sources (derived from [11]) in Fig. 11 for comparison purposes. High-quality OFCs produce superior results [11]. However, our method exhibits fractional frequency instability up to two orders of magnitude better for frequencies over 0.1 THz than photo mixing two CW lasers [12], [13], [14],. Additionally, it produces results that are comparable to those of photo mixing with a micro comb [15,16] or a quantum cascade laser [17], [18], [19],. However, neither of these methods has the tunability that our method offers.

5. Conclusion

We present a practical tunable comb-locked laser system that uses minimum optical hardware and telecom-grade components including tunable lasers and low-bandwidth electronics. Since it has no specific criteria for OFC spectral flatness or tone spacing, it could be used with various OFC generators.

The phase locking feedback architecture controls the limited bandwidth of the feedback electronics and the comparatively low stability of tuneable telecom lasers (compared to laboratory-grade single-frequency lasers). In contrast to the commonly utilized single double-integrator proportional control (PI2), we demonstrate the advantages of using two double-integrators with proportional gain (PI4). Specifically, the Alan deviation at 1 s integration time was 2×10^{-14}, and the peak-to-peak frequency error over 10 h was cut five times (to ± 0.01 Hz). With a five-fold increase in loop bandwidth, phase noise improved by 10^{-25} dB over the 10 Hz – 50 kHz frequency range for shorter observation intervals.

Our work goes beyond noise improvement by introducing a new control strategy (PI⁴), ultra-low power operation, hardware simplification, scalability for multi-laser locking, and superior long-term stability, making it highly practical for THz generation and DWDM applications.

CRediT authorship contribution statement

Win Indra: Writing – original draft, Conceptualization. **Irianto:** Writing – review & editing, Visualization, Data curation. **Jamil**
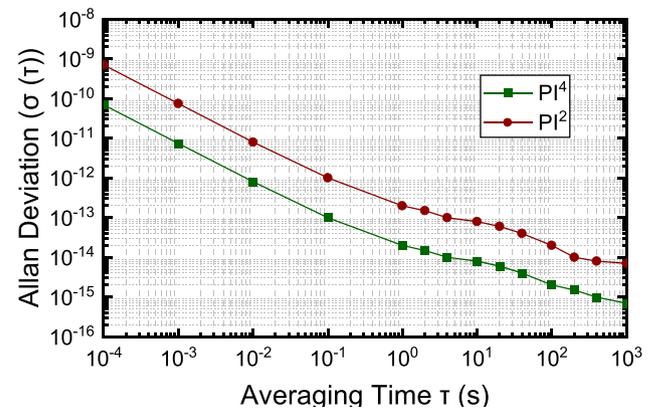

Fig. 10. Allan deviation calculated from the frequency counter data when a tunable laser is locked to the OFC using two double-integrators (PI⁴) and one double-integrator (PI²), normalized to the laser carrier frequency (192 THz).
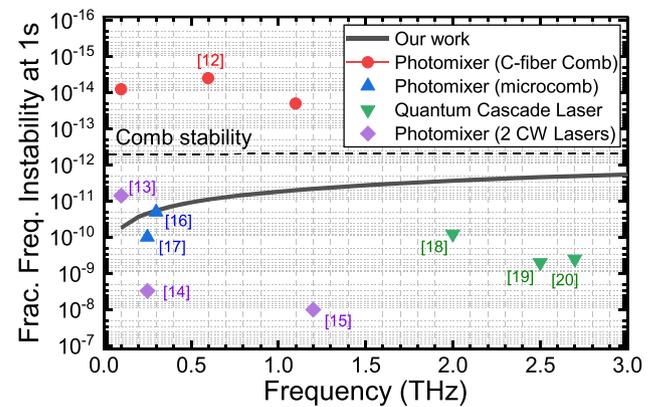

Fig. 11. Fractional frequency instability of the beat signal at 1 s averaging times for various beat frequencies expected from our system and its comparison with the state-of-the art terahertz sources (adopted from [11]).

Abedalrahim Jamil Alsayaydeh: Formal analysis. **Adam Wong Yoon Khang:** Methodology. **Nurulhalim Bin Hassim:** Software. **Safarudin Gazali Herawan:** Writing – review & editing, Validation.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Win Indra reports financial support, article publishing charges, and equipment, drugs, or supplies were provided by Indonesian Endowment Fund for Education (LPDP). If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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