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ABSTRACT

The study of biomagnetic nanofluids has gained significant attention due to their applications in biomedical engineering and thermal management systems, where precise control of heat and fluid flow is crucial. This research investigates the coupled effects of a magnetic dipole, thermal radiation, and copper nanoparticles on biomagnetic nanofluid flow over a stretching sheet to analyze their impact on velocity and temperature distribution. The governing equations are solved numerically using MATLAB's bvp4c solver over the computational domain $[0, 20]$, and results are validated against benchmark studies to ensure accuracy. Findings reveal that increasing nanoparticle volume fraction enhances thermal conductivity but reduces velocity due to increased viscosity, while stronger ferromagnetic interactions intensify localized heating, significantly altering temperature gradients. Additionally, streamline analysis illustrates the magnetic field's influence on flow structures, and surface plots provide a comprehensive visualization of heat dissipation within the nanofluid. The study also highlights the role of viscous dissipation and Prandtl number in thermal regulation, offering insights applicable to magnetic hyperthermia treatments, targeted drug delivery, and advanced cooling technologies.

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1 Introduction

The study of heat and flow transfer in biomagnetic nanofluids has gained significant attention due to its applications in biomedical engineering, industrial cooling, and energy systems. Biomagnetic fluids, such as blood containing magnetic nanoparticles, exhibit unique properties when subjected to external magnetic fields, leading to enhanced heat transfer and controlled fluid motion. The incorporation of copper nanoparticles further improves their thermal conductivity and overall transport characteristics. In recent years, researchers have extensively explored the effects of magnetic dipoles and thermal radiation on such nanofluids in various geometries and boundary conditions.

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Nasir et al. [1] numerically examined thermo-magnetic nano-blood flow under the influence of non-Darcy effects, chemical reactions, and Joule dissipation, demonstrating significant thermal enhancement due to SWCNT (Single-walled carbon nanotubes)/MWCNT (Multi-walled carbon nanotube) doping. Similarly, Islam et al. [2] analyzed Sisko fluid behavior over a stretching device and found that radiation and heat dissipation strongly impact convective transport. The biomedical and bioconvection applications of magnetic nanoparticles have been reviewed by Zainal et al. [3], emphasizing their role in hyperthermia therapy, drug delivery, and imaging technologies. Several researchers have focused on the combined effects of chemical reactions, variable thermal conductivity, and magnetic dipole interactions in different fluid systems. Vaidya et al. [4] studied Phan-Thien-Tanner peristaltic flow and observed that chemical reactions and MHD (magnetohydrodynamic) interactions significantly alter heat transfer rates. Murtaza et al. [5] numerically investigated biomagnetic fluid flow with copper nanoparticles, revealing that thermal radiation and magnetic dipole effects play a critical role in boundary layer behavior. Alam et al. [6] extended this work by applying group theoretical methods to simulate biomagnetic nanofluids over different particle shapes, highlighting the influence of thermal radiation on heat transfer performance. Jumana et al. [7] examined biomagnetic fluid motion over a moving sheet, demonstrating how particle spacing and magnetic dipole intensity affect flow characteristics. Further, Ferdows et al. [8] analyzed the impact of magnetic particle diameter and found that larger particles exhibit higher magnetic responsiveness, leading to better flow control. Jumana et al. [9] explored a dual solution of convective biomagnetic fluid considering wall transpiration and magnetization past a permeable moving flat plate.

Recent studies have also incorporated porous media effects, Joule heating, and non-Darcy interactions in biomagnetic nanofluid flow [10]. Alam et al. [11] investigated biomagnetic flow over a stretching cylinder with variable fluid properties, showing that viscosity variations significantly influence heat transfer. Studies by Ferdows et al. [12,13] further demonstrated that prescribed heat flux conditions impact boundary layer characteristics when magnetic dipoles are present. The influence of unsteady flow conditions has been analyzed by multiple researchers. Islam et al. [14] studied unsteady ferrofluid slip flow, revealing that convective boundary conditions alter velocity and temperature gradients. Murtaza et al. [15,16] applied Lie group analysis to model thin needle biomagnetic fluid flow, showing how magnetohydrodynamic (MHD) and ferrohydrodynamic interactions affect the system. Further, Mousavi et al. [17] investigated three-dimensional biomagnetic flows in non-uniform magnetic field on biomagnetic fluid flow. Kai et al. [18] explored hybrid nanofluid flow under dipole-induced vortices, demonstrating that such magnetic interactions alter convective heat transfer rates. Alam et al. [19] simulated $CoFe_2O_4$-blood nanofluid flow in rotating stretchable cylinders, highlighting complex thermal behavior under strong magnetic fields. Kamis et al. [20] numerically analyzed hybrid ferrofluid convection over inclined stretching sheets, showing that magnetic dipole effects significantly modify flow stability.

Several studies have focused on the swirling effects of magnetic particles in biomagnetic conditions. Alam et al. [21] examined swirling stretchable cylinders, revealing that exponentially stretching boundary conditions impact flow resistance and energy dissipation. Padma et al. [22] studied double-diffusive convection in ferromagnetic Carreau nanofluids for solar thermal applications, showing that magnetic dipoles influence buoyancy-driven flows. Kasiman et al. [23] applied finite element methods to model biomagnetic fluids in rectangular channels, demonstrating the importance of accurate numerical techniques in predicting transport behaviors. Allahyani et al. [24] investigated stretchable surface interactions in biomagnetic nanofluids, finding that heat generation and magnetic dipole effects are dominant at higher Reynolds numbers. Priyadharsini et al. [25] analyzed magnetic dipole effects in radiative MHD blood flow, emphasizing biomedical applications. Murtaza et al. [26] focused on three-dimensional biomagnetic flow in stretching/shrinking sheets, reporting that temperature-dependent magnetization significantly modifies the velocity field. Hybrid ferrofluid flows have also been examined for curved geometries, with Alam et al. [27] employing entropy optimization techniques to improve energy efficiency. Ahmad et al. [28] studied Jeffery slip conditions and melting effects, revealing that surface roughness alters flow stability in magnetic nanofluids. Jakeer et al. [29] applied biomagnetic nanofluid models to human circulatory flow, demonstrating potential medical applications. Advanced thermal case studies of dipole-induced vortex generation have been conducted by Kamran et al. [30], highlighting heat transfer enhancements in hybrid nanofluids. Alsenafi et al. [31] examined thin needle biomagnetic flow under stretching and shrinking conditions, providing insights into ferrohydrodynamic stability. Wang et al. [32] simulated Casson nanofluid flows over shrinking sheets, demonstrating multi-solution stability in biomagnetic fluids.

Further studies on entropy generation in biomagnetic nanofluids have shown that thermal radiation and cross-diffusion effects significantly impact energy dissipation [33]. Dong et al. [34] analyzed magnetoelectric sensors effects, optimizing biomagnetic fluid transport for energy-efficient applications. Recent work by Adnan et al. [35] has explored tetra-nanofluid thermal behavior using advanced numerical techniques. The role of nonlinear thermal radiation in MHD Casson fluids has been investigated in several studies, including those by Anantha Kumar et al. [36], who examined chemical reaction effects on stretching surfaces. Several other researchers analyzed nanoparticle thermal properties in hybrid nanofluids, providing key insights into radiative heat transfer in stretching sheet applications [37–40]. Incorporating the influence of temperature-dependent viscosity in biomagnetic fluid flow, as discussed by Ellahi [41], along with the enhanced heat transfer characteristics of hybrid nanoparticles analyzed by Zeeshan et al. [42], this study explores the coupled magnetothermal effects in biomagnetic fluid flow over a stretched sheet with copper nanoparticles.

To further strengthen the theoretical foundation and relevance of biomagnetic nanofluid studies, it is essential to consider key developments in nonlinear stretching flows, ferrohydrodynamics, and magnetic field interactions. Recent works highlight the role of nonlinear surface stretching and unsteady flow effects in applications like electrospinning and atomization, emphasizing their influence on fluid stability and heat transfer characteristics [43,44]. The fundamental principles of ferrohydrodynamics (FHD) dictate that magnetic field variations significantly alter fluid motion, yet many existing models simplify these effects, overlooking the complexities introduced by magnetic dipoles [45]. Studies on localized magnetic fields reveal their strong impact on convective heat transfer, demonstrating the necessity of incorporating dipole-induced variations for more accurate biomagnetic flow modeling [46]. Furthermore, classical electrodynamics suggests that spatially varying magnetic fields create nonlinear force distributions, influencing stability and flow structure [47]. Recent investigations into biomagnetic nanofluids with gold nanoparticles have demonstrated enhanced thermal transport, yet the potential of copper nanoparticles in ferrohydrodynamic environments remains underexplored [48].

This study addresses a critical research gap by investigating the simultaneous effects of a magnetic dipole, thermal radiation, and nanoparticle volume fraction on the heat and flow transfer characteristics of biomagnetic nanofluids over an extended sheet. While previous studies have explored these effects individually or in limited combinations, the combined influence of these parameters on velocity, temperature, and energy transport has not been comprehensively analyzed. Furthermore, most existing studies have relied on 2D profiles, neglecting detailed flow visualization techniques such as surface plots and streamlines, which are crucial for capturing the spatial variations and fluid dynamics behavior in biomagnetic nanoflows. To bridge this gap, the present study employs surface plots to illustrate temperature variations and streamline plots to visualize flow patterns, providing a more in-depth understanding of the system's behavior. Additionally, most prior numerical studies have been conducted over a limited computational domain, often restricting the analysis to smaller regions. In contrast, this study extends the domain to [0, 20], ensuring that the asymptotic behavior of velocity and temperature profiles is accurately captured, leading to more realistic and reliable predictions. The insights gained from this research have potential applications in biomedical engineering, including targeted drug delivery and hyperthermia treatments, as well as in thermal energy storage and advanced cooling systems, where precise control of heat and mass transfer in biomagnetic nanofluids is essential.

2 Articulation of the Problem

The study considers the steady, two-dimensional boundary layer flow of a biomagnetic nanofluid containing copper nanoparticles over a stretching/shrinking sheet. The flow is subjected to magnetic dipole and thermal radiation, and the governing principles are derived from ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD). The coordinate system is defined as such that the x-axis is aligned with the sheet, while the y-axis extends perpendicular to it. A magnetic dipole is located at a fixed distance from the surface, influencing the flow and thermal fields.

Governing Equations

Under the assumptions of laminar flow, incompressibility, and boundary layer approximations, the governing continuity, momentum, and energy equations are expressed as [5,40]:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_{nf}}\mu_0 M\frac{\partial H}{\partial x} \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \mu_0\left(\rho Cp\right)_{nf} T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho Cp)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

where u, v are the velocity components along x and y directions, μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the effective nanofluid density, M represents the magnetization, and H denotes the applied magnetic field strength.

Boundary Conditions

The governing equations are subjected to the following boundary conditions at the sheet surface ($y = 0$) and far from it ($y \to \infty$):

$$u = cx^m + U_{slip}, \quad v = v_w, \quad T = T_w = T_\infty + c_1 x^n, \quad at \quad y = 0 \tag{4}$$

$$u \to u_e = c_\infty x^m, \quad T \to T_\infty, \quad as \quad y \to \infty \tag{5}$$

where c is a stretching/shrinking parameter, U_{slip} represents velocity slip at the surface, v_w is the suction/injection velocity, T_w is the wall temperature, and T_∞ is the ambient temperature.

The boundary conditions in Eqs. (4) and (5) define the non-linear velocity and temperature behavior at the stretching/shrinking sheet and in the far-field region, capturing realistic physical effects. At the surface ($y = 0$), the velocity follows a non-linear stretching profile, $u = cx^m + U_{slip}$ where m determines the rate of stretching, and U_{slip} accounts for velocity slip, which is significant in microfluidic flows, rarefied gas dynamics, and polymer extrusion processes. The normal velocity $v = v_w$ represents suction ($v_w > 0$), which stabilizes the boundary layer and enhances heat transfer, or injection ($v_w < 0$), which can induce flow separation and instability. The temperature boundary condition, $T = T_\infty + c_1 x^n$, models a non-uniform surface temperature distribution, relevant in industrial processes like continuous casting and aerodynamic heating, where temperature gradients influence material properties. As $y \to \infty$, the velocity approaches $u_\infty = c_\infty x^m$, ensuring the free-stream flow follows a power-law distribution, while the temperature gradually reaches T_∞, maintaining thermal equilibrium. These non-linear boundary conditions align with studies on similarity flows and unsteady stretching sheets [43,44], emphasizing their importance in applications such as electrospinning, nanofluid heat transfer, and biomagnetic fluid transport, where precise control over velocity and temperature profiles is crucial for optimizing performance and efficiency.

Nanofluid Properties

In this study, copper nanoparticles with an average size of 25 nm are considered, ensuring optimal thermal conductivity and stable dispersion within the base fluid. This size selection aligns with previous studies on nanofluid heat transfer applications. The effective thermophysical properties of the nanofluid, incorporating the effects of nanoparticle volume fraction ϕ, are given in Table 1.

Table 1: Effective properties of nanofluids

Effective viscosity	$\mu_{nf} = \frac{\mu_f}{(1-\phi)\,2.5}$
Thermal diffusivity	$\alpha_{nf} = \frac{k_{nf}}{(\rho Cp)_{nf}}$
Effective density	$\rho_{nf} = (1-\phi)\,\rho_f + \phi\rho_s$
Thermal conductivity	$k_{nf} = k_f \frac{[k_s + 2k_f - 2\phi\left(k_f - k_s\right)}{k_s + 2k_f + 2\phi\left(k_f + k_s\right)}$
Magnetization [46,47]	$M = K\,(T_w - T)$

Where k_{nf} is the nanofluid thermal conductivity, ρ_s and k_s are the density and thermal conductivity of nanoparticles, respectively, and K is the pyromagnetic coefficient. In the magnetization equation, the magnetic field is considered a linear function of temperature because of the temperature-dependent magnetization of the fluid. This relationship is widely accepted in biomagnetic and ferrofluid research, especially with small to moderate temperature variations. This linear approximation provides a computationally efficient and physically reasonable model for analyzing magnetothermal effects in biomagnetic fluid flow.

Furthermore, the addition of nanoparticles significantly influences the fluid flow by altering its effective viscosity, thermal diffusivity, effective density, thermal conductivity, and magnetization, thereby modifying both momentum and heat transfer characteristics. The presence of nanoparticles increases the effective viscosity, introducing additional resistance to fluid motion, which can reduce velocity near the surface while stabilizing the boundary layer. Simultaneously, the effective density of the nanofluid increases due to the higher density of nanoparticles compared to the base fluid, affecting inertia and momentum transport. The thermal diffusivity is enhanced, allowing faster heat distribution within the fluid, which is particularly important in thermal management applications. The thermal conductivity of the nanofluid is significantly improved, facilitating greater heat transfer rates and reducing temperature gradients, which enhances overall energy efficiency. Additionally, in the presence of a magnetic field, the magnetization of the nanofluid plays a crucial role, as the magnetic nanoparticles respond to the applied field, altering velocity and temperature distributions through ferrohydrodynamic interactions. These combined effects make nanofluids highly effective in applications requiring precise control overheat and fluid flow, such as biomedical cooling, hyperthermia treatments, and industrial heat exchangers.

Magnetic Field Distribution

In this study, a magnetic dipole is placed at a fixed location $(0, -d)$ below the stretching sheet, which generates the magnetic field. The dipole is modeled as a point source of the magnetic field, and its influence on the fluid domain is described using the magnetic scalar potential function. Under this assumption, the magnetic potential function Ψ is given by [47]:

$$\Psi = \frac{\gamma}{2\pi}\frac{1}{x^2 + (y+d)^2} \tag{6}$$

where γ represents the magnetic field strength and d is the distance of the dipole from the surface.

Taking the negative gradient of this potential function yields the magnetic field components in the x- and y-directions, as characterized by the following field expressions:

$$\frac{\partial H}{\partial x} = \frac{\gamma}{2\pi}\left(\frac{-2x}{(y+d)^4}\right), \quad \frac{\partial H}{\partial y} = \frac{\gamma}{2\pi}\left[\frac{-2}{(y+d)^3} + \frac{4x^2}{(y+d)^5}\right] \tag{7}$$

Non-Dimensional Transformation

To simplify the problem, the following dimensionless variables are introduced:

$$\eta = \left(\frac{(m+1)\,u_w(x)}{2\nu_f x}\right)^{1/2} y, \quad \psi = \left(\frac{2\nu_f x u_w(x)}{m+1}\right)^{\frac{1}{2}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

where η is the similarity variable, ψ is the stream function, and θ is the dimensionless temperature.

Applying these transformations, the governing equations reduce to:

Dimensionless Momentum Equation

$$\frac{1}{(1-\phi)^{2.5}} f''' + \left[(1-\phi) + \phi\rho_s\rho_f\right]\left(ff'' - \left(f'^2 - A^2\right)\right) - 2\beta\theta\,(\eta+\alpha)^4 = 0 \tag{9}$$

Dimensionless Energy Equation

$$\left(\frac{k_{nf}}{k_f} + Nr\right)\theta'' + \left[(1-\phi) + \phi\,(\rho Cp)_s\,(\rho Cp)_f\right] Pr\,(f\theta' - nf'\theta) - \frac{2\beta\lambda\,(\epsilon+\theta)}{(\eta+\alpha)^3} f = 0 \tag{10}$$

Dimensionless Boundary Conditions

$$f(0) = S, \quad f'(0) = 1 + \lambda 1 f''(0), \quad \theta(0) = 1, \quad f'(\infty) = A, \quad \theta(\infty) = 0 \tag{11}$$

Dimensionless Parameters

The dimensionless parameters that emerged from the non-dimensionalization process are listed in Table 2.

Table 2: Non-dimensional parameters

Non-dimensional parameter	Symbol	Expression
Dimensionless distance	α	$d\sqrt{\frac{c}{v_f}}$
Thermal radiation parameter	Nr	$\frac{16\sigma^* T_\infty{}^3}{3k^* k_f}$
Prandtl number	Pr	$\frac{(\mu c_p)_f}{k_f}$
Ferromagnetic parameter	β	$\frac{\gamma}{2\pi}\frac{\mu_0 k\,(T_c - T_w)\,\rho_f}{\mu_f{}^2}$
Suction/Injection parameter	S	$-v_w x^{-(m-1)/2}\sqrt{\frac{2}{c\,(m+1)\,v_f}}$
Viscous dissipation parameter	λ	$\frac{c\mu_f{}^2}{\rho_f k_f\,(T_c - T_w)}$

Engineered Parameters

To enhance the quantitative analysis, the skin friction coefficient and Nusselt number are evaluated to investigate the frictional resistance and heat transfer characteristics of the biomagnetic nanofluid. The skin friction coefficient C_f is defined as:

$$C_f = \frac{\tau_w}{\rho U^2} = Re_x^{-1/2} f''(0) \tag{12}$$

where $f''(0)$ represents the dimensionless wall shear stress, and Re_x is the local Reynolds number given by $Re_x = \frac{Ux}{\nu}$.

The local Nusselt number Nu_x, which characterizes the heat transfer rate, is given by:

$$Nu_x = -Re_x^{1/2}\theta'(0) \tag{13}$$

where $\theta'(0)$ represents the dimensionless temperature gradient at the surface.

These parameters provide insights into the effects of nanoparticle volume fraction, magnetic field strength, and thermal radiation on frictional resistance and thermal performance. This mathematical model describes the coupled transport phenomena in biomagnetic nanofluid flow influenced by thermal radiation and a magnetic dipole. The dimensionless equations serve as the foundation for numerical solutions using MATLAB's bvp4c solver, which will be analyzed in the subsequent sections.

3 Method of the Solution

The governing equations are solved numerically using the *bvp4c* solver in MATLAB, which is specifically designed for solving boundary value problems involving coupled ordinary differential equations. The system of nonlinear differential equations obtained from the transformed governing equations is first converted into a system of first-order equations, and appropriate boundary conditions are imposed. The *bvp4c* solver is chosen due to its efficiency in handling two-point boundary value problems, ensuring smooth and stable solutions while capturing the asymptotic behavior of the velocity and temperature profiles.

To apply *bvp4c*, the third-order momentum equation and second-order energy equation are rewritten as a system of first-order equations. Defining new variables:

$$f' = g, g' = h, h' = -\frac{(1-\phi)^{2.5}\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)\left(fh - g^2 + A^2\right) - 2\beta\theta\,(\eta+\alpha)^4}{(1-\phi)^{2.5}} \tag{14}$$

$$\theta' = \varphi, \varphi' = -\frac{(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}Pr(f\varphi - ng\theta) - 2\beta\lambda(\epsilon+\theta)\,(\eta+\alpha)^3 f}{\left(\frac{k_{nf}}{k_f}+Nr\right)} \tag{15}$$

where $g = f'$, $h = f''$, $\theta' = \varphi$, and $\phi^- = \theta^{--}$. The boundary conditions in this transformed form are:

$$f\,(0) = S, \quad g\,(0) = 1 + \lambda_1 h\,(0)\,, \quad \theta\,(0) = 1, \quad g\,(\infty) = A, \quad \theta\,(\infty) = 0 \tag{16}$$

One of the key advantages of using bvp4c is its adaptive mesh refinement, which allows for high accuracy while minimizing computational effort. Unlike shooting methods, which can struggle with highly nonlinear systems, bvp4c employs a continuation approach that iteratively adjusts the initial guess and refines the solution until the residuals meet a predefined tolerance. This method is particularly useful for solving fluid flow problems where the velocity and temperature fields exhibit complex behavior due to the combined effects of magnetization, thermal radiation, and nanoparticle volume fraction.

The computational domain is chosen as $0 \le \eta \le 200$ to ensure that the temperature profiles asymptotically approach their far-field boundary conditions without unnecessary computational expense. A fine initial mesh is used to capture critical flow and heat transfer variations near the boundary layer while maintaining numerical stability. The numerical results are validated by comparing solutions obtained with different mesh densities, confirming the accuracy and convergence of the solver. This approach enables efficient and reliable analysis of the impact of the magnetic dipole, thermal radiation, and nanofluid properties on the biomagnetic fluid flow over an extended sheet.

4 Results and Discussion

This study aims to investigate the coupled effects of a magnetic dipole, thermal radiation, and copper nanoparticles on biomagnetic nanofluid flow over a stretching sheet. The focus is on understanding how varying physical parameters influence velocity, temperature, and streamline structures, with applications in biomedical engineering and industrial heat transfer. A comprehensive analysis of the impact of various pertinent parameters (volume fraction of nanoparticles, viscous dissipation parameter, ferromagnetic parameter, Prandtl number, dimensionless distance) has been observed in the graphs presented in Figs. 1–9. The analysis is carried out by taking the numerical range of these parameters as: $0.01 \le \varnothing \le 0.3$; $0.01 \le \lambda \le 0.3$; $0.5 \le \alpha \le 2$; $3 \le \beta \le 9$; $10 \le Pr \le 25$ [5,11,48]. Copper nanoparticles are employed in this study, which have an average diameter of 25 nm.

This size is consistent with previous studies on nanofluid heat transfer applications. The values of various thermo-physical characteristics of blood and copper nanoparticles are explored in Table 3.

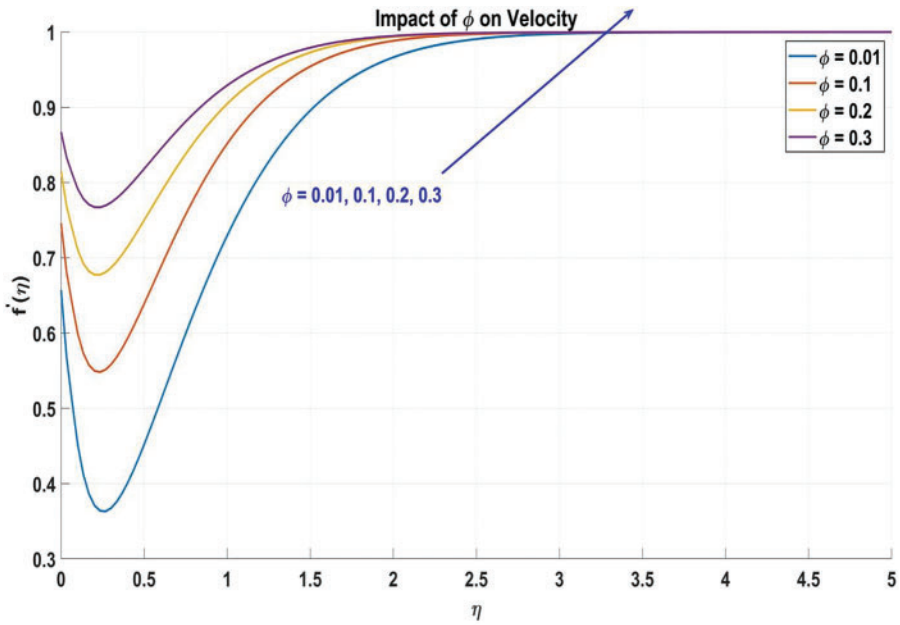

Figure 1: Volume fraction of nanoparticles vs. velocity of nanofluid

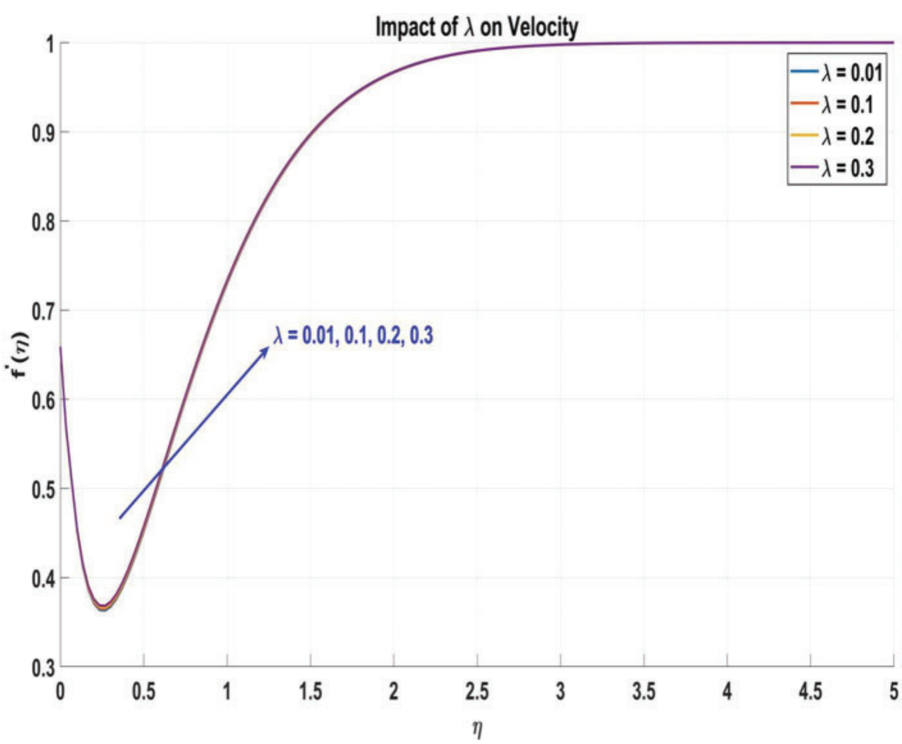

Figure 2: Viscous dissipation parameter vs. velocity of nanofluid

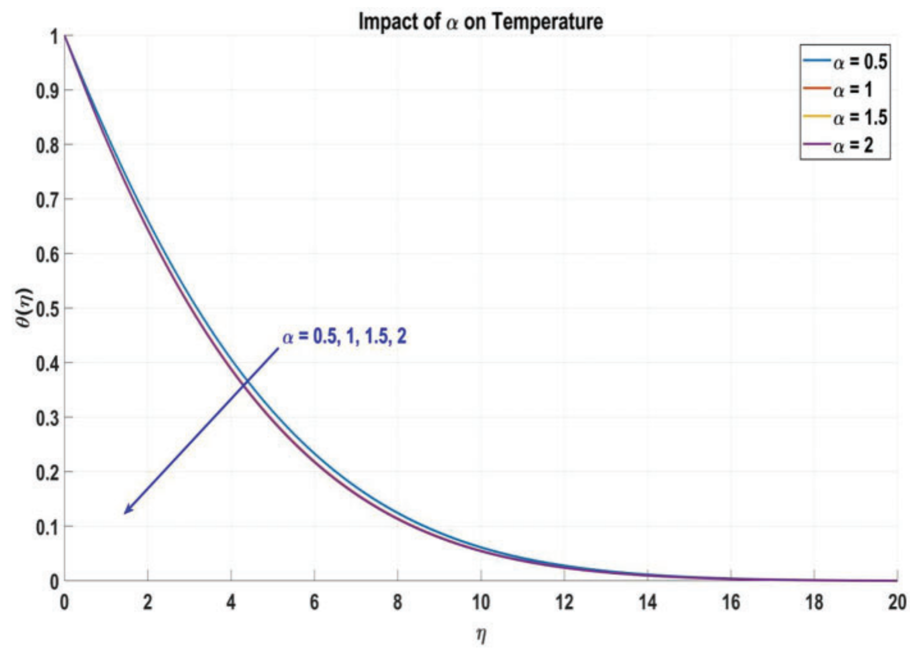

Figure 3: Dimensionless distance vs. temperature of nanofluid

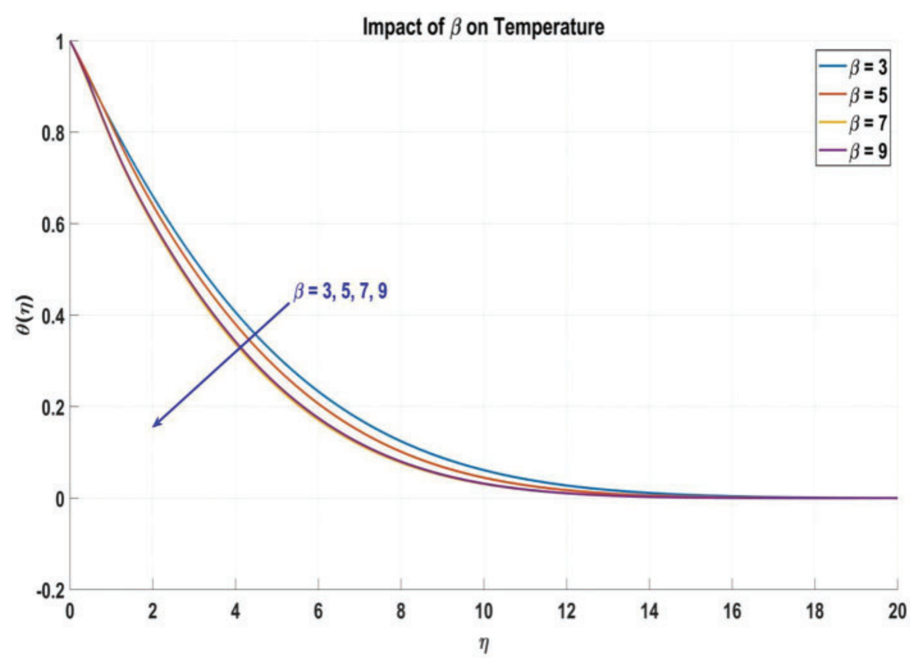

Figure 4: Ferromagnetic parameter vs. temperature of nanofluid

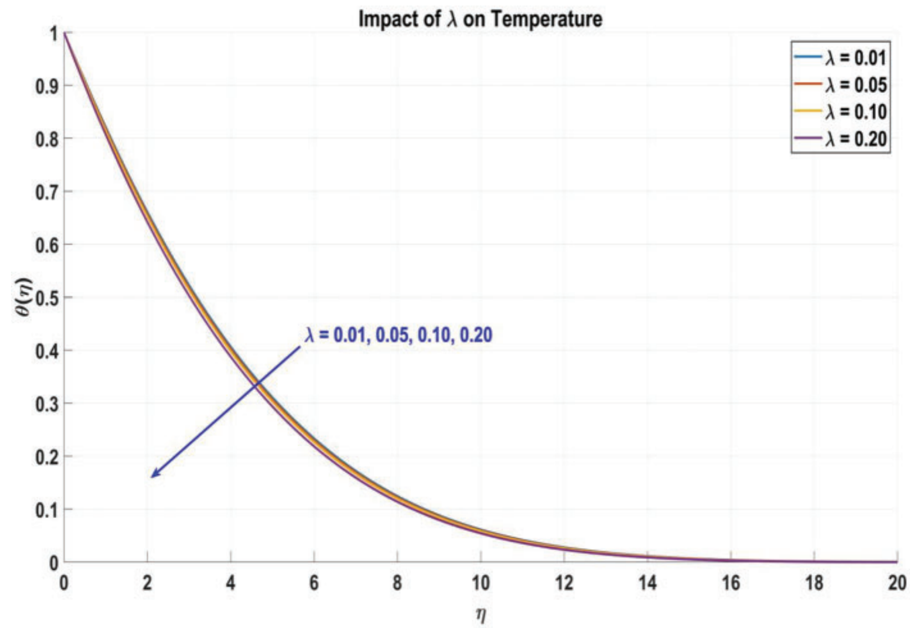

Figure 5: Viscous dissipation parameter vs. temperature of nanofluid

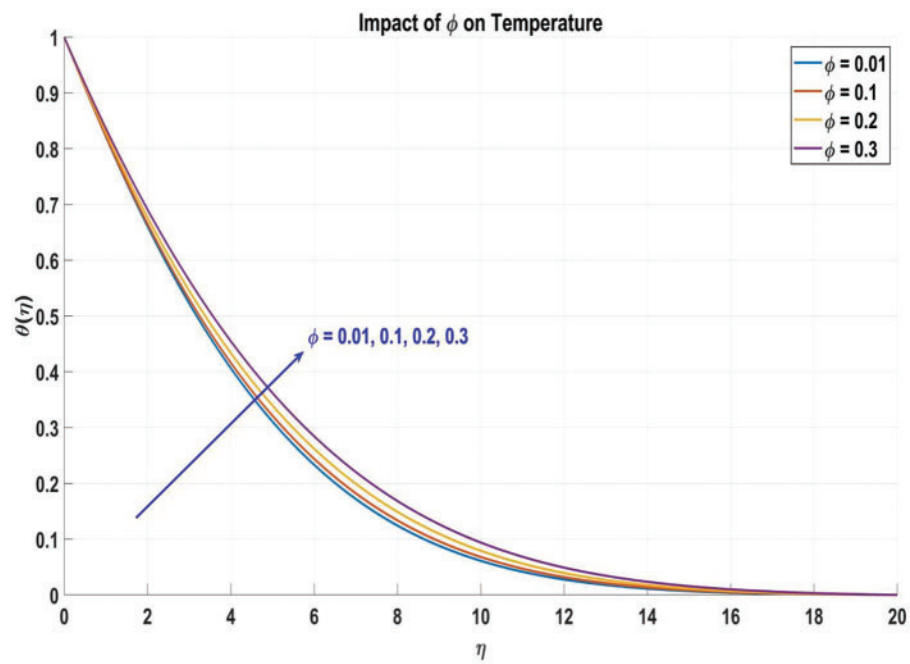

Figure 6: Volume fraction of nanoparticles vs. temperature of nanofluid

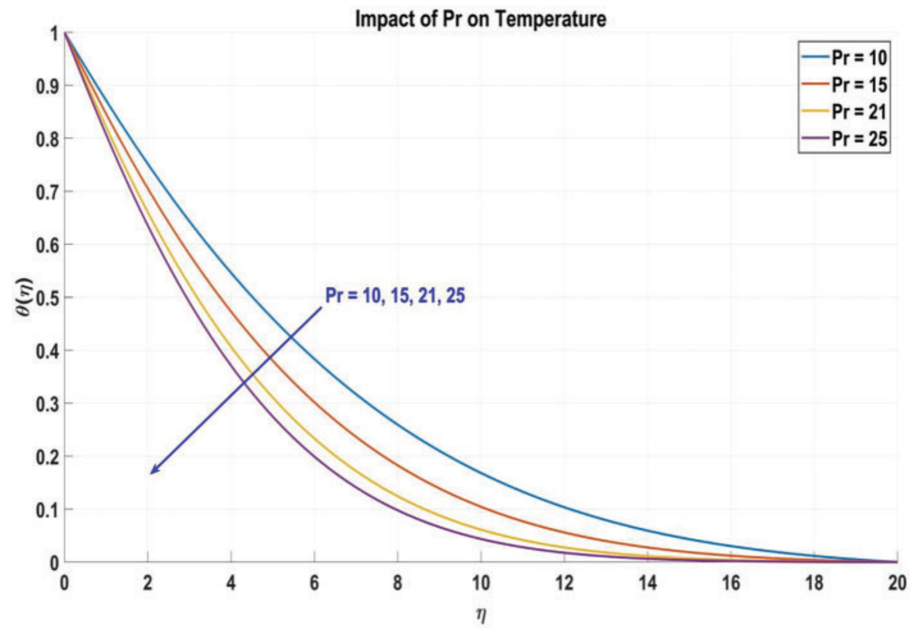

Figure 7: Prandtl number vs. temperature of nanofluid

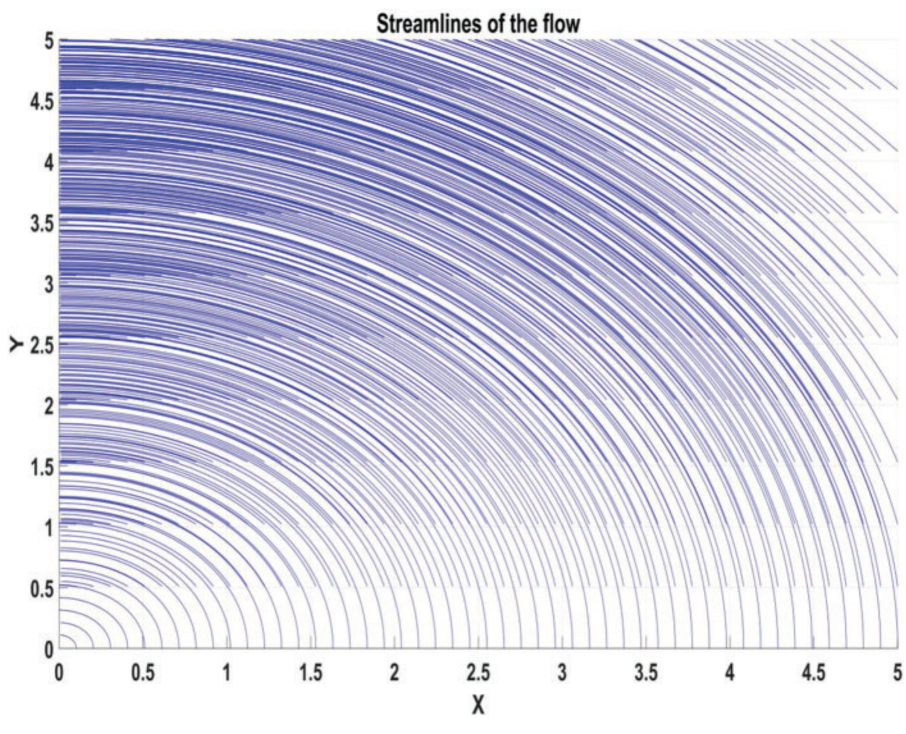

Figure 8: Streamlines of ferrofluid flow

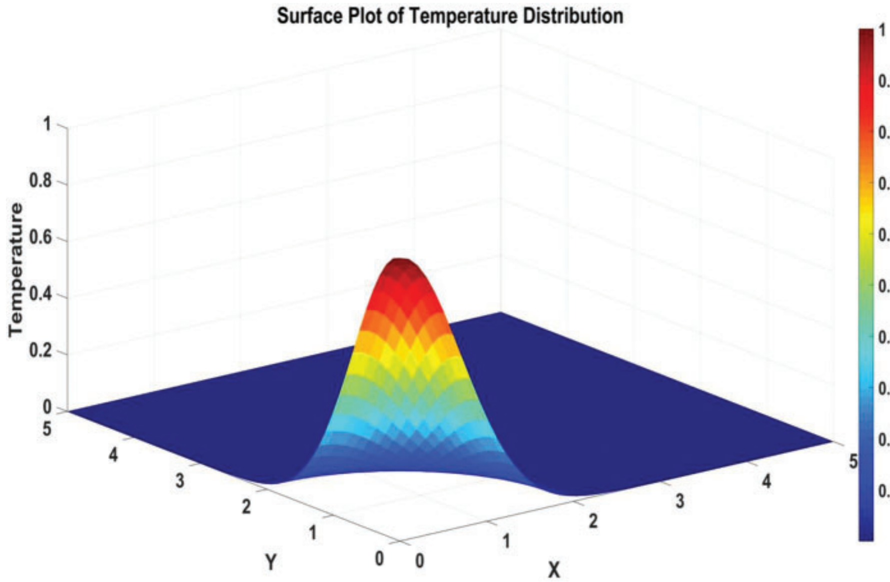

Figure 9: Surface plot for distribution of temperature

Table 3: Thermophysical properties of nanoparticles and human blood

	Density (ρ)	Specific heat (C_p)	Thermal conductivity (k)
Unit	kgm^{-3}	$Jkg^{-1}K^{-1}$	$WM^{-1}K^{-1}$
Copper nanoparticles	8933	385	400
Human blood	1053	3594	0.492

Fig. 1 reveals the impact of the volume fraction of nanoparticles on the velocity profile of the nanofluid. As the volume fraction of nanoparticles increases, the effective viscosity of the nanofluid rises due to the increased presence of solid particles in the base fluid, which enhances internal friction and introduces additional resistance to fluid motion. This increased resistance generally leads to a reduction in velocity near the surface of the stretching sheet, particularly in high-shear regions where momentum transfer is dominant. However, the presence of nanoparticles also improves the thermal conductivity of the nanofluid, which enhances heat dissipation and modifies temperature gradients. This, in turn, can influence buoyancy-driven secondary flows in cases where thermal convection effects are significant. Physically, this phenomenon plays a crucial role in biomedical applications, such as targeted drug delivery, where magnetic nanoparticles are introduced into blood flow to transport medication to specific locations. In such applications, controlling the velocity distribution ensures uniform dispersion and prevents particle aggregation, which is essential for maintaining treatment efficacy. Additionally, in magnetic hyperthermia therapy, where nanoparticles are used to generate localized heating for cancer treatment, optimizing their transport dynamics is crucial to achieving uniform heat distribution while avoiding excessive resistance to blood flow.

Fig. 2 illustrates the effect of the viscous dissipation parameter on the velocity profile of the nanofluid. Viscous dissipation refers to the conversion of mechanical energy into thermal energy due to internal friction within the fluid. As the viscous dissipation parameter increases, the energy lost due to internal fluid motion rises, which alters the momentum balance and increases local fluid heating. This effect becomes particularly significant in high-shear regions, where strong velocity gradients intensify frictional heating, leading to a more pronounced deceleration of the nanofluid.

Physically, this phenomenon is crucial in applications where fluid movement and heat generation are closely coupled, such as in biomedical cooling systems, hyperthermia treatments, and microfluidic devices. In magnetic hyperthermia therapy, where nanofluids containing magnetic nanoparticles are used to generate localized heating, excessive viscous dissipation could lead to unintended overheating, potentially damaging healthy tissues. Similarly, in nanofluid-based cooling systems, such as microchannel heat sinks or electronic cooling, excessive dissipation can reduce overall efficiency by increasing fluid temperature beyond the desired operating range. In biomagnetic fluid dynamics, where external magnetic fields influence nanoparticle-laden blood flow, understanding viscous dissipation effects helps in optimizing fluid transport to prevent excessive energy loss while ensuring effective heat regulation.

Fig. 3 presents the variation of temperature with a dimensionless distance from the surface of the stretching sheet. As the fluid moves away from the heated surface, thermal energy is gradually dissipated into the surrounding fluid, leading to a decline in temperature. The rate of this decline is strongly influenced by the thermal conductivity of the nanofluid, which determines how efficiently heat is transferred within the fluid. A higher thermal conductivity leads to a more gradual decline in temperature, while lower thermal conductivity results in a steeper temperature gradient. The imposed boundary conditions also play a crucial role, as they dictate the initial heat flux at the surface and the asymptotic behavior of the temperature far from the sheet. From a biomedical perspective, this temperature distribution is particularly significant in magnetic hyperthermia treatments, where magnetic nanoparticles are used to induce localized heating in cancerous tissues. In such applications, a controlled and predictable temperature decline is essential to ensure that heat remains concentrated in the targeted area while preventing unintended damage to surrounding healthy tissues. Similarly, in nanofluid-based heat exchangers used in industrial and electronic cooling systems, managing temperature gradients is crucial for optimizing thermal performance while minimizing excessive heat loss.

Fig. 4 explores the effect of the ferromagnetic parameter on the temperature profile of the nanofluid. When an external magnetic field is applied to the biomagnetic fluid, the magnetic nanoparticles experience a force that aligns them with the field lines, significantly influencing both the velocity and thermal fields. This alignment enhances the formation of structured flow regions, which can modify heat transport properties. As the ferromagnetic parameter increases, the interaction between magnetic forces and nanoparticle motion becomes stronger, resulting in enhanced heat conduction. This leads to a rise in temperature near the surface while altering the temperature gradient further into the fluid domain. From a physical perspective, this phenomenon arises due to the combination of magnetoconvection and ferrohydrodynamic interactions. The external magnetic field induces a magnetophoretic effect, which causes nanoparticles to migrate toward regions of high field intensity. This migration can increase local nanoparticle concentration, enhancing thermal conductivity in those regions and leading to a higher temperature near the surface. Additionally, the interaction between magnetic body forces and viscous resistance can modify the momentum boundary layer, indirectly affecting heat transfer rates. This effect has significant biomedical and industrial applications. In magnetic hyperthermia therapy, precise control over temperature distribution is crucial to target cancer cells while avoiding overheating of healthy tissues. An optimal ferromagnetic parameter ensures that heat generation is localized, improving treatment efficacy.

Fig. 5 investigates the influence of viscous dissipation on the temperature profile of the nanofluid. Viscous dissipation refers to the internal generation of heat caused by frictional forces within the fluid, where kinetic energy is converted into thermal energy due to the continuous deformation of fluid layers. As the viscous dissipation parameter increases, more energy is absorbed by the fluid, leading to a significant rise in temperature. This effect is particularly pronounced near the stretching sheet, where the shear stress is highest, resulting in stronger energy dissipation and localized heating. From a physical perspective, this phenomenon occurs because higher viscosity and velocity gradients amplify energy dissipation, increasing thermal energy accumulation within the boundary layer. The heat generated due to viscous dissipation competes with conductive heat transfer, meaning that if dissipation is strong, it can dominate the heat balance, leading to higher overall temperatures. In biomagnetic and industrial systems, controlling viscous dissipation is crucial to preventing overheating and thermal instability. This effect is highly relevant in high-shear-rate applications, such as blood flow through narrow arteries, where increased viscosity and frictional heating can elevate local blood temperature, potentially affecting tissue viability. Similarly, in industrial fluid transport, excessive viscous dissipation can cause undesirable heating, reducing efficiency in cooling systems and thermal management devices.

Fig. 6 illustrates the impact of the volume fraction of nanoparticles on the temperature distribution within the nanofluid. Since nanoparticles generally possess higher thermal conductivity than the base fluid, an increase in their volume fraction enhances the effective thermal conductivity of the nanofluid, leading to improved heat transfer capabilities. As a result, the temperature profile within the fluid rises, as heat is conducted more efficiently from the heated surface into the fluid domain. However, when the nanoparticle concentration becomes excessively high, particle agglomeration may occur due to van der Waals forces, leading to an increase in viscosity and a reduction in thermal conductivity enhancement, which can negatively impact heat transfer efficiency. From a physical perspective, the improved heat conduction with increasing nanoparticle volume fraction is due to the increased solid-liquid interfacial area, which facilitates rapid thermal energy exchange. However, excessive nanoparticle loading thickens the boundary layer and increases fluid resistance, which can lead to an overall reduction in convective heat transfer efficiency. The optimal balance between enhancing thermal transport and avoiding excessive flow resistance is crucial in applications where precise temperature control is required.

Fig. 7 demonstrates the variation of temperature concerning the Prandtl number of the nanofluid. The Prandtl number (Pr) is a dimensionless parameter that represents the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity, characterizing the relative thickness of the momentum and thermal boundary layers. A higher Prandtl number indicates that momentum diffusivity dominates over thermal diffusivity, meaning that heat conduction is slower compared to velocity diffusion. As a result, fluids with high Prandtl numbers exhibit steeper temperature gradients, leading to a more rapid decline in temperature as the fluid moves away from the heated surface. Conversely, lower Prandtl numbers correspond to fluids with higher thermal diffusivity, allowing heat to spread more efficiently and resulting in a more gradual temperature decay. From a physical standpoint, this behavior arises because fluids with high Prandtl numbers, such as oils and polymeric nanofluids, have lower thermal diffusivity, which confines heat within a thin thermal boundary layer near the surface. This effect is particularly useful in applications where localized heating is required. On the other hand, fluids with low Prandtl numbers, such as liquid metals, exhibit more uniform heat distribution due to their high thermal diffusivity. In biomagnetic fluid dynamics, where ferrofluids are utilized for therapeutic applications, tuning the Prandtl number is essential for controlling heat penetration into biological tissues. For example, in magnetic hyperthermia therapy, where magnetic nanoparticles generate heat under an external field, a carefully chosen Prandtl number ensures that temperature elevation remains localized to the tumor region without excessive heat spreading into healthy tissue.

Fig. 8 presents the streamline patterns of ferrofluid flow in the presence of a magnetic dipole. The visualization of these streamlines provides insight into the flow structure, including recirculation zones and the influence of magnetic forces on fluid motion. Due to the interaction between the magnetic field and the fluid, distinct flow patterns emerge, which can significantly impact heat and mass transfer characteristics. From a physical perspective, such flow structures are crucial in designing magnetic fluid-based biomedical devices, such as drug carriers, where precise control of fluid movement is essential for effective drug delivery. Additionally, in industrial applications, streamline analysis aids in optimizing fluid transport in systems where magnetic forces are employed to regulate flow behavior.

Fig. 9 presents a surface plot illustrating the spatial distribution of temperature within the nanofluid. This three-dimensional representation provides a comprehensive view of how temperature varies across the flow domain, offering deeper insight into heat transfer mechanisms. High-temperature regions are concentrated near the surface of the stretching sheet, where thermal energy is introduced, and the temperature gradually decreases as the fluid moves away due to convective and conductive heat dissipation. The shape and intensity of these temperature distributions are influenced by the combined effects of magnetic field strength, thermal radiation, and nanoparticle concentration. From a physical standpoint, the observed temperature distribution results from the interplay between forced convection (due to stretching motion) and thermal diffusion. The presence of nanoparticles enhances thermal conductivity, leading to more efficient heat transfer. Additionally, a stronger magnetic field influences the movement of magnetic nanoparticles, altering local heat transfer rates by modifying velocity gradients and boundary layer thickness. Thermal radiation effects contribute to further temperature enhancement, particularly in high-energy applications. It is noteworthy that the nonlinear system (9)–(11) may exhibit multiple solutions for certain parameter values. The presence of multiple solutions can lead to significant physical implications, including boundary layer separation and instability. Future work should include stability analysis to determine which solution is physically realizable.

5 Validation of Results

To ensure the accuracy and reliability of the numerical solutions obtained in this study, a comparative analysis has been conducted against previously published results. Specifically, the computed values of the local Nusselt number have been compared with the results reported by Mukhopadhyay [40] and Murtaza et al. [5] for different values of the Prandtl number Pr. The corresponding numerical values are presented in Table 4.

Table 4: Values of Nusselt number for several values of the Prandtl number

Prandtl number (**Pr**)	Mukhopadhyay [40]	Murtaza et al. [5]	Present study
1	0.9547	0.9542	0.9547
2	1.4714	1.4748	1.4714
3	1.8961	1.8937	1.8961

This comparative study demonstrates that the present numerical solutions exhibit an excellent agreement with previously published findings. The significance of this validation lies in establishing the credibility of the implemented numerical approach, which is based on the MATLAB bvp4c solver. The close correspondence between the present results and the benchmark values ensures that the governing equations, boundary conditions, and physical assumptions have been correctly implemented. This also confirms that the mathematical model accurately captures the effects of thermal and fluid flow parameters, making it a reliable tool for further analysis.

6 Conclusions

The analysis of biomagnetic nanofluid flow over a stretching sheet has provided valuable insights into the effects of various governing parameters on velocity, temperature, and flow structure. By incorporating the influence of nanoparticle volume fraction, viscous dissipation, ferromagnetic effects, and Prandtl number, the study highlights the intricate interplay between thermal and fluid dynamics under the influence of a magnetic dipole. The results are particularly relevant to biomedical and industrial applications, where precise control over fluid transport and heat transfer is crucial. Based on the findings, the key observations can be summarized as follows:

- The interplay between magnetic dipole effects, thermal radiation, and copper nanoparticles significantly alters the velocity and temperature fields, revealing intricate magnetothermal interactions in biomagnetic nanofluid flow.
- Increasing the nanoparticle volume fraction enhances thermal conductivity, leading to improved heat transfer, yet introduces additional viscosity that reduces the fluid's velocity near the stretching sheet.
- Viscous dissipation emerges as a critical factor in thermal regulation, generating internal heat that raises the temperature, particularly in high-shear regions, making it an essential consideration in biomedical cooling and hyperthermia applications.
- The ferromagnetic parameter strongly influences heat conduction, as magnetic forces align nanoparticles along field lines, modifying temperature gradients and reinforcing localized heating effects relevant to targeted therapy and magnetic fluid engineering.
- The Prandtl number dictates the thermal boundary layer thickness, where higher values steepen temperature gradients, a key aspect in designing nanofluid-based cooling systems and biomedical heat transfer applications.
- Streamline analysis uncovers how magnetic field interactions reshape flow structures, offering valuable insights into optimizing ferrofluid transport in drug delivery, hyperthermia, and industrial thermal systems.
- Surface temperature distributions reveal the spatial dynamics of heat dissipation, guiding precise thermal control strategies for biomedical, energy, and microfluidic applications.
- The findings provide a strong foundation for future studies exploring solution multiplicity and stability, ensuring that observed nonlinear behaviors are physically realizable in biomagnetic nanofluid systems.

The results of this study have direct implications in biomedical engineering, particularly in magnetic hyperthermia treatments and targeted drug delivery, where controlling nanoparticle-induced heat transfer is crucial for localized temperature regulation. Additionally, in nanofluid-based cooling systems, such as microchannel heat exchangers, optimizing ferrohydrodynamic interactions ensures efficient heat dissipation. These findings contribute to the development of advanced thermal management systems, biomagnetic fluid devices, and energy-efficient transport mechanisms.

An important aspect of nonlinear boundary layer equations is the potential for multiple solutions. This study did not explicitly analyze solution multiplicity; however, previous research on free convection problems with variable heat flux suggests that such behavior may occur. Further numerical experiments and stability analyses are necessary to confirm the uniqueness or bifurcation of solutions in this system. Further investigations could also incorporate three-dimensional effects, unsteady flow conditions, and hybrid nanofluid models to enhance the applicability of this study to real-world biomedical and engineering systems.

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