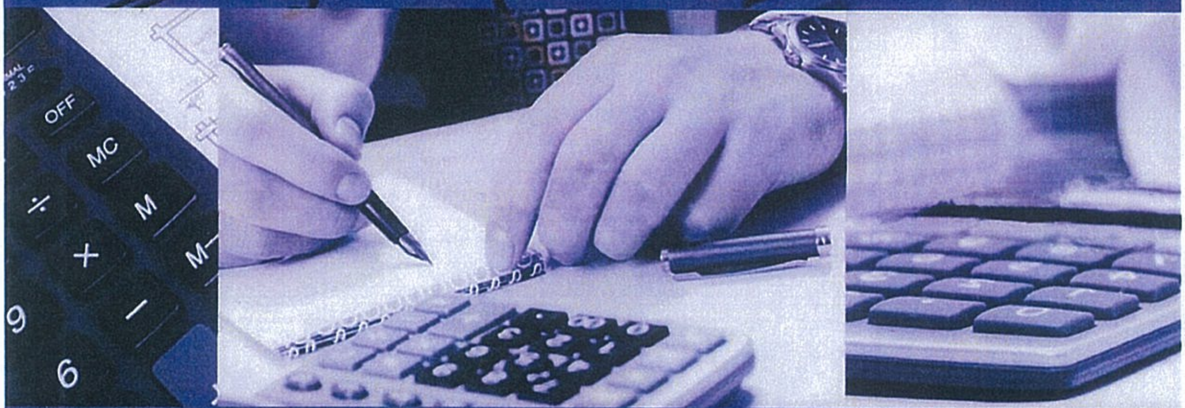
TEACHING AND LEARNING SERIES : FACULTY OF ELECTRICAL ENGINEERING

SOLUTION MANUAL OF DIFFERENTIAL EQUATIONS

Module 4



Tay Choo Chuan
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Preface

One of the most challenging aspects of mathematics learning is to give students suitable examples and exercises which can improve their understanding.

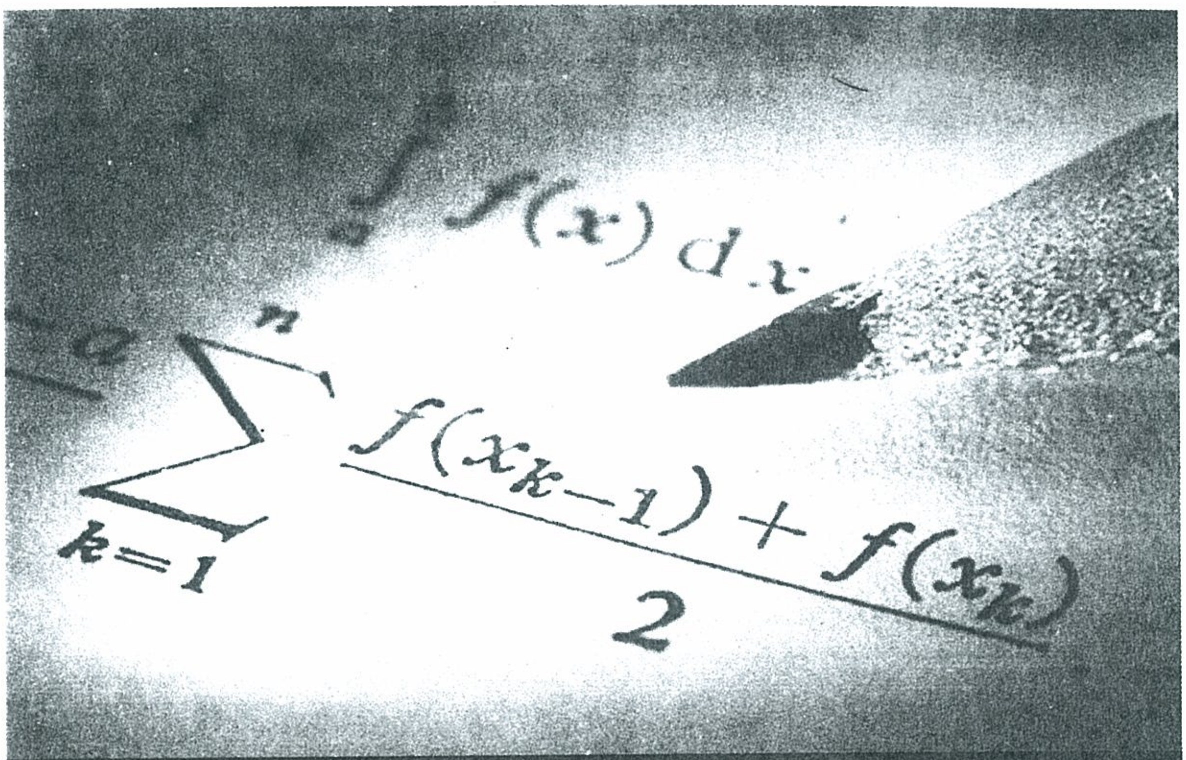
This solution manual of Differential Equation is designed to serve as a study guide for engineering students of Universiti Teknikal Malaysia Melaka (UTeM). The topics are based on the syllabus of Differential Equation teaching in UTeM.

The material in this book will cover questions and answers for:

- Second Order Linear Differential Equation
- Laplace Transform
- Foureir Series
- Partial Differential Equation

An excellent student must have an initiative to learn before being taught by lecturers. By using this manual, students can be more prepared before attending a tutorial session. The examples are presented in a sequence of steps with full details so that students can follow systematically.

In preparing this manual, I would like to thank all the individuals involved. Such comments and feedback are always welcome.



CHAPTER ONE

SECOND ORDER LINEAR DIFFERENTIAL EQUATION

CHAPTER ONE

SECOND ORDER LINEAR DIFFERENTIAL EQUATION

After completing these tutorials, students should be able to:

Tutorial 1

- find the general solution of the given homogeneous differential equations
- solve homogeneous differential equations of the given initial value problem

Tutorial 2

- find the general solution of the given non-homogeneous differential equations by using the method of undetermined coefficients
- solve non-homogeneous differential equations of the given initial value problem by using the method of undetermined coefficients

Tutorial 3

- find the general solution of the given non-homogeneous differential equations by using the method of variation of parameters
- solve non-homogeneous differential equations of the given initial value problem by using the method of variation of parameters

APPENDICES

A. Table of General Solution, $y(x) = y_h(x) + y_k(x)$

	If m values are	Then general solution, $y_h(x)$
1.	Real & Different	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real but Repeated	$y = Ae^{m_1 x} + Bxe^{m_1 x}$
3.	Complex conjugates, $(a \pm b\, i)$	$y = e^{ax}(A\cos bx + B\sin bx)$

Table 1.1

Form of $f(x)$	Roots	$y_k(x)$
$\alpha_n x^n + \alpha_{n-1}x^{n-1} + \ldots + \alpha_1 x + \alpha_0$	$m_1 \neq 0$ and $m_2 \neq 0$	$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$
$\alpha_n x^n + \alpha_{n-1}x^{n-1} + \ldots + \alpha_1 x + \alpha_0$	$m_1 = 0$ or $m_2 = 0$ (either one)	$x(a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0)$
$ke^{\alpha x}$	$m_1 \neq \alpha$ and $m_2 \neq \alpha$	$Ce^{\alpha x}$
$ke^{\alpha x}$	$m_1 = \alpha$ or $m_2 = \alpha$ (either one)	$Cxe^{\alpha x}$
$ke^{\alpha x}$	$m_1 = m_2 = \alpha$	$Cx^2 e^{\alpha x}$
$k\cos\alpha x$ atau $k\sin\alpha x$	$m_1 \neq i\alpha$ and $m_2 \neq i\alpha$	$p\cos\alpha x + q\sin\alpha x$
$k\cos\alpha x$ atau $k\sin\alpha x$	$m_1 = i\alpha$ or $m_2 = i\alpha$ (either one)	$x(p\cos\alpha x + q\sin\alpha x)$

Table 1.2

B. Table of Laplace Transform

$f(t)$	$L\{f(t)\} = F(s)$
k	$\frac{k}{s}$
$t^n, \quad n = 1,2,3,....$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$kos\ at$	$\frac{s}{s^2+a^2}$
$\sin at$	$\frac{a}{s^2+a^2}$
$kosh\ at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t), \quad n = 1,2,3,....$	$(-1)^n \frac{d^n F}{ds^n}$
$y^n(t), \quad n = 1,2,3,....$	$s^n Y(s) - s^{n-1} y(0) - s^{n-2} y'(0) ... - sy^{(n-1)}(0) - y^{(n-1)}(0)$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as} F(s)$
$\delta(t-a)$	e^{-as}

Table 1.3

C. Formulas of Fourier Series

FOURIER SERIES

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n kos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right)$$

$$a_0=\frac{1}{L}\int_{-L}^{L}f(x)dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)kos\frac{n\pi x}{L}dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx$$

FOURIER COSINE SERIES

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n kos\frac{n\pi x}{L}$$

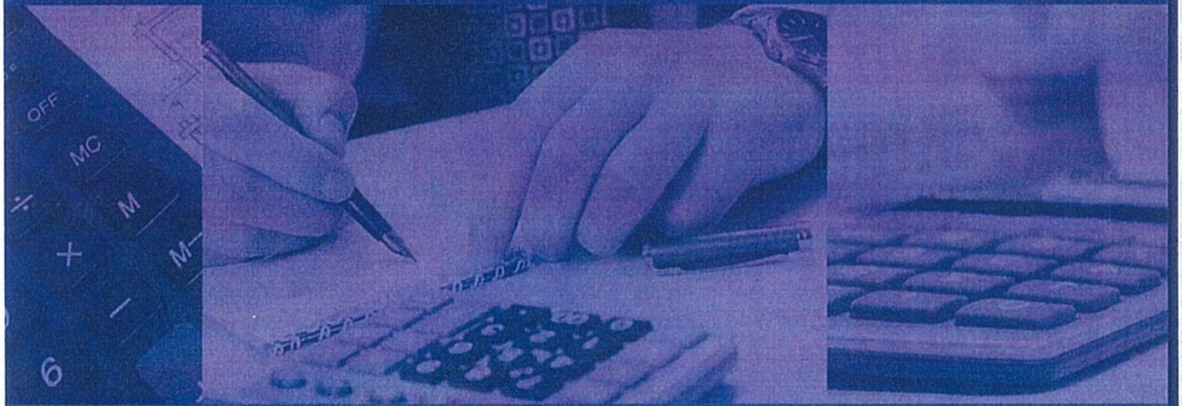
$$a_0=\frac{2}{L}\int_{0}^{L}f(x)dx$$

$$a_n=\frac{2}{L}\int_{0}^{L}f(x)kos\frac{n\pi x}{L}dx$$

FOURIER SINE SERIES

$$f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

$$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}dx$$



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