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Simpler UNBabc Mapping Function for Global Positioning System (GPS) Tropospheric Delay

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Abstract **- The $UNBabc(E)$ mapping function models should be simplified to allow faster calculation and also better understanding of the models. Many modern mapping function models use mapping functions in the form of continued fractions which is quite tedious in calculation. There are 11 mathematical operations for $UNBabc(E)$ to be carried out before getting the mapping function scale factor. The mapping functions for $UNBabc(E)$ models for hydrostatic and non hydrostatic components are given in a form of continued fraction are to be simplified. $UNBabc(E)$ mapping function models are selected due to its ability to calculate mapping function value down to 2 degree of elevation angle. By using linear, hyperbolic, logarithm and also regression method, the mapping function models can be simplified and at the same time can produce the same result. The calculation of sum of errors shows that the deviation of the simplified models from the original models is not significant. The computation time between the original model and simplified model shows that the simplified $UNBabc(E)$ model is 6.5 times faster than the original model for both hydrostatic and non hydrostatic component. As the coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function scale factor value plays an important role for getting the total tropospheric delay value.**

Keywords: **tropospheric, zenith, mapping function**

I. INTRODUCTION

Recently, the developed tropospheric delay models use mapping functions in the form of continued fractions. The Saastamoinen [3] model does not use a mapping function in the same sense as the models with continued fractions. Most of the modern models have separated mapping functions for the hydrostatic and the wet component, in a form of continued fraction [2].

The calculation for finding the mapping function scale factor, which is in a form of continued fractions are quite tedious. There are many mathematical operations (11 operations for UNBabc and 26 operations for Neill mapping function) to be done before getting the mapping function scale factor.

There is a need to simplify the mapping function models to allow faster calculation and also better understanding of the models. The mapping functions such as UNBab, UNBabc and Neill models for hydrostatic and non hydrostatic components are given in a form of continued fraction, whereby the elevation angle is the variable as shown in Figure 1.

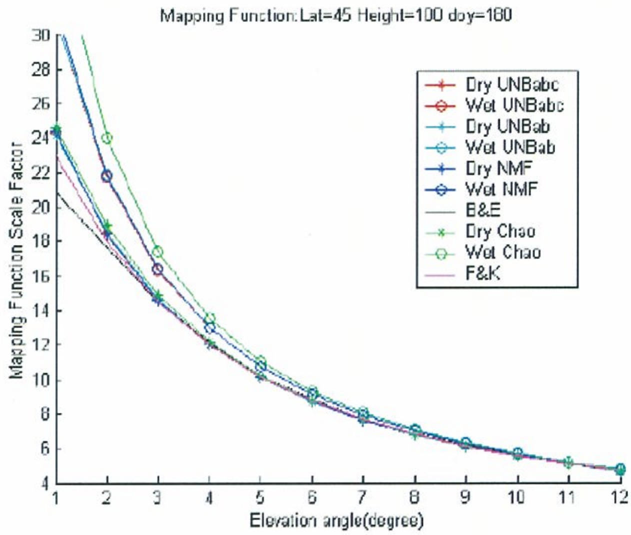

Figure 1: Comparison of mapping function values [1]

All mapping function graphs shown in Figure 1 are in a shape of a parabolic. These graphs give very close mapping function values when the elevation angles more than 10 degree, however, for the elevation angles less than 10 degrees, each mapping function model give difference scale factor. These mapping function models are very tedious in calculating the value of mapping function, due to its continued fraction form.

In this study, UNBabc mapping function for both component, either hydrostatic or non hydrostatic will be selected to be simplified, due to its ability to calculate mapping function value down to 2 degree of elevation angle. However, the graphs can also be obtained by using other form of equation which is simpler than the established equations.

At 90 degree the mapping function should be normalized to unity [1]. As a coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function scale factor depend on the elevation angles, whereby at 90 degree of elevation angle, the mapping function scale factor value is 1. So, this value will give minimum value of tropospheric delay.

In this paper, UNBabc mapping function model for hydrostatic and non hydrostatic components will be used to be simplified to a simpler equation. This model is selected due to its ability to achieve mapping function scale factor value down to 2 degree of elevation angle. This simpler equation can be used to calculate the mapping function scale factor by varying the elevation angles in the model.

II. Simplification of UNBabc Mapping Function Models

UNBabc mapping function model performed by [1] has 11 operations in a form of continued fraction as given in equation (1) below. UNBabc has been separated into hydrostatic and non hydrostatic components. The difference between the two components are their constants values, a, b and c.

$$m_h(\varepsilon) = \frac{1+\cfrac{a_i}{1+\cfrac{b_i}{1+c_i}}}{\sin\varepsilon + \cfrac{a_i}{\sin\varepsilon + \cfrac{b_i}{\sin\varepsilon + c_i}}} \quad \text{where i = h or nh} \qquad (1)$$

From the ray tracing delay values, (Guo, 2003) has introduced UNBabc which has a 3-terms continued fraction form. From a series of analyses, he concluded that parameter a is sensitive to the orthometric height (H) and latitude (ϕ) of the station whereas parameters b and c could be represented by constants. The least-squares estimated parameters for the hydrostatic function are:

a_h= (1.18972-0.026855 H+ 0.10664 cosϕ)/1000

b_h= 0.0035716

c_h= 0.082456

and for the non-hydrostatic component;

a_{nh}= (0.61120-0.035348 H-0.01526 cosϕ)/1000

b_{nh}= 0.0018576

c_{nh}= 0.062741

Regression method is used to find the same type of graph for the original UNBabc(h) mapping function. However there is a slight difference for some points on the graph. From the statistical analysis, the difference between the original and the simplified model is small and not significant.

A. *Simplification of hydrostatic UNBabc (UNBabc(h)) mapping function*

UNBabc(h) mapping function model has been named as R, while the simplified models have been named as R1, R2 and R3. These three simplified models and also UNBabc(h) mapping function model produce a graph of a parabolic shape. However there is a slight difference between the UNBabc(h) model and the simplified models.

The simplified models (R1, R2 and R3) have been generated using regression method, which give the model in a form of :

$$R1 = A * X^B \qquad (2)$$

where: $R1$: modified UNBabc(h)

A, B : constant

X : elevation angle (independent variable).

R1 model is simpler than the original UNBabc(h) mapping function, in term of equation type. By using these simplified models, we can reduce the computation time from 11 operations to only 2 operations. Model R1 has been generated from regression method, whereby model R2 and R3 have been generated based on R1 model. Model R2 is formed by fixing the value of constant B and changing the value of constant A, while model R3 is formed by fixing the value of constant A and changing the value of constant B. Model R2 and R3 are formed when they give unity when the elevation angle, X is 90 degree.

B. *Sum of Error Calculation For UNBabc(h) Mapping Function*

Sum of error method can be used to show how far the simplified models deviate from the original model. The smaller deviation will show better correlation, which means the simplified model is closer to the original model.

From Table 1 (Attachment 1) although the sum of error is small (1.815), the R1 model has not been selected, because at 90 degree elevation angle, the value is 0.865, but the condition needs at 90 degree elevation angle, the mapping function scale factor should be unity, 1. Although the R2 model meets the requirements, but the model gives the biggest value for sum of error (15.249) among the others, which most of the points are scattered quite far from the original UNBabc(h) mapping function model.

So, R3 = 33.595*X^(-0.781) model has been selected as the simplification mapping function model for UNBabc(h) due the smallest sum of error (2.545) compared to the others and its mapping function gives unity at 90 degree elevation angle as given in Figure 2 below.

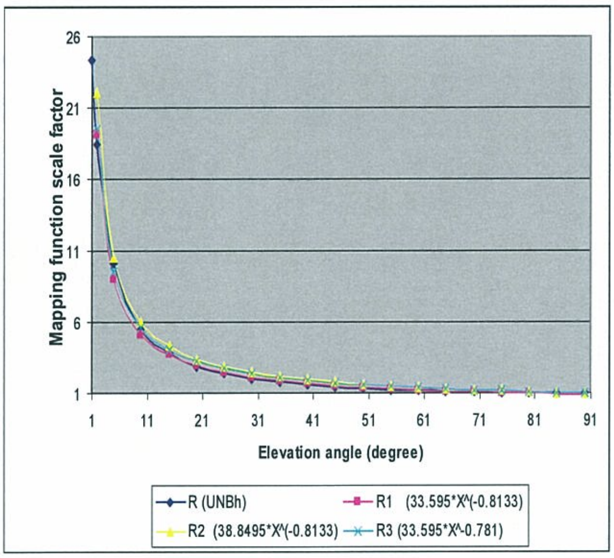

Figure 2. Graph of UNBabc(h) mapping function by regression.

III. Simplification of Non Hydrostatic UNBabc (UNBabc(nh)) Mapping Function

UNBabc(nh) mapping function model has been named as S, while the simplified models have been named as S1, S2 and S3. These four models give a shape of parabola graph. However there is a slight difference between the S model and the three simplified models. The simplified models (S1, S2 and S3) have been generated using regression method, which give the model in a form of :

$$S1 = A * X^{B} \qquad (3)$$

where: $S1$: modified UNBab(h)

A, B : constant

X : elevation angle (independent variable)

These simplified models are simpler than the original UNBab(nh) mapping function. By using these simplified models, we can reduce the computation time from 11 operations to only 2 operations. Model S1 has been generated from regression method, whereby model S2 and S3 have been generated based on S1 model. Model S2 is formed by fixing the value of constant B and changing the value of constant A, while model S3 is formed by fixing the value of constant A and changing the value of constant B. As a constraint for Model S2 and S3, they give unity values when the elevation angle, X is 90 degree.

A. *Calculation of Sum of Error For UNBabc(nh) Mapping Function*

Sum of error method can be used to show how the simplified models deviate from the original model. Smaller deviation is better, which shows that the simplified model is closer to the original model.

From Table 2 (Attachment 2), although the sum of error is small (1.815), the S1 model has not been selected, because at 90 degree elevation angle, the value is 0.863, but the condition needs at 90 degree elevation angle, the mapping function scale factor should be unity, 1. Model S1 also give bigger value of the sum of error (6.207). Although the S3 model meets the requirements, but the model gives bigger value for sum of error (4.307) than model S2, which most of the points are scattered quite far from the original UNBabc(nh) mapping function model.

So, S2 = 40.585*X^(-0.823) model has been selected as the simplification mapping function model for UNBab(nh) due the smallest sum of error (3.443) compared to the others and it's mapping function gives unity at 90 degree elevation angle as given in Figure 3 below:

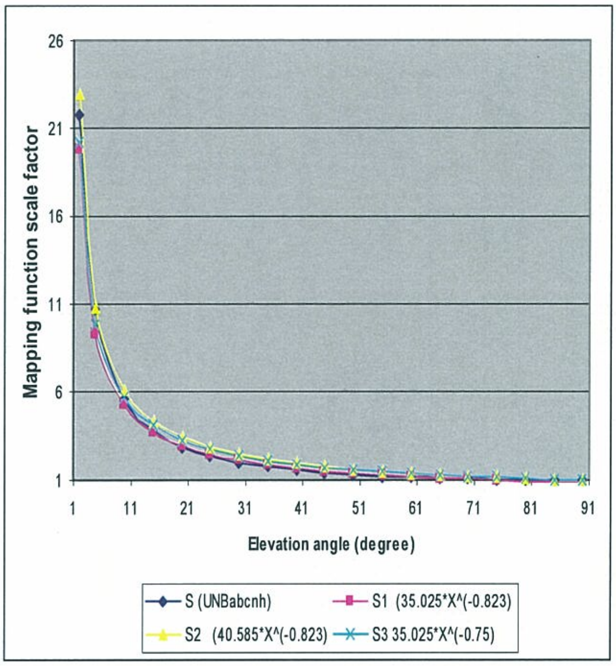

Figure 3. Graph of UNBabc(nh) mapping function (S), S1, S2 and S3 by regression.

IV. Discussion and Conclusion

The original UNBabc mapping function as given in equation (1) is in a form of continued fraction, has 11 operations.

For hydrostatic component, the regression method gives a simpler parabolic model, R1 in a form of equation

(2), which has 2 constants namely as A and B. The R2 equation has been formed by changing A value while B is unchanged. The R3 equation has been formed by changing B value while A is unchanged. The calculation of sum of error shows that R3 model can give smaller value than R2 value. So, R3 is similar to the original hydrostatic UNBabc mapping function model, R.

For non hydrostatic component, the regression method gives a simpler parabolic model, S1, which has 2 constants namely as A and B. The S2 equation has been formed by changing A value while B is unchanged. The S3 equation has been formed by changing B value while A is unchanged. The calculation of sum of error shows that S2 model can give smaller value than S3 value. So, S2 is similar to the original non hydrostatic UNBabc mapping function model, S.

By using regression method, the UNBabc mapping function either for hydrostatic and also non hydrostatic components can be simplified to a simpler form as given in equation (2), which has only 2 operations. The simplification of UNBabc mapping functions can reduce the computation time by inserting only once the elevation angle as in equation (2) and (3) rather than 3 times in equation (1). The simplified models also can give simpler parabolic equation model. The simplified equations give similar result with the original UNBabc mapping function, for both hydrostatic and also non hydrostatic components.

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Attachment 1

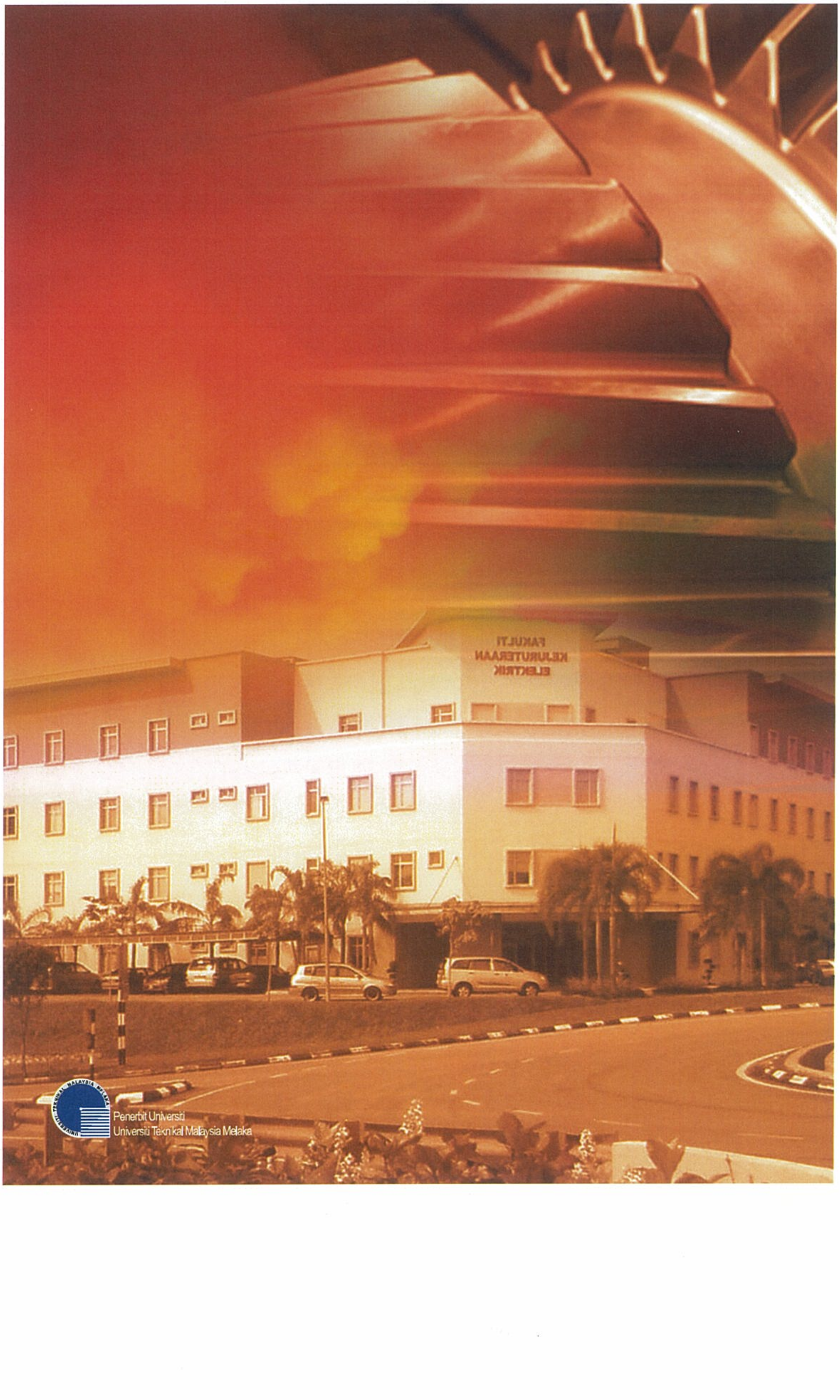
TABLE I.
SUM OF ERROR FOR UNBABC(H), R AND SIMPLIFIED MODELS (R1,R2, R3)

X	R	R1	R2	R3	R - R1	(R - R1)^2	R - R2	(R - R2)^2	R - R3	(R - R3)^2
(E)	(UNBabc(nh))	(33.595*X^ (-0.8133))	(38.8495*X^ (-0.8133))	(33.595*X^ (-0.781))						
2	18.467	19.118	22.108	19.551	-0.651	0.423	-3.641	13.256	-1.084	1.174
5	10.132	9.074	10.493	9.558	1.058	1.118	-0.362	0.131	0.573	0.329
10	5.551	5.164	5.972	5.563	0.387	0.150	-0.421	0.177	-0.012	0.000
15	3.800	3.713	4.294	4.053	0.086	0.007	-0.494	0.244	-0.253	0.064
20	2.897	2.939	3.398	3.237	-0.041	0.002	-0.501	0.251	-0.340	0.115
25	2.353	2.451	2.834	2.719	-0.098	0.010	-0.481	0.232	-0.366	0.134
30	1.993	2.113	2.444	2.359	-0.121	0.015	-0.451	0.203	-0.366	0.134
35	1.739	1.864	2.156	2.091	-0.125	0.016	-0.417	0.174	-0.352	0.124
40	1.553	1.672	1.934	1.884	-0.119	0.014	-0.381	0.145	-0.331	0.109
45	1.412	1.520	1.757	1.718	-0.107	0.011	-0.345	0.119	-0.306	0.094
50	1.304	1.395	1.613	1.583	-0.091	0.008	-0.309	0.095	-0.278	0.078
55	1.220	1.291	1.493	1.469	-0.071	0.005	-0.273	0.074	-0.249	0.062
60	1.154	1.203	1.391	1.373	-0.048	0.002	-0.236	0.056	-0.218	0.048
65	1.103	1.127	1.303	1.289	-0.024	0.001	-0.200	0.040	-0.186	0.035
70	1.064	1.061	1.227	1.217	0.003	0.000	-0.163	0.026	-0.153	0.023
75	1.035	1.003	1.160	1.153	0.032	0.001	-0.125	0.016	-0.118	0.014
80	1.015	0.952	1.101	1.096	0.064	0.004	-0.085	0.007	-0.081	0.007
85	1.004	0.906	1.048	1.046	0.098	0.010	-0.044	0.002	-0.042	0.002
90	1.000	0.865	1.000	1.000	0.135	0.018	0.000	0.000	0.000	0.000
						1.815		**15.249**		**2.545**

Attachment 2

TABLE II.
SUM OF ERROR FOR UNBABC(NH), S AND SIMPLIFIED MODELS (S1,S2, S3)

X (E)	S (UNBabc(nh))	S1 (35.025*X^ (-0.823))	S2 (40.585*X^ (-0.823))	S3 35.025*X^ (-0.75))	S - S1	(S - S1)^2	S - S2	(S - S2)^2	S - S3	(S - S3)^2
2	21.767	19.799	22.941	20.252	1.969	3.876	-1.174	1.378	1.515	2.295
5	10.742	9.314	10.792	9.817	1.428	2.039	-0.051	0.003	0.925	0.855
10	5.655	5.265	6.101	5.676	0.390	0.152	-0.445	0.198	-0.021	0.000
15	3.833	3.771	4.370	4.120	0.062	0.004	-0.537	0.288	-0.287	0.083
20	2.911	2.976	3.448	3.282	-0.065	0.004	-0.537	0.289	-0.371	0.138
25	2.360	2.477	2.870	2.752	-0.117	0.014	-0.510	0.260	-0.392	0.153
30	1.996	2.132	2.470	2.382	-0.135	0.018	-0.474	0.224	-0.386	0.149
35	1.741	1.878	2.176	2.109	-0.136	0.019	-0.434	0.189	-0.368	0.135
40	1.554	1.682	1.949	1.898	-0.128	0.016	-0.395	0.156	-0.343	0.118
45	1.413	1.527	1.769	1.729	-0.113	0.013	-0.356	0.127	-0.316	0.100
50	1.305	1.400	1.622	1.591	-0.095	0.009	-0.317	0.101	-0.286	0.082
55	1.220	1.294	1.500	1.476	-0.074	0.005	-0.279	0.078	-0.255	0.065
60	1.154	1.205	1.396	1.378	-0.050	0.003	-0.242	0.058	-0.223	0.050
65	1.103	1.128	1.307	1.293	-0.025	0.001	-0.204	0.042	-0.190	0.036
70	1.064	1.061	1.230	1.220	0.003	0.000	-0.166	0.027	-0.155	0.024
75	1.035	1.003	1.162	1.155	0.032	0.001	-0.127	0.016	-0.120	0.014
80	1.015	0.951	1.102	1.097	0.065	0.004	-0.086	0.007	-0.082	0.007
85	1.004	0.905	1.048	1.046	0.099	0.010	-0.044	0.002	-0.042	0.002
90	1.000	0.863	1.000	1.000	0.137	0.019	0.000	0.000	0.000	0.000
						6.207		3.443		4.307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