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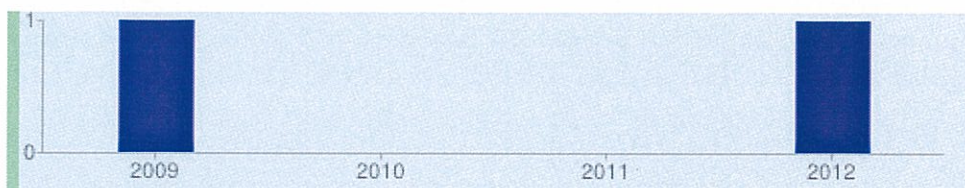
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Sliding Mode Control Design for Active Suspension on A Half-Car Model

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Abstract- The purpose of this paper is to present a new robust strategy in controlling the active suspension system. The strategy utilized the proportional integral sliding mode control scheme. A half-car model is used in the study and the performance of the controller is compared to the linear quadratic regulator and with the existing passive suspension system. A simulation study is performed to prove the effectiveness and robustness of the control approach.

Keywords

Sliding Mode Control, Active Suspension, Robust Control.

I. INTRODUCTION

An ideal suspension should isolate the vehicle body from road disturbances and inertial disturbances associated with cornering and braking or acceleration [1]. Furthermore, the suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. These objectives can be achieved by minimizing the vertical car body acceleration. An excessive wheel travel will result in non-optimum attitude of tyre relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tyre-to-road contact must be maintained on four wheels.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [2]. Thus the passive suspension system, which approach optimal characteristics had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers [3,4,5,6,7]. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, active suspension systems include sensors to measure suspension variables such as

body velocity, suspension displacement, wheel velocity and wheel or body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control strategies such as optimal state-feedback [3,5], backstepping method [4], fuzzy control [8] and sliding mode control [9] have been proposed in the past years to control the active suspension system. The sliding mode control has relatively simpler structure and it guarantees the system stability.

In this paper we will discuss a control scheme that can improve further the ride comfort and road handling of the active suspension system. The proposed control scheme differs from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral sliding mode control (PISMC). The additional integral in the proposed sliding surface provides one more degree of freedom and also reduce the steady state error. A computer simulation will be performed to demonstrate the effectiveness and robustness of the proposed control scheme.

II. HALF-CAR SUSPENSION MODEL

Consider the model of a passenger's car subject to irregular excitation from a road surface as shown in Fig. 1. This model has been used by [8] in the design of active suspension. From the figure, the state-space model of a half-car model can be easily obtained as in appendix A, where m_b is the mass for the vehicle body, I_b is the mass moment of inertia for the vehicle body, m_{wf} and m_{wr} are the masses for the vehicle front/rear wheels respectively, x_c is the vertical displacement of the vehicle body at the center of gravity, x_{bf} and x_{br} are the vertical displacements of the vehicle body at the front/rear suspension locations respectively, x_{wf} and x_{wr} are the vertical displacements of the vehicle body at the front/rear wheels respectively, θ_0 is the rotary angle of the vehicle body at the center of gravity, f_f and f_r are the active controls at the front/rear suspensions respectively, w_f and w_r

are the irregular excitations from the road surface respectively, and L_f and L_r are the distances of the front/rear suspension locations respectively, with reference to the center of gravity of the vehicle body and $L_f + L_r = L$. Let the state variables are defined as:

$$x_1 = x_{bf}, \quad x_2 = x_{wf}, \quad x_3 = x_{br}, \quad x_4 = x_{wr}, \quad x_5 = x_{bf},$$

$x_6 = x_{wf}, \quad x_7 = x_{br}, \quad x_8 = x_{wr}$. The state equation shows that the disturbance input is not in phase with the actuator input, i.e. $\text{rank}[B] \neq \text{rank}[B, f(t)]$. Therefore the system does not satisfy the matching condition. Thus the proposed controller must be robust enough to overcome the mismatched condition inherent so that the disturbance would not have significant effect on the performance of the system.

We start the analysis by presenting the half-car dynamics as in appendix A in the following form,

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t) \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the control input, and the continuous function $f(t)$ represents the uncertainties with the mismatched condition. The following assumptions are taken as standard:

Assumption i: There exists a known positive constant such that $\|f(t)\| \leq \beta$, where $\|\cdot\|$ denotes the standard Euclidean norm.

Assumption ii: The pair (A,B) is controllable and the input matrix B has full rank.

In the following section, the proposed control technique is presented.

III. VARIABLE STRUCTURE CONTROL DESIGN

Variable Structure Control utilizes a high-speed switching control law to drive the plant's state trajectory onto a specified and user-chosen surface in the state space called the sliding surface, and to maintain the plant's state trajectory on this surface for all subsequent time. This surface is called the sliding surface because if the state trajectory of the plant is "above" the surface, a control path has one gain and a different gain if the trajectory drops "below" the surface. The plant dynamics restricted to this surface represent the controlled system's behavior. By proper design of the sliding surface, VSC attains the conventional goals of control such as stabilization, tracking and regulation. The design of VSC consists of two steps:

- (i) The design of the switching surface so that the plant restricted to the sliding surface has desired system response.
- (ii) The construction of the control scheme to drive the plant's state trajectory to the sliding surface.

In this study, the PI sliding surface defined as follows is used:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau) d\tau \quad (2)$$

where $C \in \mathcal{R}^{1 \times n}$ and $K \in \mathcal{R}^{m \times n}$ are constant matrices. The matrix K satisfies $\lambda(A+BK) < 0$ and C is chosen so that CB is nonsingular. It is well known that if the system is able to enter the sliding mode, hence $\dot{\sigma}(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can

thus be obtained by letting $\dot{\sigma}(t) = 0$ [10] i.e.,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0 \quad (3)$$

If the matrix C is chosen such that CB is nonsingular, this yields the equivalent control of

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(t) \quad (4)$$

Substituting equation (4) into system equation (1) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C\}f(t) \quad (5)$$

The uncertain system in equation (5) is boundedly stable on the sliding surface $\sigma(t) = 0$ [11]. For the system with uncertainties satisfy the matching condition, i.e. $\text{rank}[B, f(t)] = \text{rank}[B]$, then equation (5) can be

reduced to $\dot{x}(t) = (A + BK)x(t)$ [12]. Thus asymptotic stability of the system during sliding mode is assured.

The control scheme that drives the state trajectories of the system in equation (1) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter is now be designed. For the uncertain system in equation (1) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t) = -(CB)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB)^{-1} \frac{\sigma(t)}{|\sigma(t)| + \delta} \quad (6)$$

where $\phi \in \mathcal{R}^{m \times m}$ is a positive symmetric design matrix, k and δ are the positive constants.

It has been proved in [13] that the hitting condition of the sliding surface (2) is satisfied if $\|A + BK\| \|x(t)\| \geq \|f(t)\|$.

IV. SIMULATION AND DISCUSSION

The mathematical model of the system defined in equation (1) and the proposed proportional integral sliding mode controller (PISMC) in equation (6) were simulated on computer. For comparison purposes, the performance of the PISMC is compared to the linear

quadratic regulator (LQR) control approach. We assume a quadratic performance index is in the form of:

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (7)$$

where Q is a symmetric positive semi-definite matrix and R is a symmetric positive definite matrix. Then the optimal linear feedback control law is obtained as

$$u = -Kx \quad (8)$$

where K is the designed matrix gain.

The numerical values for the model parameters are taken from [8], and are as follows:

$$m_b = 430 \text{ kg}, I_b = 600 \text{ kgm}^2, m_{wf} = 30 \text{ kg},$$

$$m_{wr} = 25 \text{ kg}, k_{bf} = 10000 \text{ kN/m},$$

$$k_{br} = 6666.67 \text{ kN/m}, k_{wff} = k_{wrr} = 152 \text{ kN/m},$$

$$c_{bf} = 500 \text{ N/(m/sec)}, c_{br} = 400 \text{ N/(m/sec)},$$

$$L_f = 0.871 \text{ m}, L_r = 1.469 \text{ m}, v = 20 \text{ m/s}.$$

In the study, the following set of typical road disturbance is used:

$$r(t) = \begin{cases} a(1 - \cos(8\pi t)) / 2 & \text{if } 0.50 \leq t \leq 0.75 \\ & \text{and } 3.00 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases}$$

where a denotes the bump amplitude (see Fig.2). This type of road disturbance has been used by [4,14] in their studies. Furthermore, the maximum travel distance of the suspension travel used is $\pm 8\text{cm}$ as suggested by [4].

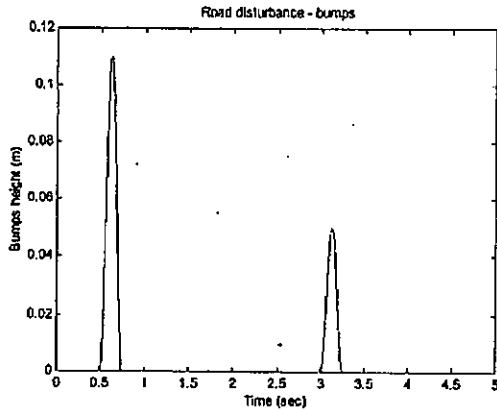


Fig. 2. Typical road disturbance

In the design of the LQR controller, the weighting matrices Q and R are selected as follows:

$$Q = \text{diag}[q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8] \quad \text{where}$$

$$q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = q_8 = 100 \quad \text{and}$$

$$R = \text{diag}(r_1, r_2) \quad \text{where } r_1 = r_2 = 1 \times 10^{-2}. \quad \text{Thus, the}$$

designed gains for the LQR controller for the front and rear suspensions can be calculated as $k_{1f} = 49.8$, $k_{1r} = 30.8$, $k_{2f} = -210.4$, $k_{2r} = 1.4$, $k_{3f} = -20.7$, $k_{3r} = 74.6$, $k_{4f} = -4.2$, $k_{4r} = -367.1$, $k_{5f} = 2654.7$, $k_{5r} = -80$, $k_{6f} = -229.3$, $k_{6r} = 6.1$, $k_{7f} = -69.1$, $k_{7r} = 6.1$, $k_{8f} = 3.2$ and $k_{8r} = -305.1$. The values of the matrix K used for the PISMIC is similar to the values calculated for the LQR controller i.e.

$$K = \begin{bmatrix} -49.8 & 210.4 & 20.7 & 4.2 & -2654.7 & 229.3 & 69.1 & -3.2 \\ -30.8 & -1.4 & -74.6 & 367.1 & 80 & -6.1 & -6.1 & 305.1 \end{bmatrix}$$

and thus

$$\lambda(A+BK) = \{-28.37, -35.64, -1327.8, -0.1 \pm j0.87, -0.05 \pm j0.69\}$$

In this simulation the following values are selected for the PISMIC:

$$C = \begin{bmatrix} 10 & 2 & 1 & 2 & 1 & 1 & 1 & 5 \\ 1 & 2 & 20 & 2 & 0.1 & 5 & 0.4 & 0.1 \end{bmatrix},$$

$$\phi = \text{diag}[1000, 1000], k_1 = k_2 = 100, \delta_1 = \delta_2 = 1.$$

Fig. 3 shows that the state trajectories slid onto the sliding manifold and remains in it thereafter. Therefore the hitting condition is satisfied. The controller input signals that drives the state trajectories onto the sliding manifold is shown in Fig.4.

In order to fulfill the objective of designing an active suspension system i.e. to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and the wheel deflection. Fig. 5(a) and Fig. 5(b) shows the suspension travels of both controllers for an active suspension system and a passive suspension system for comparison purposes. The result shows that the suspension travel within the travel limit i.e. $\pm 8\text{cm}$, and the result also shows that the active suspension utilizing the PISMIC technique perform better as compared to the others. Fig. 6(a) and Fig. 6(b) illustrates clearly how the PISMIC can effectively absorb the vehicle vibration in comparisons to the LQR method and the passive system. The body acceleration in the PISMIC design system is reduced significantly, which guarantee better ride comfort. Fig. 7(a) and 7(b) shows that the wheel deflection is also smaller using the proposed controller. Therefore it is concluded that the active suspension system with the PISMIC improves the ride comfort while retaining the road handling characteristics, as compared to the LQR method and the passive suspension system.

V. CONCLUSION

The paper presents a robust strategy in designing a controller for an active suspension system which is

based on variable structure control theory, which is capable of satisfying all the pre-assigned design requirements within the actuators limitation. The mathematical model of a half car is presented in a state space form. A detailed study of the proportional integral sliding mode control algorithm is presented and the mismatched condition problem in the mathematical model is overcome by the proposed controller. The performance characteristics and the robustness of the active suspension system are evaluated by two types of controllers, and then compared with the passive suspension system.

The result shows that the use of the proposed proportional integral sliding mode control technique proved to be effective in controlling the vehicle and is more robust as compared to the linear quadratic regulator method and the passive suspension system.

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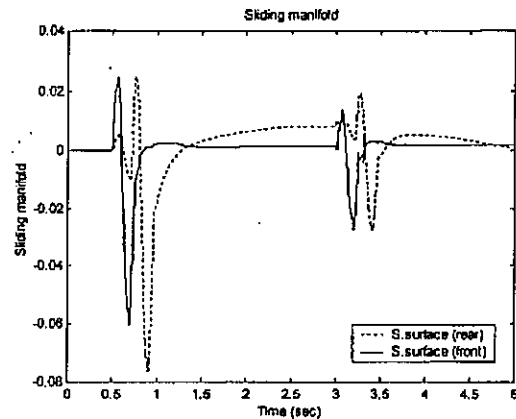


Fig. 3. The sliding manifold

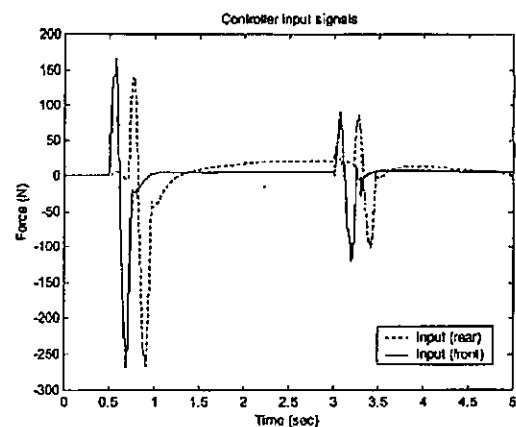


Fig. 4. The controller input signals

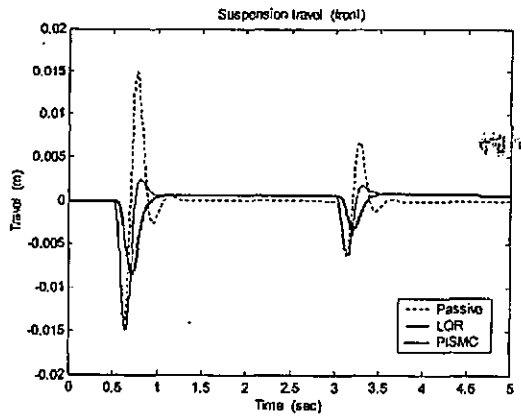


Fig. 5(a). The travel of front suspension

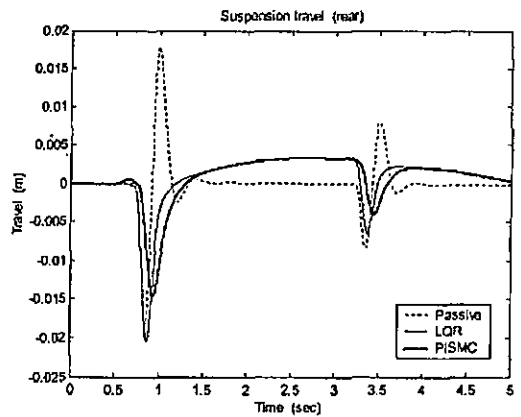


Fig. 5(b). The travel of rear suspension

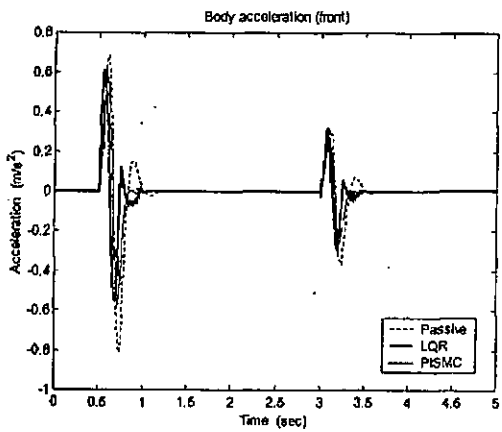


Fig. 6(a). The acceleration of front body

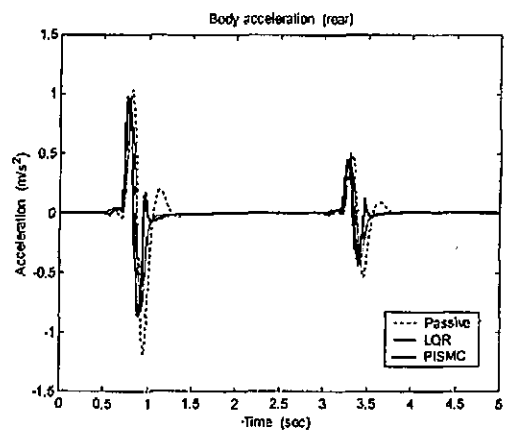


Fig. 6(b). The acceleration of rear body

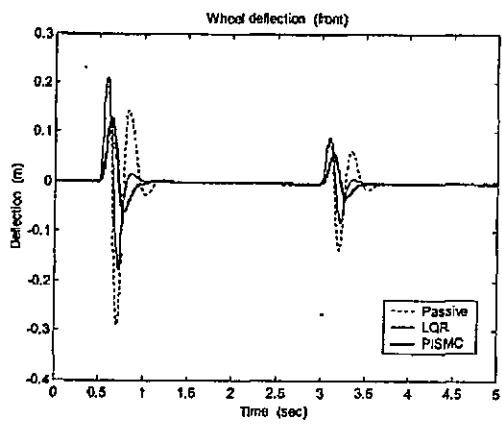


Fig. 7(a). The deflection of front wheel

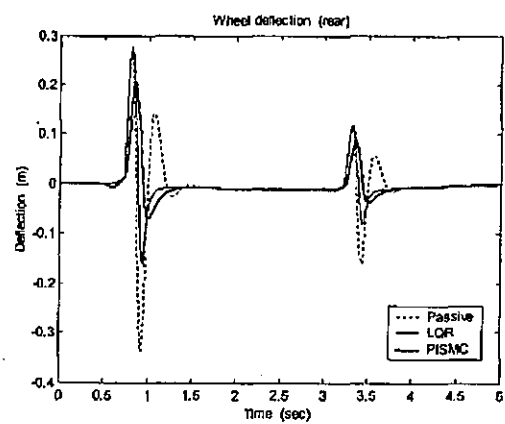


Fig. 7(b). The deflection of rear wheel

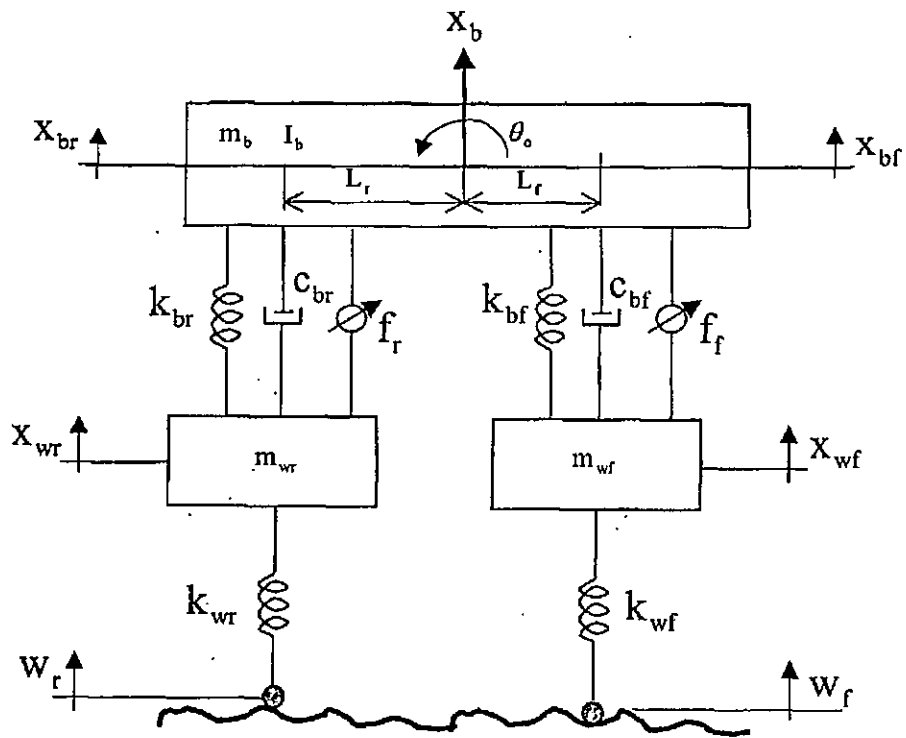


Fig. 1. A Half Car Model

Appendix A

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0}_0 & \mathbf{I} \\ \mathbf{A}_{p1} & \mathbf{A}_{p2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L}{m_b(L_f+L_r)} + \frac{L_f^2 L}{I_b(L_f+L_r)} & \frac{L}{m_b(L_f+L_r)} & \frac{L_f L L_r}{I_b(L_f+L_r)} \\ \frac{-1}{m_{wf}} & 0 \\ \frac{L}{m_b(L_f+L_r)} & \frac{L_f L L_r}{I_b(L_f+L_r)} & \frac{L}{m_b(L_f+L_r)} + \frac{L_r^2 L}{I_b(L_f+L_r)} \\ 0 & \frac{-1}{m_{wr}} \end{bmatrix} \begin{bmatrix} f_f \\ f_r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{k_{wf}}{m_{wf}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_{wr}}{m_{wr}} & 0 \end{bmatrix} \begin{bmatrix} w_f \\ w_f \\ w_r \\ w_r \end{bmatrix} \quad (1)$$

where: $\mathbf{O}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$$\mathbf{A}_{p1} = \begin{bmatrix} \frac{-Lk_{bf}}{m_b(L_f+L_r)} & \frac{L_f^2 Lk_{bf}}{I_b(L_f+L_r)} & \frac{Lk_{bf}}{m_b(L_f+L_r)} + \frac{L_f^2 Lk_{br}}{I_b(L_f+L_r)} & \frac{-Lk_{br}}{m_b(L_f+L_r)} + \frac{L_f L L_r k_{br}}{I_b(L_f+L_r)} & \frac{Lk_{br}}{m_b(L_f+L_r)} & \frac{L_f L L_r k_{br}}{I_b(L_f+L_r)} \\ \frac{k_{bf}}{m_{wf}} & \frac{-(k_{bf} + k_{wf})}{m_{wf}} & 0 & 0 & 0 & 0 \\ \frac{-Lk_{bf}}{m_b(L_f+L_r)} + \frac{L_f L_r Lk_{bf}}{I_b(L_f+L_r)} & \frac{Lk_{bf}}{m_b(L_f+L_r)} & \frac{L_f L_r Lk_{bf}}{I_b(L_f+L_r)} & \frac{-Lk_{br}}{m_b(L_f+L_r)} & \frac{L_f^2 Lk_{br}}{I_b(L_f+L_r)} & \frac{Lk_{br}}{m_b(L_f+L_r)} + \frac{L_r^2 Lk_{br}}{I_b(L_f+L_r)} \\ 0 & 0 & 0 & \frac{k_{br}}{m_{wr}} & \frac{-(k_{br} + k_{wr})}{m_{wr}} & 0 \end{bmatrix}$$

and

$$\mathbf{A}_{p2} = \begin{bmatrix} \frac{-Lc_{bf}}{m_b(L_f+L_r)} & \frac{L_f^2 Lc_{bf}}{I_b(L_f+L_r)} & \frac{Lc_{bf}}{m_b(L_f+L_r)} + \frac{L_f^2 Lc_{br}}{I_b(L_f+L_r)} & \frac{-Lc_{br}}{m_b(L_f+L_r)} + \frac{L_f L L_r c_{br}}{I_b(L_f+L_r)} & \frac{Lc_{br}}{m_b(L_f+L_r)} & \frac{L_f L L_r c_{br}}{I_b(L_f+L_r)} \\ \frac{c_{bf}}{m_{wf}} & \frac{-c_{bf}}{m_{wf}} & 0 & 0 & 0 & 0 \\ \frac{-Lc_{bf}}{m_b(L_f+L_r)} + \frac{L_f L_r Lc_{bf}}{I_b(L_f+L_r)} & \frac{Lc_{bf}}{m_b(L_f+L_r)} & \frac{L_f L_r Lc_{bf}}{I_b(L_f+L_r)} & \frac{-Lc_{br}}{m_b(L_f+L_r)} & \frac{L_f^2 Lc_{br}}{I_b(L_f+L_r)} & \frac{Lc_{br}}{m_b(L_f+L_r)} + \frac{L_r^2 Lc_{br}}{I_b(L_f+L_r)} \\ 0 & 0 & 0 & \frac{c_{br}}{m_{wr}} & \frac{-c_{br}}{m_{wr}} & 0 \end{bmatrix}$$