Stabilization of Uncertain Systems using Backstepping and Lyapunov Redesign

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Abstract— **This article presents stabilization method for uncertain system using backstepping technique and Lyapunov redesign. The design begins by obtaining stabilizing function for unperturbed system using control Lyapunov function. As such, the stabilizing function guarantees asymptotic stability in the sense of Lyapunov. The control Lyapunov function is re-used in the re-design phase whereby the nonlinear robust function is then augmented with pre-designed stabilizing function for robustness toward uncertainties. Lyapunov redesign is used in designing overall robust stabilizing function which guarantees asymptotic stability toward uncertainties and also toward any perturbed states within asymptotic stability region.**

I. Introduction

Backstepping is one of control technique used for nonlinear systems. Many current works on backstepping reveal that the technique increasingly widespread. Backstepping technique also has been used for adaptive control in [1], for position control of electro-hydraulic actuator in [2], for wheeled mobile robot in [3], for spacecraft attitude control in [4-6], for quadrotor in [4-8] and for ship course control in [9]. Backstepping has also been integrated with artificial intelligence methods to improve robustness such as in fuzzy system [10] and artificial neural network [11].

Bakcstepping technique is a flexible concept as compared with its counterpart such as sliding mode control [12]. Often the discontinuous control law offered by sliding mode technique require fast switching and robust only during sliding phase, backstepping offers smooth and proper control law. The flexibility of this technique give advantages to designer when some special criteria of the control law is emphasized. For instance, backstepping can avoid elimination of some useful nonlinear terms. The control law also can be easily be bounded to respect system admissible set of inputs. Moreover in backstepping, an advanced mathematics can be employed in the design process such that the most feasible and less complex control law is obtained. This may require comparing squares or Young's inequality [13].

In this paper, with consideration of system like $\dot{x} = f(x, u)$ with $x \in \mathcal{R}^n$, we discus on how the recursive method of backstepping technique can be applied with Lyapunov redesign to achieve robust control law for system with $n \geq 2$. Backstepping and Lyapunov redesign has been used for Ball and Beam system and two wheeled mobile inverted pendulum in [14] and for uncertain nonlinear system with mixed matched mismatched uncertainties in [13, 15].

In our case, with the appearance of uncertainties cum external disturbance, we design the feedback stabilizing function for unperturbed nominal system based-on positive definite, smooth and proper function such that the derivative of such function about system under control is negative definite. The guaranteed asymptotic stability of the closed-loop unperturbed system is obtained easily as long as the necessity and sufficiency of Lyapunov theorem is fulfilled. The trivial part in the design phase may occur during Lyapunov redesign when the augmented control law is about to design. The augmented control law may be a saturation-like function [15], nonlinear damping function [13] or others.

II. Problem Formulation

Our design procedure is based on the following class of system [12]:

$$\dot{x} = Ax + B(u + \rho(x, u, t)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $\rho(\cdot): \mathbb{R}^n \text{ x } \mathbb{R}^m \text{ x } \mathbb{R}^+ \rightarrow \mathbb{R}^m$ is the lumped uncertainty which matched to the control input. $A \in \mathbb{R}^{nxn}$ and $B \in \mathbb{R}^{nxm}$ is the system characteristic and input matrix with full rank $m < n$. For design purpose, let consider

$$A \triangleq f(x) \tag{2a}$$

$$B \triangleq g(x) \tag{2b}$$

with vector f and g are smooth, that is $f(0) = 0$ for nonlinear case. System (1) is affine in control with matched uncertainties. Thus the control design begins with stabilization of unperturbed nominal subsystem using Lypaunov, hence back step for higher dimension and redesigned for uncertainties compensation. The design procedure is depicted in Fig. 1.

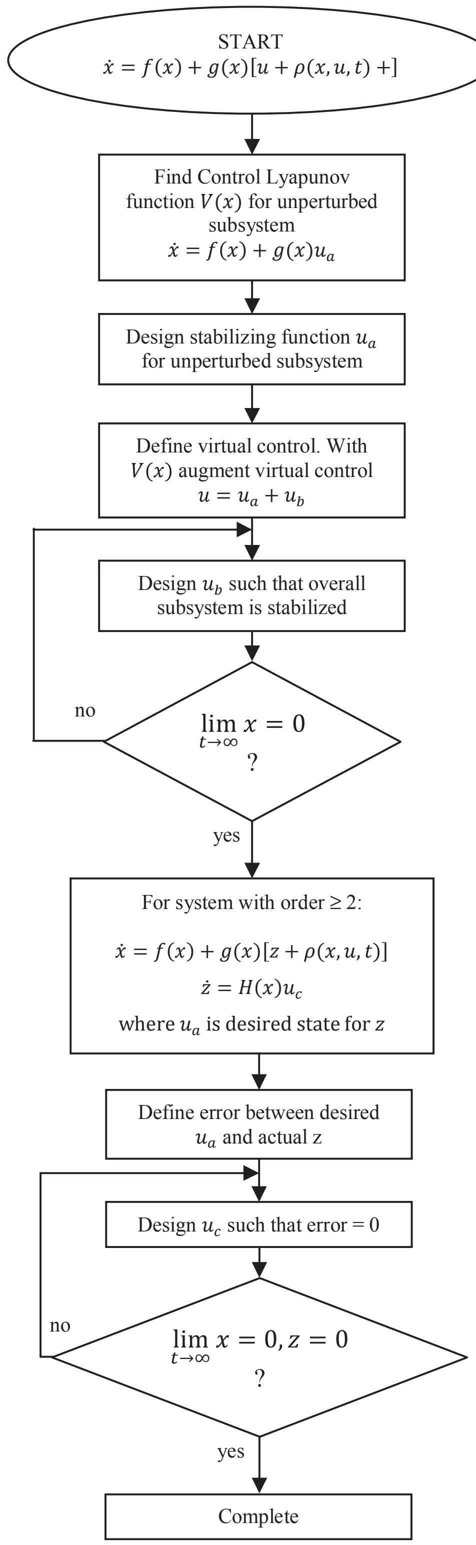

Figure 1: Design methodology

For system with order ≥ 2, the stabilizing function for x-subsystem is the desired state for z-subsystem. As such, our control problem is to diminish the error dynamics between the actual state and the desired state. Asymptotic stability can be achieved if the control law manage to perish the error such that the state trajectories approaching equilibrium or at least sail to the asymptotic stability region and stay thereafter.

III. BACKSTEPPING AND LYAPUNOV RE-DESIGN CONTROL SCHEME

Consider numerical system [12] in strict feedback form be:

$$\dot{x}_1 = x_2 \tag{3a}$$

$$\dot{x}_2 = x_1 + \sin(120\pi t) + 0.2x_2\cos(x_1) + 2u \tag{3b}$$

With Lyapunov function $V_1(x_1) = \frac{{x_1}^2}{2}$, It is easy to see that for any $K_1 \in \mathbb{R}^+ > 0$, feedback control $x_2 \triangleq x_{2(des)} \triangleq -K_1x_1$ asymptotically stabilize x_1-subsystem as:

$$V_1(x_1) = \frac{{x_1}^2}{2} \quad \Longrightarrow \dot{V}_1(x_1) = -K_1{x_1}^2 \leq 0 \tag{4}$$

An error dynamics z between the desired state $x_{2(des)}$ and actual state x_2 is defined as:

$$z = x_2 - x_{2(des)} \tag{5}$$

With lumped cosine terms in system parameters, we consider $\Delta(t) \triangleq \sin(120\pi t)$ as exogenous time varying disturbance, therefore the actual system (3a) and (3b) can be represented in new coordinate:

$$\dot{x}_1 = -K_1x_1 \tag{6a}$$

$$\begin{aligned}\dot{z} = \ & x_1 + 0.2z\cos(x_1) - 0.2x_{2(des)}\cos(x_1) \ldots \\ & +K_1 + 2u + \Delta(t)\end{aligned} \tag{6b}$$

To stabilize z-subsystem, we consider Lyapunov function:

$$V_2(x_1, z) = \frac{{x_1}^2 + z^2}{2} \tag{7}$$

Hence for $K_2 \in \mathbb{R}^+ > 0$, equation (7) gives choice for stabilizing function for unperturbed z-subsystem. We arrive the stabilizing function:

$$\begin{aligned}u_{des}(x_1, z) = & -\frac{1}{2}(K_1 + K_2z) - x_1 - 0.1z\cos(x_1) \ldots \\ & -0.1x_{2(des)}\cos(x_1)\end{aligned} \tag{8}$$

which yields

$$\dot{V}_2(x_1, z) = -K_1{x_1}^2 - K_2z^2 \leq 0 \tag{9}$$

z-subsystem suffered from uncertain $\Delta(t)$ which behaves as an exogenous time varying disturbance. As such, we redesign the stabilizing function $u_{des}(x_1, z)$ to reach robust stabilization with final control law u. For $\varepsilon > 0\ a > 0$ We then augment the control law:

$$u = -\frac{1}{2}(K_1 + K_2 z) - x_1 - 0.1z\cos(x_1)\ \ldots$$

$$-0.1x_{2(des)}\cos(x_1)\ \ldots \tag{10}$$

$$-\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]\frac{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]}{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]+\beta e^{-at}}$$

With that, the asymptotic stability is proven by derivative of re-used $V_2(x_1, z)$ about z-subsystem:

$$\dot{V}_2(x_1, x_2)$$

$$= -K_1{x_1}^2 - K_2 z^2 \ldots$$

$$+2z\left[-\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]\frac{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]}{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]+\beta e^{-at}}\ldots + \frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]$$

$$= -K_1{x_1}^2 - K_2 z^2 \ldots$$

$$+2z\left[-\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\frac{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]}{2z\left[\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]+\varepsilon e^{-at}}\ldots + \frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right]$$

$$= -K_1{x_1}^2 - K_2 z^2 + \frac{\left\|2z\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right\|\beta e^{-at}}{\left\|2z\frac{\sin(120\pi t)-\mathrm{x}_1}{2}\right\|+\beta e^{-at}}$$

$$< -K_1{x_1}^2 - K_2 z^2 + \beta e^{-at} \leq 0 \tag{11}$$

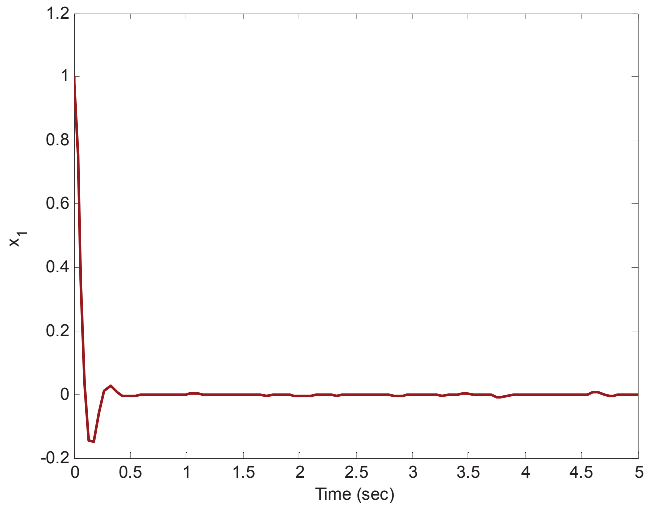

Figure 2: System trajectory

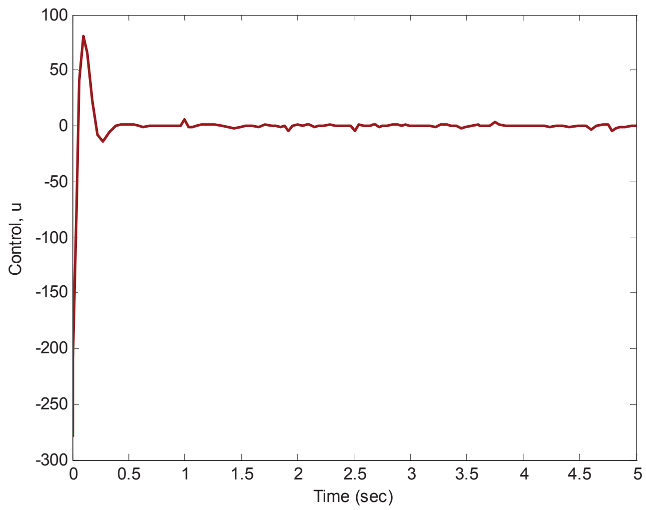

Figure 3: Control signal for backstepping and Lyapunov redesign

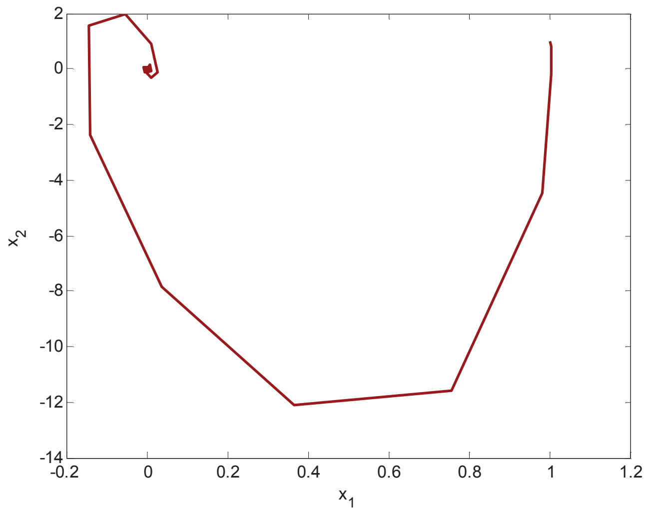

Figure 3: Overall system trajectory

IV. Conclusion

The uncertain system introduced in [12] has been stabilized using backstepping with Lyapunov redesign. Upon perturbation in initial states, the control law guarantees asymptotic stabilization within 1 seconds. The magnitude of control law has been limited via the augmentation of saturation-like function with the pre-designed stabilizing function for unperturbed subsystem. However, the control law require two control parameters which are need to be tuned systematically. The sufficiency of systematic trying-and-error tuning paradigm is proven by trajectory pattern for x_1-subsystem in Fig. 2 as well as overall system trajectories in Fig. 3.

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